A New Variable Tap-length LMS Algorithm and Application in Fixed Satellite Communications

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Abstract—A new variable tap-length LMS algorithm is proposed in this paper. This algorithm can effectively obtain both fast convergence rate and small steady state error by non-linear weighting L errors of the current moment and past moments. The algorithm analysis is also given which the ideal tap-length is obtained by increasing the error information and eliminating the effect of correlated noise sequences. Then the new algorithm is applied to the fixed satellite communications system in which the channel is affected by weather conditions attenuation and multipath effect. Simulation results are presented to support the analysis. The new algorithm converges faster than usual algorithms under same steady state performance. The signal BER performance is greatly improved with the new adaptive filter algorithm equalizer when the signal passes simulation model of fixed satellite communications system.

Keywords-variable tap-length LMS algorithm; fixed satellite; adaptive equalization;

I. INTRODUCTION

In the high speed broadband fixed satellite digital communications systems, transmitting signal reaches the terminal through more than one path, which is known as multipath propagation. Each path will have different amplitude attenuation and phase delay, so that the signal is expanded in the time domain at the receiver, which results in inter-symbol interference (ISI). ISI and channel fading affect the efficiency and quality of communication. Therefore, it is necessary to use equalization algorithm to mitigate inter-symbol interference and signal distortion caused by the fixed satellite channel.

One of the most popular filter algorithm in adaptive signal processing is the least mean square (LMS) algorithm[1,2]. It has been extensively applied in many areas due to its simplicity, robustness and implementing easily. Within the LMS algorithm the tap-length is an important parameter that influences the algorithm’s convergence performance. For fixed tap-length LMS algorithm, the choice of the tap-length reflects a tradeoff between steady state error and the speed convergence. A small tap-length gives small steady state error but also a longer convergence time constant. In order to solve the contradictions between convergence speed and steady state error of the fixed tap-length LMS algorithm, reference [3] proposed a variable tap-length LMS algorithm. However, the [4] analysis points out that this algorithm in [3] is vulnerable to the effect of independent noise, and proposes the MVSS LMS algorithm which the adaptive errors is not correlative when it is close to the optimal parameters and the correlation is also small in the process of convergence, leading to that tap-length reduces too fast and converges too slow.

II. FIXED SATELLITE COMMUNICATIONS CHANNEL

The atmosphere (including rain, clouds, oxygen and scintillation) will cause changes of the signal amplitude, phase, polarization and downlink beam incidence angle, which lead to the decline of the signal transmission quality and the increase of bit error rate. Among all weather conditions, rain causes the most serious propagation loss in fixed satellite communications at Ka-band[6]. Based on the satellite channel propagation characteristics measurements statistical model[5], multipath effect fading can be expressed as a complex Gaussian random process, in which the envelope is Rayleigh distribution and phase is uniformly distributed between 0 and $\pi$. The satellite communications channel total attenuation is blessed with non-frequency selective, which is slow enough and is only affected by the weather conditions[7].

<table>
<thead>
<tr>
<th>Weather Conditions</th>
<th>Envelope</th>
<th>Phase</th>
</tr>
</thead>
<tbody>
<tr>
<td>clear sky</td>
<td>0.413</td>
<td>0.00087</td>
</tr>
<tr>
<td>cumulus cloudy</td>
<td>0.346</td>
<td>0.00272</td>
</tr>
<tr>
<td>thunder shower</td>
<td>0.436</td>
<td>0.01386</td>
</tr>
<tr>
<td>light snow</td>
<td>0.488</td>
<td>0.00034</td>
</tr>
<tr>
<td>blowing snow</td>
<td>0.500</td>
<td>0.00021</td>
</tr>
<tr>
<td>rain</td>
<td>0.662</td>
<td>0.0200</td>
</tr>
</tbody>
</table>

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When the signal passes through the channel, the equivalent low-pass received signal (not including the Gaussian white noise) is simplified as follows:
\[ x(t) = \kappa \exp(j\phi)u_i(t) \quad 0 \leq t \leq T , \]  
where \( T \) is modulation symbol width, \( u_i(t) \) signal source outputs in time domain expression, \( \kappa, \phi \) are signal envelope and phase real Gaussian random variables respectively for the equivalent low-pass communications channel. For fixed satellite channel at Ka-band under various weather conditions, the signal envelope and phase Gaussian model parameters are shown in Table 1 [8].

### III. A NEW VARIABLE TAP-LENGTH LMS ALGORITHM

The adaptive filtering or system equalization problem being considered is to try to adjust a set of filter weights so that the system output tracks a desired signal. Least mean square algorithm is built upon minimum mean square error between system output tracks a desired signal. Least mean square considered is to try to adjust a set of filter weights so that the mean square value of estimate error is the smallest, least mean square error is expressed as
\[ E\{e^2(n)\}_{\text{min}} = E\{d^2(n)\} - r^TW_{\text{opt}}(n). \]

The usual weight update recursion is of the form
\[ W(n+1) = W(n) + \mu(n)e(n)X(n), \]
where \( \mu(n) \) is the tap-length factor. If \( \mu(n) \) is a constant, the algorithm becomes to fixed tap-length LMS(FSS_LMS) filtering algorithm. Otherwise, the algorithm is variable tap-length LMS(VSS_LMS) algorithm. In order to ensuring algorithm convergence in the process of iteration, The step size must satisfy \( 0 < \mu < 1/\lambda_{\text{max}} \), where \( \lambda_{\text{max}} \) is input signal \( X(n) \) maximum eigenvalue of autocorrelation matrix.

Within the limits of convergence, for fixed tap-length LMS algorithm, the larger tap-length is, the faster equalization algorithm convergence speed and tracking capability is. However, if the initial tap-length is set too large, it will lead to large oscillation and steady state performance deterioration during the convergence process, and the mean square error will also become large. Reducing tap-length can improve steady state accuracy for the algorithm, but the convergence performance and tracking performance will be deteriorated. In order to avoiding the conflict between convergence rate and steady state error, the variable tap-length update equation is given by [3]
\[ \mu(n+1)' = \alpha \mu(n) + \gamma e^2(n) \]  
with \( 0 < \alpha < 1 \), \( \gamma > 0 \) and
\[ \mu(n+1) = \begin{cases} \mu_{\text{max}} & \text{if } \mu(n+1)' > \mu_{\text{max}} \\ \mu_{\text{min}} & \text{if } \mu(n+1)' < \mu_{\text{min}} \\ \mu(n+1)' & \text{otherwise} \end{cases} \]
where \( \mu \) is restricted ranging from \( \mu_{\text{min}} \) to \( \mu_{\text{max}} \) to guarantee stability of the algorithm, and \( 0 < \mu_{\text{min}} < \mu_{\text{max}} \). The initial tap-length is usually taken to be \( \mu_{\text{max}} \), but results show that the algorithm is not sensitive to the choice. The tap-length is controlled by the size of prediction error and the parameters \( \alpha \), \( \gamma \) in [3]. It can be seen that error is large at the early stage of algorithm, so a larger tap-length is taken advantage. Thereafter output signal gradually approaches to the ideal value, and the error decreases, then the tap-length is also reduced ensuring a small detuning. The constant \( \mu_{\text{max}} \) is chosen to ensure that the mean square error of the algorithm remains bounded, and \( \mu_{\text{min}} \) is chosen to provide a minimum level of tracking ability.

This algorithm is nearly perfect. Unfortunately, the usage of the instantaneous squared error as a measure of the optimum results is significantly degradation in the presence of noise.
The desired output signal becomes to
\[ d(n) = W_{opt}^T(n)X(n) + N(n), \] (9)
where \( N(n) \) independent Gaussian white noise that is not related to input signal \( X(n) \). Taking advantage of \( \Delta(n) = Q^T \Delta(n) \) and \( X'(n) = Q^T X(n) \), where \( \Delta(n) = W(n) - W_{opt}(n) \) is the transformed weight vector. Putting (5) and (9) into (8) yields
\[ \mu(n+1) = \alpha \mu(n) + \gamma [d(n) - W^T(n)X(n)]^2 \\
= \alpha \mu(n) + \gamma \Delta^T(n)X(n)X^T(n)\Delta(n) \\
+ \gamma N^2(n)\Delta^T(n)X(n). \] (10)

Then we have
\[ E\{\mu(n+1)\} = \alpha E\{\mu(n)\} + \gamma (E\{N^2(n)\}) \]
\[ + E\{\Delta^T(n)\Delta'(n)\} , \] (11)
where it is assumed that the \( \Delta(n) \) and \( X'(n) \) is mutual independence. Clearly, \( E\{\Delta^T(n)\Delta'(n)\} \) influences the closeness of adaptive system to the optimal solution. However, due to the presence of \( E\{N^2(n)\} \), the tap-length update deteriorates at all stages of adaptation but particularly near the optimum. The tap-length is no longer an accurate reflection of the adaptive state. The performance of algorithm is greatly reduced with a poor signal, so that weight vector is hard to close to optimal value. To avoid this sensitivity to noise, a new algorithm is proposed. The idea is based on the fact that the error is poor and correlation among successive samples is small near the optimum. Thus, we use an estimate of the autocorrelation between present moment error and past \( L \) errors rather than time-averaged of \( e(n)e(n-1) \) in [4] to control tap-length updating. The new algorithm tap-length iterative formula is defined as the following equation:
\[ p(n) = \beta p(n-1) + (1-\beta)e(n)e(n-1) \]
\[ + \sum_{i=1}^{L} e^{-i+1} e^3(n-i+1) \] (12)
and
\[ \mu(n+1) = \alpha \mu(n) + \gamma p^2(n) \]. (13)

Define
\[ \lambda_n = \sum_{i=1}^{L} e^{-i+1} e^3(n-i+1), \] (14)
where \( \alpha (0 < \alpha < 1) \), \( \beta (0 < \beta < 1) \) and \( \gamma (\gamma > 0) \) are the same as those of the MVSS_LMS algorithm. The autocorrelation of the present error and the past \( L \) errors not only rejecting the independent noise sequence effect, but also is an efficient measure of the closeness to the optimum. \( \lambda_n \) is patching factor. Nonstationary environment and large error will cause the tap-length to increase to provide faster tracking, while stationary environment and small error will result in a decrease in the tap-length to yield smaller misadjustment. Through non-linear weighting \( L \) error values of the present moment \( e(n) \) and past moments \( e(n-1) \ldots, e(n-L+1) \), \( p(n) \) is close to 0 and \( \mu \) is close to 0 and achieving smaller misadjustment values at the time of steady state. The error coefficient is related to length between the corresponding time and the current time. The longer the length is, the smaller the error coefficient is. Therefore the independence noise has a little influence on new algorithm. The new algorithm has the attractive property of achieving a small final steady state error while providing fast convergence at early stages of adaptation, and it allows the adaptive filter to track changes in the system.

IV. SIMULATION RESULTS

In this section, we describe simulations performed to verify the theory developed in the previous sections, and to compare experimentally the performance of the new variable step size LMS algorithm to that of the MVSS_LMS algorithm proposed in [4] and the fixed step size (FSS) algorithm. To validate our analysis, we consider an application of the new LMS adaptive filter to fixed satellite communications system. First of all, the convergence performance of the new variable step size (NVSS) LMS algorithm and other algorithms is simulated and analyzed, and then the bit error rate (BER) performance of the satellite communications system based on the new LMS equalization algorithm is also researched.

![Figure 1. Convergence characteristic of three kinds of equalization algorithm.](image)

In the simulations presented here, the unknown system to be modeled is a FIR filter which the dimension is 8, and defined as \( W^* = [0.8783 \, -0.5806 \, 0.6537 \, -0.3223 \, 0.6577 \, -0.0582 \, 0.2895 \, -0.2710] \). Both system and the adaptive filter are excited by a correlated signal \( x(n) \) generated by [4]
\[ x(n) = 0.9x(n-1) + a(n), \] (15)
where $a(n)$ is a zero-mean, uncorrelated Gaussian noise of unity variance. Parameters of these algorithms are selected to produce a comparable level of steady state error. Results are obtained by averaging 150 independent runs.

In Fig. 1. Curve 1 describes the convergence rate of the fixed tap-length LMS algorithm when the tap-length is $\mu = 1 \times 10^{-2}$, it is not until iterative 300 times that the curve becomes convergent. It is noticed that the misadjustment level of the fixed tap-length algorithm with a small tap-length is achieved, but the convergence rate of this algorithm with a large iterative times is also achieved. Curve 2 describes the convergence speed of the MVSS_LMS algorithm, in which the parameters are $\alpha = 0.95$, $\beta = 0.95$, $\gamma = 1 \times 10^{-3}$, $\mu_{\text{max}} = 0.2$ and $\mu_{\text{min}} = 1 \times 10^{-3}$. As can be seen, the algorithm converges fast in the initial stage, but ultimately convergent result is not ideal. It is not until 400 iterations that the algorithm is convergent. Curve 3 describes the convergence speed of the newly proposed algorithm simulation result, where the parameters are $\alpha = 0.95$, $\beta = 0.95$, $\gamma = 1 \times 10^{-3}$, $\mu_{\text{max}} = 0.2$ and $\mu_{\text{min}} = 1 \times 10^{-3}$. It converges after 50 iterations, and the steady-state performance is perfect.

Under ideal conditions, the fixed satellite communications channel declines slowly enough, and without the satellite data processing. Transmitter and repeater amplifiers don’t have linear distortion \cite{6}. In different weather conditions, we just simply need to change the two Gaussian process parameters and the real constant generator parameters. A complex Gaussian process simulates multi-path effects.

In order to analyze the effects of new LMS filtering algorithm on BER performance of the fixed satellite communications system, Fig. 2 shows the behavior of the channel BER performance with the NVSS_LMS filtering algorithm equalizer compared to AWGN channel and weather attenuation channel of fixed satellite. Sending signals is the Bernoulli random binary sequence. The number of data is the $10^{5}$. The modulation method of signal is BPSK. Take thunder shower for example, setting the corresponding parameters according to the conclusion given by \cite{6}. Equalizer weighted tap coefficients $M$ are 7. The value for $\alpha = 0.95$ appears to be a good choice for all the experiments, while the value for $\beta = 0.95$ and $\gamma = 1 \times 10^{-3}$ which control algorithm convergence time is chosen arbitrarily. $L = 3$ is the past error numbers. Maximum and minimum tap-length respectively are set $\mu_{\text{max}} = 1 \times 10^{-2}$, $\mu_{\text{min}} = 1 \times 10^{-5}$. For comparison easily, the ideal AWGN channel BER performance curve is given. From Fig. 2 we can see that the coherent BPSK signal sent by modulator passes through the fixed satellite channel, inciting severe distortion and inter symbol interference. When the BER is $P_e = 1 \times 10^{-4}$, signal corresponding loss of signal to noise ratio(SNR) is up to 10dB if the NVSS_LMS algorithm equalizer is not applied in the communications system, otherwise the loss of SNR reduce to 2.8dB.

V. CONCLUSION

The adaptive new variable tap-length LMS filter algorithm has not only less computational complexity, but also has better convergence and steady state error performance than other algorithms. The analysis and simulation results indicate that the algorithm converges faster than the fixed tap-length LMS algorithms. The loss of signal to noise ratio is small under certain bit error rate conditions, and it has achieved the attractive result in the fixed satellite communications system.

REFERENCES