SMSE-Based DSA Radar Waveforms

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Abstract— Previous communications systems research has demonstrated that the Spectrally Modulated, Spectrally Encoded (SMSE) framework is well suited to SDR platforms and operation in both contiguous and non-contiguous spectrum given its ability to generate a wide variety of multicarrier waveforms such as OFDM, NC-OFDM, MC-CDMA, NC-MC-CDMA, CI/MC-CDMA, NCCI/MC-CDMA, and TDCS. In this paper, SMSE waveforms are extended for use in sparse frequency radar systems. Sparse frequency radar can be capable of dynamically sensing available spectrum, and tailoring the transmitted waveforms to suit the instantaneous RF environment conditions, simultaneously transmitting energy in multiple noncontiguous spectrum bands. Conventional radar performance metrics are reviewed, and a new metric is introduced which specifically addresses performance of non-contiguous spectrum waveforms through comparison to contiguous spectrum waveforms. Through simulation, various examples are shown to illustrate and characterize the impairments that are introduced in processing these classes of non-contiguous spectrum waveform returns, specifically the generation of large range sidelobes.

I. INTRODUCTION

A. Cognitive Radio

Cognitive Radio (CR) is an intelligent radio which is capable of setting and configuring its own parameters including waveform, protocol, transmitting and receiving carrier frequency and networking, autonomously. Specifically, a cognitive radio detects spectrum holes (unused frequency bands) and transmits its own transmission over those spectrum holes without interfering with the primary users. As spectrum conditions change, the CR adapts its frequency use over time through a process called dynamic spectrum access (DSA). The innovation which makes engineers see Cognitive Radios as a possible technology is the Software Defined Radio (SDR) where the software meets the antenna. SDR is one of the latest and most evolving technologies in the communications industries in civilian, military as well as commercial sectors. An entirely hardware based radio gives no flexibility because of the fixed characteristics of the modules performing the radio functions. However, in a radio system built with SDR, the flexibility is very high. Many different waveforms have been proposed for overlay cognitive radio such as non-contiguous OFDM (NC-OFDM), non-contiguous MC-CDMA (NC-MC-CDMA) and transform domain communication system (TDCS). In our previous work, we have proposed a cognitive centric overlay/underlay waveform design through spectrally modulated spectrally encoded (SMSE) framework to implement such multi-function waveform which can be adapted to support any kind of multi-carrier based cognitive radio waveforms [3][4].

B. Prior Relevant Research

Related research efforts have addressed the design of sparse frequency radar waveforms and return processing. These are generally applied to foliage penetration (FOPEN), are often ultra-wide bandwidth (UWB), and operate in the VHF and UHF bands [7][9]. These bands are chosen due to their relative immunity to absorption by foliage, however, challenges exist due to the presence of many other RF systems operating in these bands. Nearby spectral occupants may include high power commercial communications systems, television broadcasts, military radios, navigation services, etc. Other sparse frequency radar systems are geared towards over the horizon (OTH) radar applications in the HF band, where the nature of the propagation conditions creates gaps in reception as a function of frequency, in addition to spectrum congestion from other users [3][13][14][15]. Sparse frequency waveforms offer a partial solution to operating in congested or dynamically occupied bands, but challenges remain nevertheless. In addition, other research has addressed a class of cognitive radars, some of which employ Knowledge Based techniques to adaptively construct waveforms, spatial response patterns, signal processing methods, and operating parameters, but not towards DSA [17].

C. Noncontiguous Radar Challenges

One of the challenges of operating radar in discontinuous, or spectrally fragmented bands, is the elevated range sidelobes that result as a function of the spectrum fragmentation [11]. In addition, practical difficulties exist due to non-constant power output at the power amplifier, leading to undesirable high peak to average power ratios (PAPR). Prior work in this area includes examination of adaptive techniques to build transmit waveforms based on an iterative algorithm with multiple penalty functions. These penalty functions serve to minimize transmitted energy in particular bands, while maintaining near-constant power output [7]. A second receive-side algorithm uses penalty functions to minimize range sidelobes. Other papers examine complex genetic algorithms and Particle
Swarm Optimization (PSO) methods to generate optimal waveforms and transmit/receive filters that minimize range sidelobes [11].

The remainder of this paper is organized as follows. The use of the SMSE framework is proposed to generate non-contiguous spectrum waveforms for radar use rather than the traditional communications use. Specifically, the SMSE framework is employed to generate multi-function multi-carrier transmission waveforms over non-contiguous frequency bands for the radar implementation. Combined with a spectrum sensing engine, the cognitive radar detects the availability of each and every subcarrier in the operational bandwidth. By turning off those subcarriers occupied by the primary users, the cognitive radar implements a non-contiguous SMSE transmission. The mathematical foundations of the SMSE framework are examined to show how it is inherently suitable for use with software defined transceiver platforms. In addition, metrics are nominated by which spectrum fragmentation and the resultant sidelobe effects are measured in order to more uniformly quantify the performance of the various techniques. Lastly, the elevated range sidelobe issues that are a result of spectrum fragmentation are examined.

II. SMSE FRAMEWORK AND ITS APPLICATION TO DSA

Previous work provides a general analytic framework for SMSE signals that accommodates multi-carrier, CR-based waveforms [2][8][18]. Specifically, an arbitrary CR waveform can be expressed in terms of its amplitude (A), phase (Θ) and frequency (F) characteristics. These three factors aid in SMSE wave design through six design variables, namely data modulation (d), Code (c), Window (w), orthogonality (o) and two frequency allocation variables. Considering Nf total frequency components, the coding \( c = \{c_1, c_2, ..., c_{N_f}\} \), \( c_i \in \mathbb{C} \), where \( \mathbb{C} \) denotes complex values, data modulation, \( d = \{d_1, d_2, ..., d_{N_f}\} \), \( d_i \in \mathbb{C} \), and windowing, \( w = \{w_1, w_2, ..., w_{N_f}\} \), \( w_i \in \mathbb{C} \) vectors account for component-by-component amplitude and/or phase variations. A phase only variable \( \phi = \{\phi_1, \phi_2, ..., \phi_{N_f}\}, \phi_i \in \mathbb{C} \) is used for orthogonality between symbol streams and facilitate multiple access.

The analytic SMSE framework development begins by considering data, code and window variables. The \( m^{th} \) frequency component of the \( k^{th} \) symbol is given by

\[
S_{k}[m] = c_m d_{m,k} w_m e^{j(\theta_{d_{m,k}} + \theta_{c_{m}} + \theta_{w_{m}})} \tag{1}
\]

Where \( m = 0, 1, ..., N_f - 1 \) is the frequency index and \( c_m, d_m \) are magnitude and phase design variables.

The expression in (1) is next modified to incorporate frequency and orthogonality variables. Frequency component selection is a function of two factors, including an available variable \( a = \{a_1, a_2, ..., a_N\}, a_i \in \{0,1\} \) and a use variable

\[
u = \left[u_1, u_2, ..., u_{N_f}\right], u_i \in \{0,1\} \].

Given an \( N_f \)-point fast Fourier transform (FFT) process, \( N_f \) frequency components or spectral bands are available for waveform design. It is important to note that the frequency assignment variable takes on binary values 0 or 1 indicating the spectrum availability for secondary users. As a direct result, this pool of frequencies is reduced by component selection to create a number of CR available frequencies and usable frequencies. The \( m^{th} \) component of the \( k^{th} \) CR symbol corresponds to

\[
S_{k}[m] = a_m u_m c_m d_{m,k} w_m e^{j(\theta_{d_{m,k}} + \theta_{c_{m}} + \theta_{w_{m}} + \theta_{e_{m,k}})} \tag{2}
\]

Where the product \( a_i u_i \in \{0,1\} \). The discrete time domain SMSE waveform is obtained by taking the Inverse Discrete Fourier Transform (IDFT) of (2) according to

\[
s_{k}[n] = \frac{1}{N_f} \text{Re} \left\{ \sum_{m=0}^{N_f-1} a_m u_m c_m d_{m,k} w_m e^{j\left(2\pi f_m t_n + \theta_{d_{m,k}} + \theta_{c_{m}} + \theta_{w_{m}} + \theta_{e_{m,k}} \right)} \right\} \tag{3}
\]

Where \( t_k \leq t + T, f_m \leq f + c + \Delta f \), \( T \) is the symbol duration and \( \Delta f = 1/T \) is the frequency resolution. The SMSE framework provides a unified expression for generating and implementing a host of multi-carrier type waveforms (e.g., OFDM, MC-CDMA, CI/OFDM, TDCS, etc) and satisfies current CR goals of exploiting unused spectral bands. Table 1 shows how selection of the SMSE variables can be used to construct a variety of waveforms. The SMSE framework can be extended to exploit both unused and underused spectrum to generate both overlay-CR and underlay-CR type waveforms [16].

<table>
<thead>
<tr>
<th>Operation</th>
<th>Basic OFDM</th>
<th>MC-CDMA</th>
<th>CI/OFDM-CDMA</th>
<th>TDCS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data Modulation</td>
<td>MPSK</td>
<td>MPSK</td>
<td>MPSK</td>
<td>MPSK</td>
</tr>
<tr>
<td>Coding</td>
<td>( \theta_{o} = 0 )</td>
<td>( \theta_{o} = 0 )</td>
<td>( \theta_{o} = 0 )</td>
<td>( \theta_{o} = 0 )</td>
</tr>
<tr>
<td>Window</td>
<td>( \theta_{w} = 0 )</td>
<td>( \theta_{w} = 0 )</td>
<td>( \theta_{w} = 0 )</td>
<td>( \theta_{w} = 0 )</td>
</tr>
<tr>
<td>Orthogonality</td>
<td>( \theta_{o} = 0 )</td>
<td>( \theta_{o} = 0 )</td>
<td>( \theta_{o} = 0 )</td>
<td>( \theta_{o} = 0 )</td>
</tr>
<tr>
<td>Frequency Assignment</td>
<td>( a )</td>
<td>( u )</td>
<td>( u )</td>
<td>( u )</td>
</tr>
<tr>
<td>Frequencies Used</td>
<td>( a )</td>
<td>( u )</td>
<td>( u )</td>
<td>( u )</td>
</tr>
</tbody>
</table>

A. Extension of SMSE to DSA Radar

DSA radar has the promise of providing a means to maintain or improve radar performance in the face of rising spectrum demand and congestion. Applications such as foliage penetration, over the horizon sensing, stealth techniques defeat, ground penetration and early warning ballistic missile search generally rely on radar operation in RF bands that are characterized by high primary user congestion. DSA techniques provide a potential means to operate in these bands by dynamically selecting available and non-contiguous spectrum.
The SMSE framework provides an attractive approach for generating non-contiguous spectrum waveforms for radar use. It supports the efficient generation of multi-function, multi-carrier transmission waveforms over non-contiguous frequency bands. Combined with a spectrum sensing engine, DSA radar detects the availability of unused spectrum in the operational bandwidth. By “turning off” those subcarriers which reside on frequencies occupied by the primary users, the cognitive radar implements a non-contiguous SMSE-based transmission.

B. Metrics for Measuring Non-continuous DSA Radar Performance

A fundamental question related to the use of non-continuous DSA radar is its relative performance. In this section a number of quantitative metrics are proposed that can be used to compare non-contiguous spectrum radar to a continuous spectrum baseline in assessing performance. But before radar performance can be compared, it is important to create a quantitative definition for discontinuous spectrum, which will be called spectrum fragmentation. A sum of squares metric has been conceived to reflect the amount of non-contiguous spectrum that is present within the total spectrum which penalizes for the size of spectrum gaps relative to the total bandwidth. It is this size of these spectrum gaps which cause challenges with radar signal processing, and will be discussed in more detail later in the paper.

Figure 1 below shows some examples where each vector represents the total spectrum segmented into discrete bins of equal size. The blank squares indicate open spectrum bins and “X’s” indicate unavailable spectrum bins.

![Figure 1 Fragmentation Example](image)

**Figure 1 Fragmentation Example**

For all applications and analyses involving fragmentation quantification the definition of percentage spectrum fragmentation (% SF) as defined below will be used:

\[
\% SF = 100 \times \left( \frac{\sum_{k=1}^{N} p_k^2}{M^2} \right) \tag{4}
\]

Where M corresponds to the total number of spectral bins, N is the number of unavailable spectrum gap segments, and \( p_k \) is the number of unavailable contiguous spectrum bins in the \( k \)th segment. In example A above, there are two spectrum gaps (\( N = 2 \)), one spanning one bin at index 4, and one spanning two bins at indexes 9 and 10. There are 15 total spectral bins (\( M=15 \)). Therefore the % SF for each example is as follows:

\[
\text{Ex. A} = 100 \times \left( \frac{1^2 + 2^2}{15^2} \right) = 2.2\% \\
\text{Ex. B} = 100 \times \left( \frac{3^2 + 2^2 + 6^2}{15^2} \right) = 21.8\% \\
\text{Ex. C} = 100 \times \left( \frac{1^3}{15^2} \right) = 75.1\% .
\]

Returning to the quantification of radar performance, the use of the primary radar performance measurements are proposed as metrics (target presence, range, velocity, angles of arrival, etc.), quantified in traditional electrical quantities (amplitude, time delay, Doppler shift, phase, etc.) when compared to the continuous spectrum equivalent. The following terms as applied to complex ambiguity function (CAF) analysis are proposed to adequately compare continuous spectrum radar to non-continuous radar:

- **PSL** (Peak Sidelobe Ratio): the radar return peak to highest sidelobe amplitude ratio. Higher sidelobes can lead to reduced performance in terms of masking multiple radar returns, and can lead to large errors if the wrong peak is chosen as a legitimate return. Larger values indicate better performance.

- **FWHM** (Full Width at Half Maximum): this is a metric that quantifies the 3dB main lobe width. Smaller values lead to higher accuracy in the dimension in which it is measured (range or Doppler).

- **ISL** (Integrated Sidelobe Level): the ratio of the range response energy past the first minimum to that within the mainlobe. (In other words, the ratio of all the off-peak correlation energy to the energy in the main lobe correlation peak.) It is desirable for this value to be a large negative number.

C. Observations on the Effects of Spectrum Fragmentation on CAF-related Parameters

The effects of non-contiguous spectrum operation can be illustrated through simulation. In the following analysis, a coded waveform is generated in both continuous and various non-continuous spectrum occupancy configurations, followed by CAF-based return analysis. In these examples, the propagation path is assumed to be multipath and clutter free, with very high return S/N in order to focus on the effects of fragmented spectrum operation.

Due to the nature of spectrally noncontiguous signals (which will be explored in detail later in the paper), in this analysis the PSL will be computed in both the range (PSL\(_{\text{range}}\)) and Doppler (PSL\(_{\text{doppler}}\)) dimensions. Figure 2 shows an example of the Cross Ambiguity Function (CAF) for a spectrally contiguous radar waveform return. Comparing this to the CAF computed with a spectrally fragmented waveform in Figure 3, there is a large change in range sidelobe levels, but the Doppler sidelobe levels are unaffected.
Similarly, if one were to fragment the waveform in time, there would be increased Doppler sidelobes. In the case of the FWHM metric, the shape of this ambiguity function peak is not necessarily symmetric in range and Doppler, as each dimension is influenced by different phenomena (aside from the spectrum fragmentation effects). For example, long pulse duration will result in higher Doppler resolution, and therefore narrow width in the Doppler domain, but the range domain will be unaffected. Similarly, changing the bandwidth of each radar pulse changes the width of the pulse only in the range domain, where a wider bandwidth creates a narrower time domain mainlobe width. These phenomena of time and frequency relationships have been exhaustively evaluated in prior literature [10]. In this paper, two variations of FWHM defined as $\text{FWHM}_{\text{range}}$ and $\text{FWHM}_{\text{doppler}}$ are proposed for each of the dimensions in the CAF.

III. SMSE WAVEFORM RADAR ANALYSIS

Referring to the previous SMSE Framework Section, simulate SMSE waveforms are developed for radar use. Radar-specific spectral encoding can be simulated through parameters $c_m$ and $\theta_m$ which can be taken from any set of defined encoding schemes including binary phase coding, PRN coding, OFDM coding, etc. Ultimately, the choice of the coding will be driven by a variety of operational requirements such as range and Doppler resolution, available bandwidth, PRI, available spectrum mask, carrier frequency, radar range, maximum radar and Doppler expected, and maximum unambiguous range and Doppler. The simulations in this paper utilize SMSE-based coding borrowed from a 3GPP specification [1]. This specification describes the generation of Gold scrambling codes, which have properties that are desirable for the radar waveform sequences: good autocorrelation and cross-correlation properties. The pulse compression is determined by the selected chipping rate of the sequence.

The complete set of parameters chosen for use in these simulations are described below in Table 2. Note that these properties are somewhat arbitrarily chosen and could be easily modified to meet a particular radar application area in terms of individual operational requirements.

<table>
<thead>
<tr>
<th>Property</th>
<th>Attribute</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coding Vector (c)</td>
<td>25$^{th}$ Order Gold Code</td>
</tr>
<tr>
<td>Orthogonality Vector (o)</td>
<td>Rectangular</td>
</tr>
<tr>
<td>Window Vector (w)</td>
<td>Rectangular</td>
</tr>
<tr>
<td>FFT size</td>
<td>256</td>
</tr>
<tr>
<td>Modulation</td>
<td>QPSK</td>
</tr>
<tr>
<td>Oversampling ratio</td>
<td>8</td>
</tr>
<tr>
<td>OFDM Cyclic Prefix</td>
<td>4 samples</td>
</tr>
<tr>
<td>Sampling Period</td>
<td>20 nanoseconds</td>
</tr>
</tbody>
</table>

Figure 4 through Figure 11 demonstrate SMSE generated pulse radar returns with varying degrees of spectrum fragmentation and their effects on the CAF. These are presented in terms of their Power Spectral Density, and the CAF, with an increasing level of spectrum fragmentation.
Figure 5 Contiguous Spectrum CAF

Figure 6 PSD Waveform with 11% SF

Figure 7 Waveform CAF with 11% SF

Figure 8 PSD Waveform with 32.4% SF

Figure 9 Waveform CAF with 32.4% SF

Figure 10 PSD Waveform with 87.11% SF
Careful examination of the CAFs presented above reveal steadily increasing range sidelobe levels (PSL\textsubscript{range}) as the spectrum fragmentation increases. This phenomena is summarized in Figure 12 which shows the PSL\textsubscript{range} as a function of fragmentation for all the simulated scenarios. It is apparent that even in moderately fragmented spectrum, signal processing techniques must be employed in order to deal with the dramatically increased range sidelobe phenomenon.

The behavior of the frequency domain sidelobes (PSL\textsubscript{doppler}) is unaffected by degree of fragmentation as illustrated in Figure 13.

Also notice in Figure 14 that the mainlobe width (FWHM\textsubscript{range}) decreases as spectrum fragmentation increases, which is an interesting phenomenon. This leads to the conclusion that despite the fact that the aggregate bandwidth has decreased, the range resolution, which is inversely proportionally to FWHM\textsubscript{range}, actually improves. This observation has many implications towards achieving high range resolution even in the presence of low aggregate bandwidth. Ultimately, there are many tradeoffs to be considered when designing these types of waveforms. One consideration is the fact that given a fixed pulse duration, lower power is transmitted due to the lower bandwidth use, which has implications on the radar range. Alternatively, one could choose to increase the transmit power or increase pulse duration to alleviate this impact if the application allows for it.

Figure 15 shows the impact of the increasing spectrum fragmentation on the FWHM\textsubscript{doppler} metric, which is a measure of the CAF lobe width in the Doppler dimension. As expected, this metric is unaffected by the spectrum fragmentation.
A. Effects of Windowing

An initially obvious option for reducing the time domain sidelobes would be to apply windowing in the frequency domain (as opposed to the time domain when used for spectral analysis), on each spectral segment. Applying this technique to a contiguous frequency band does cause a significant reduction in sidelobe time-domain energy.

Application of a Hamming window to the frequency domain data yields time-domain sidelobes at approximately 40dB down from the mainlobe, compared with approximately 13dB down without the window. The spectrum of a frequency domain windowed signal is shown in Figure 16. Also, as expected, this comes at the expense of widening in the mainlobe width by a factor of two, as shown in the CAF in Figure 17.

Unfortunately, in the case of fragmented spectrum, applying this same window function either on each sparse frequency segment or on the entire transmitted bandwidth has no significant effect on PSL_range. Detailed inspection of the PSL_range changes that result from frequency domain windowing are shown in Figure 18. Unfortunately, the only scenario which benefits is the case of the spectrally contiguous case. In all other cases, the PSL_range remains unacceptably high for many applications, and must be dealt with through other techniques.

B. Research Applicable to Sidelobe Reduction

Identified existing research literature that addresses the problem of PSL reduction in fragmented spectrum has been focused on HF radar and UWB radar, which by their nature must cope with a fragmented spectrum to operate \cite{3,6,7,9,11,15}. In HF, the propagation environment creates gaps in usable spectrum, and in UWB, certain bands must be excluded per FCC guidelines. Many of the research techniques involve the reduction of sidelobe energy through special filtering or through specialized waveform design criteria. Some of these techniques will be recreated and
examined for effectiveness in near-term research. An ongoing research area for the authors is to develop techniques for effective range sidelobe reduction within the flexible SMSE waveform generation context. Scenarios will be evaluated by their degree of spectrum fragmentation as defined in this paper.

IV. CONCLUSION

DSA radar is a promising technology to use in key applications to combat ever-increasing spectrum congestion. The SMSE framework provides an efficient method to generate multi-carrier waveforms for DSA radar applications, applicable to both overlay and underlay use cases. DSA radar performance can be measured by comparing it to its continuous spectrum equivalent with respect to traditional radar performance parameters. In processing non-contiguous radar returns, many parameters are unaffected, but severe range sidelobes are created. Straightforward data windowing in the time and frequency domain is not an effective approach to reducing the sidelobes. Existing published literature and ongoing research may provide techniques to reduce sidelobe levels that should have applicability to non-contiguous spectrum SMSE-generated radar waveforms as well.

ACKNOWLEDGMENT

This work was supported by USAF AFRL/RYRE under Phase I & II SBIR program AF083-160 entitled “Cognitive Radio Technology”.

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