Research on the Cooperative BSs’ Number of Coordinated Multi-Point Transmission with Perfect Feedback

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Abstract—The paper studies regional planning issues of Coordinated Multi-Point (CoMP) Transmission with perfect feedback under single-user scenario. Given the downlink received signal expression and the definition of penalty factor, the net ergodic capacity optimization problem is derived. For this optimization problem, the capacity of non-CoMP and CoMP is analyzed and the optimization goal is simplified. Then the relation between net ergodic capacity and cooperative BSs’ number is analyzed. Simulation results show that the theoretical analysis curves are consistent with the results obtained by Monte Carlo simulation.

Keywords – Coordinated Multi-Point; Net Ergodic Capacity; penalty factor; perfect feedback

I. INTRODUCTION

In traditional non-cooperative communication, it is typical to use spatial multiplexing division in the specified cell or cluster to multiplex spectrum resources for anti-jamming. Although the total channel interference is low in this case, frequency reuse is obviously not trustworthy due to the limited spectrum resources. With the Code Division Multiple Access (CDMA) system allowing global frequency reuse, the cell-edge user (CEU) will be more seriously interfered. This situation not only results in severe loss of total throughput, but also cannot meet the fairness among the users.

Aiming at this problem, cooperative communications have been proposed as a positive anti-interference scheme [1]-[6]. A clustered BS coordination strategy is proposed for a large cellular MIMO network in [7], which includes full intra-cluster coordination to enhance the sum-rate and limited inter-cluster coordination to reduce interference for the cluster edge users. For inter-cluster coordination, the coordination area is chosen to balance fairness for edge users and to achieve higher sum rate. It is shown that a small cluster size about 7 cells is sufficient to obtain most of the sum rate benefiting from clustered coordination while greatly relieving channel feedback requirement. Similarly, a downlink transmission mode selection method is proposed in [8], aiming at improving cell-edge throughput without reducing cell-center user rate. To reduce the adverse impact of overhead, only the following users are selected for CoMP transmission: those can obtain significant performance gain from CoMP and those can increase the multiplexing gain. Paper [9] introduces a CoMP Cooperation Set (CCS) approach where the selection range of CCS serving one user equipment (UE) is a smaller cluster rather than the whole network in UE-specific method.

Enhanced Multimedia Broadcast Multicast Service (E-MBSFN) [10], as a multi-cell transmission system, introduces a single frequency network transmission, namely realizing synchronized transmission using the same block of time and frequency in multi-cells. Based on the semi-dynamic cooperation idea [11] and MBSFN regional planning, region-partitioning problem of coordinated multi-point transmission is provided and investigated in this paper. First, we define the net ergodic capacity optimization problem with penalty factor, whose optimization variables are the number of the coordinated BSs. And then, the relation between net ergodic capacity and cooperative BSs’ number is obtained.

The paper is organized as follows. In Section II, the system model is introduced. Section III formulates the problem and proposes the solutions. In section IV, simulation results are shown and analyzed. The paper is concluded in Section V.

Notations: $\|\cdot\|$,$\|\cdot\|_F$ denote the spectral norm and the Frobenius norm respectively. The transpose, the conjugate (Hermitian) transpose are written as $\%(\cdot)$ and $(\cdot)^H$ respectively. $\text{diag}(a_1, \cdots, a_k)$ is a diagonal matrix with elements $a_1, \cdots, a_k$ on the main diagonal. $E\{\cdot\}$ is the expectation operator.

II. SYSTEM MODEL

The system model is based on the single-user scenario. The model of the multi-user scenarios is easily obtained by generalizing the single-user scenario. On the principle of cooperative cell clustering[8] and for single cluster collaboration model, a typical double-cell cellular collaboration system composed by seven hexagonal cell is considered. The single-user downlink model is shown in Figure 1. The cell edge user UE simultaneously receives the transmission signal from the serving cell Cell 0 and two cooperative cells i.e. Cell 1and Cell 2. The transmission signals of the remaining cells are interference sources. While the cell center user UE only receives the transmission signal from Cell 0.

For the network consisting of seven cells, i.e., the base station number $M$ is 7, each base station is configured with $N_t$ transmitting antennas, and the mobile users are distributed.
randomly and uniformly in each cell. Each base station can select several base stations to collaborate with. Furthermore, the user is equipped with a single receiving antenna, that is \( N_r = 1 \). For the primary cell user, the received signal is

\[
y = h_n^w x_0^w + \sum_{m=1}^{6} h_m^w x_m^w + n
\]  

(1)

where, \( x_m \) denotes the downlink single-stream data sent from the base station to the user of the camped cell, and \( x_m \in \mathcal{C} \mathcal{V}(0,1); h_m \) is the \( 1 \times N_t \) channel vector from the base station to this user including large-scale fading and small-scale fading; \( w_m^H \) is the \( N_t \times 1 \) precoding matrix from the base station to the user, and \( n \) is the additive Gaussian white noise with the mean of 0 and variance of 1.

![CoMP model](image)

**Fig. 1** The model of CoMP based on the single-user scenario

When the user without using CoMP technology is in the center region, the received signal \( y^N \) can be organized into

\[
y^N = h_n^w x_0^w + \sum_{m=1}^{6} h_m^w x_m^w + n
\]  

(2)

the first item in the right hand side of Eq. (2) is equal to the useful signal, which is the desired signal, and the second and third items represent the interference caused by the neighboring cells and the additive channel noise respectively. Then the SINR of the receive signal is

\[
\text{SINR}^N = \frac{E \left\{ \left\| h_n^w x_0^w \right\|^2_F \right\}}{E \left\{ \left\| \sum_{m=1}^{6} h_m^w x_m^w \right\|^2_F \right\} + \sigma^2}
\]  

(3)

When the user is within the edge region, it needs to use the CoMP technology. Here, \( \Psi \) is defined as the cooperative set of the user; the interfering set is \( \Phi \); the size of \( \Psi \) is referred to as \( M_t \), which means the number of the cooperated base station. If using joint processing, the serving base station needs to share channel information and data information through the backhaul link with the cooperative cell, and \( y^C \) can be defined as

\[
y^C = h_n^w x_0^w + \sum_{m \in \Psi} h_m^w x_m^w + \sum_{m \notin \Psi} h_m^w x_m^w + n
\]  

(4)

\[
= H w x_0^w + \sum_{m \in \Psi} h_m^w x_m^w + n
\]

where \( H = [h_0, h_1, \cdots, h_{M_t}] \), \( w = [w_0^H, w_1^H, \cdots, w_{M_t}^H]^T \). The right hand side of the Eq. (4) shows the received useful signal, the interference term and noise term sequentially. The received SINR is

\[
\text{SINR}^C = \frac{E \left\{ \left\| H w x_0 \right\|^2_F \right\}}{E \left\{ \left\| \sum_{m \notin \Psi} h_m^w x_m^w \right\|^2_F \right\} + \sigma^2}
\]  

(5)

According to the Shannon theorem , the instantaneous capacity of the user can be expressed as

\[
C = \log_2(1 + \text{SINR})
\]  

(6)

![CoMP BSs](image)

**Fig. 2** Illustration of the cooperative BSs selected by the principle of proximity

Referring to zoning standard of the traditional relay systems and distributed antenna systems [17], the solution of this problem is given as follows: First, a radius \( r \) is provided to draw the boundaries of CoMP and non-CoMP area. Then, in the CoMP area we select the cooperative BSs by the principle of proximity, and the boundary is divided by the connection lines between base stations, as shown in Figure 2.

Due to cellular system with good symmetry and assuming all users and base stations are uniformly distributed , the approximately equal probability density function for \( (\rho, \theta) \) is

\[
f(\rho, \theta) \approx \frac{2\sqrt{3}}{9\pi^2} \rho \quad 0 \leq \rho \leq R, 0 \leq \theta \leq 2\pi
\]  

(7)

Ergodic capacity of the user in the cell is given by the hexagonal
The net ergodic capacity can be expressed as
\[
    \overline{C}_{\text{net}} = 6 \int_0^\frac{\pi}{6} \int_0^{\sqrt{3}R} \rho f(\rho, \theta) d\rho d\theta + 6 \int_0^\frac{\pi}{6} \sqrt{3}R \rho f(\rho, \theta) d\rho d\theta
\]
where \( C_{\text{NC}} \) shows the subscriber capacity located at the region of non-CoMP and CoMP respectively, and
\[
    \overline{C}_{\text{net}} = \frac{C_{\text{NC}}}{2} \left[ \log_2 (1 + \text{SINR}_{\text{NC}}) \right] + \frac{C_{\text{Co}}}{2} \left[ \log_2 (1 + \text{SINR}_{\text{Co}}) \right]
\]

Taking hardware complexity and bit overhead of CoMP into account, a penalty factor \( \alpha (\alpha \leq 1) \) is introduced to express the equivalent capacity loss. The net ergodic capacity can be expressed as
\[
    \overline{C}_{\text{net}} = 6 \int_0^\frac{\pi}{6} \int_0^{\sqrt{3}R} \rho f(\rho, \theta) d\rho d\theta + \frac{\pi}{6} \int_0^{\sqrt{3}R} \rho f(\rho, \theta) d\rho d\theta
\]

Consider this downlink system, the coherence time interval is referred to as \( T \). According to specific scenario of this paper, additional bit overhead and transmission delay will be created due to CoMP between base stations, so penalty factor is given by
\[
    \alpha = f(M_c, N_s) = 1 - \frac{M_c N_s T_0}{T}
\]
where, \( T_0 \) is additional bit overhead and transmission delay interval.

Due to cellular system with good symmetry and assuming that all users and base stations are uniformly distributed, it is sufficient to study a single one-twelfth of the covered triangle area for evaluating the ergodic capacity of the user status in all position. Removing the constant factor, the mathematical model is given by
\[
    \max_{M_c} \int_0^\frac{\pi}{6} \int_0^{\sqrt{3}R} \rho f(\rho, \theta) d\rho d\theta + \frac{\pi}{6} \int_0^{\sqrt{3}R} \rho f(\rho, \theta) d\rho d\theta
\]
\[
    \text{s.t.} \quad \left\{ \begin{array}{l}
    0 < r < \sqrt{3}R / 2, \\
    \overline{C}_{\text{net}}, C_{\text{Co}}
    \end{array} \right.
\]

where, \( r \) is the radius to divide the CoMP area and non-CoMP area; \( M_c \) is the number of cooperation cells.

### III. THEORETICAL ANALYSIS

When the user is in the service cell-center area, i.e. \( (\rho, \theta) \in S_N \), the ergodic capacity of non-CoMP is

\[
    \overline{C}_N = \frac{C_{\text{NC}}}{2} \left[ \log_2 (1 + \text{SINR}_{\text{NC}}) \right] + \frac{C_{\text{Co}}}{2} \left[ \log_2 (1 + \text{SINR}_{\text{Co}}) \right]
\]

In this mode, precoding vector \( \mathbf{w}_m^H (m = 0, \cdots, 6) \) is random variable independent of channel information as well as data information, and obeys independent complex Gaussian distribution, and \( E\left( \| \mathbf{w}_m^H \|_F^2 \right) = P_m \), where \( P_m \) is the transmitted power of the \( m \)th base station. Therefore, by calculating the expectation of internal data information and precoding vector, we get

\[
    E\left( \sum_{m=1}^{6} \mathbf{h}_m \mathbf{w}_m^H \right) = 0
\]

Without considering shadow fading, the channel fading variable is denoted by \( \mathbf{h}_m \), \( \mathbf{f}_m \) is fast fading random vectors that obeys complex Gaussian distribution.

\[
    \overline{C}_N = \frac{C_{\text{NC}}}{2} \left[ \log_2 (1 + \frac{P_0 L_0 \| \mathbf{f}_m \|_F^2}{\sum_{m=1}^{6} P_m L_m \| \mathbf{f}_m \|_F^2 + \sigma^2}) \right] + \frac{C_{\text{Co}}}{2}
\]

where, \( L_m \) is \( \| f_m \|_F^2 \) represents power loss factor generated by path loss.

When a user is at the edge of the cell, it chooses the nearest coordinated base stations to send downlink data, which add an additional optimization variable, and this optimization variable is a global precoding vector. The remaining cells are non-cooperative cells, and the user receives a signal from them as the interference signal, which is denoted by

\[
    y^C_{\text{int}} = \sum_{m \in \Psi} \mathbf{h}_m \mathbf{w}_m^H \psi_m
\]

The precoding vector and transmission signal are independent random vectors, i.e.
The capacity of the edge user is as follows.

For non-cooperative cell, let 

\[ C_{\text{Neterg}} = E_{\text{th}} \log_2 \left( 1 + \frac{|\mathbf{Hw}|^2}{\sum_{m' \in \Psi} |\mathbf{P}| |\mathbf{f}_{m'}|^2 + \sigma^2} \right) \]

Based on the assumption that the power allocation on each antenna are equal, the non-cooperative cell interference is

\[ I = \sum_{m' \in \Psi} \frac{P_{N_t}}{N_t} |\mathbf{f}_{m'}|^2 = \sum_{m' \in \Psi} P_{N_t} |\mathbf{f}_{m'}|^2 \]

Substituting Eq. (20) into Eq. (5) and (9), the state ergodic capacity of the edge user is

\[ C^C = E_{\text{th}} \log_2 \left( 1 + \frac{|\mathbf{Hw}|^2}{\sum_{m' \in \Psi} |\mathbf{P}| |\mathbf{f}_{m'}|^2 + \sigma^2} \right) \]

Assume that the dividing radius \( r \) is a constant, and that \( C_{\text{Neterg}} \) is a function with a variable \( M_c \), which is the cooperative BSs number. After Eq. (21) are substituted into the target function (13), the derivative of \( C_{\text{Neterg}} \) can be obtained as follows

\[ \frac{\partial C_{\text{Neterg}}}{\partial M_c} = \int_0^r \int_0^{\frac{4\pi}{3}} \frac{\partial C^N}{\partial M_c} \rho \rho^2 d\rho d\theta + \alpha \int_0^r \int_0^{\frac{4\pi}{3}} \frac{\partial C^C}{\partial M_c} \rho \rho^2 d\rho d\theta \]

\[ \frac{\partial C^N}{\partial M_c} = \frac{\partial C^C}{\partial M_c} = \frac{\partial}{\partial M_c} \log_2 \left( 1 + \sum_{m' \in \Psi} |\mathbf{P}| |\mathbf{f}_{m'}|^2 + \sigma^2 \right) \]

Equality (24) holds if and only if \( \mathbf{w} \) is linear with \( \mathbf{H}^* \). Assuming that the transmission power of each BS is equal to \( P \), constraints relaxation is \( |\mathbf{w}|^2 = M_c P \). Beamforming vector satisfying the equation should be obtained correspondingly as 

\[ \mathbf{w} = \sqrt{M_c P} \mathbf{H}^* \]

Therefore,

\[ \frac{\partial C_{\text{Neterg}}}{\partial M_c} = \frac{\partial}{\partial M_c} \log_2 \left( 1 + \sum_{m' \in \Psi} |\mathbf{H}|^2 |\mathbf{f}_{m'}|^2 + \sigma^2 \right) \]

By swapping the order of the derivative and expectation, and applying the Jensen inequality, omitting the tedious process of calculation, the final second derivation is

\[ -2 \frac{\rho^2 E\left( |\mathbf{H}|^2 \right) P}{|\mathbf{H}|^2} \]

Obviously, the secondary derivation result is negative, i.e. \( \frac{\partial C_{\text{Neterg}}}{\partial M_c} \) decreasing from positive to negative. \( r \) and \( N_{t\,T_0} / T \) have an impact on decreasing speed. The physical meaning of the formula is as follows. When the selected number of cooperative BSs is smaller than a threshold, \( C_{\text{Neterg}} \) increases with the increasing of \( M_c \); moreover, when it exceeds the threshold, \( C_{\text{Neterg}} \) decreases with the increasing of \( M_c \). The changing speed is decided by \( r \) and \( N_{t\,T_0} / T \).

The value \( r \) represents the cell edge region. If the edge region is close to the serving BS, it will result in that the user do not need to cooperate, which increases complexity. If the edge region is far from the serving BS, it results in that the user can not cooperate and the interference from neighbor BS is strong enough to make the user performance degraded. Therefore, the value cannot be too large or too small. \( N_{t\,T_0} / T \) is the unit value of the penalty factor, and changes the optimal value \( M_c \).

**IV. NUMERICAL RESULTS**

The system simulation scenario and parameters are shown as follows.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Channel model</td>
<td>COST231 Hata Model</td>
</tr>
<tr>
<td>Cell Radius</td>
<td>1000m</td>
</tr>
<tr>
<td>standard deviation of shadowing</td>
<td>8dB</td>
</tr>
<tr>
<td>Antenna Gain</td>
<td>10dB</td>
</tr>
<tr>
<td>Carrier frequency</td>
<td>1.9 GHz</td>
</tr>
<tr>
<td>Channel bandwidth</td>
<td>20MHz</td>
</tr>
<tr>
<td>Path loss factor</td>
<td>-3.7</td>
</tr>
</tbody>
</table>
According to the COST231 Hata model, the path loss model is
\[
P_{\text{PL}}[\text{dB}] = (44.9 - 6.55 \log_{10}(h_{\text{bs}})) \log_{10}(\frac{d}{1000}) + 45.5 + (35.46 - 1.1h_{\text{ms}}) \log_{10}(f_c) - 13.82 \log_{10}(h_{\text{ms}}) + 0.7h_{\text{ms}} + C
\]
where, \(h_{\text{bs}}, h_{\text{ms}}\) are heights of BS’s and MS’s antenna; \(f_c\) is carrier frequency, in unit of MHz; \(d\) is the horizontal distance between the BS and MS; \(C\) is a constant. Parameters for urban macrocells are
\[
h_{\text{bs}} = 32\text{ m}, h_{\text{ms}} = 1.5\text{ m}, f_c = 1900\text{ MHz}, C = 3\text{ dB}.
\]
The correction model of the path loss is
\[
P_{\text{PL}} = 34.5 + 35 \log_{10}(d), \quad d \geq 35\text{ m}.
\]

Fig. 3 and Fig. 4 show the curves of the ergodic capacity with non-CoMP and CoMP of different located users.

![Fig. 3 The comparison of capacity for nonCoMP and different coordinated BSs with the penalty factor of 0.1.](image)

![Fig. 4 The comparison of capacity for the different penalty factors with two coordinated BSs.](image)

According to Fig. 3 and Fig. 4, the conclusions are stated below.

(1) With the penalty factor of 0.1 and the cell radius of 1000m, the user does not need to select any BS to cooperate with within 250m from the cell center; it needs to choose one BS within around 250m to 450m from the center, select two BSs within around 450m to 600m from the center, select three BSs within about 600m to 650m from the center, and select four BSs as distance beyond 650m.

(2) The number of cooperative BSs has a certain relationship with the cell radius.

(3) When the user is in the same position, different penalty factor will bring in the different number of cooperated base stations. Generally speaking, the greater the penalty factor, the less cooperated base stations are chose.

V. CONCLUSIONS

This paper studies the number of the coordinated BSs of CoMP transmission with perfect feedback under single-user scenario. To solve the tradeoff between the advantage and disadvantage of CoMP, a penalty factor is introduced to express the equivalent capacity loss. The net ergodic capacity optimization problem is derived and simplified. We can draw conclusions from the above facts that the number of cooperative BSs has a certain relationship with the cell radius and different penalty factor will bring in the different number of cooperated base stations when the user is in the same position.

ACKNOWLEDGMENT

This work is supported by National Nature Science Foundation of China (No. 60972023), Research Fund of National Mobile Communications Research Laboratory, Southeast University (No. 2011A06), and also supported by STITP of Nanjing University of Posts and Telecommunications (No. SZD2013020 & XYB2013138).

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