A New Precoding Scheme Using Interference Alignment on Modulation Signal for Multi-User MIMO Downlink

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Abstract—In this paper, we propose an improved precoding scheme using interference alignment on modulation signal for multi-user MIMO downlink transmission. The simulation shows that the new precoding scheme can significantly reduce interference and provide much better bit error rate performance than the conventional precoding schemes for multi-user MIMO.

Keywords—multi-user MIMO downlink; precoding; interference alignment; multiplexing.

I. INTRODUCTION

In the mobile communication system, multiple transmit antennas are used at the base station to improve performance and increase capacity. Because only one or a few receive antennas can be used at the mobile stations, the spatial multiplexing scheme is used at the base station and multiple data streams are transmitted simultaneously to different mobile users. Conventionally, the number of mobile users to be served is limited by the number of transmit antennas at the base station. If there are \( N_T \) transmit antennas at the base station, the number of mobile users that can be simultaneously served is not more than \( N_T \). When the number of users is larger than the number of transmit antennas, the system performance is dominated by interference, the bit error rate (BER) performance using the conventional precoding schemes such as signal-to-leakage ratio (SLR), minimum mean square error (MMSE) and zero-forcing (ZF) [1][2][3][4] will have an error floor. In this paper, we propose a new precoding scheme using interference alignment in modulation signal domain for the multi-user (MU) MIMO. In order to avoid the interference between these transmitted streams, the base station sacrifices half the signal space and only sends a real signal. At the receiver side, all interfering signals are aligned and are orthogonal to the desired signal. Each receiver discards the imaginary part of the received signal that contains all the interference and detects the real part signal that contains the useful information. It is found that this new precoding scheme using one-dimensional precoding and one-dimensional modulation for interference alignment performs much better than the conventional schemes using two-dimensional precoding and two-dimensional modulation especially when the number of users is larger than the number of transmit antennas.

II. AN IMPROVED IA-MU-MIMO PRECODING SCHEME WITH PHASE ROTATION

Suppose that the base station has \( N_T \) transmit antennas and there are \( K \) users in the system. Each user has only one receive antenna. The base station sends \( K \) different data streams to the \( K \) users simultaneously. We propose a new precoding scheme as shown in Fig. 1. Each mobile user only detects the real part signal of the rotated received signal that contains the useful information, and discards the imaginary part of the rotated received signal that contains all the interference. The rotation phases are parameters that can be optimized to reduce interference and improve system performance.

We assume that the channel state gain vector corresponding to the \( N_T \) transmit antennas for the \( m \)-th user is \( H_m = [h_{m1}, h_{m2}, \ldots, h_{mN_T}] \), \( 1 \leq m \leq K \), where \( h_{m1}, h_{m2}, \ldots, h_{mN_T} \) are independent complex Gaussian variables with zero means and unit variances. The precoding vector for the \( m \)-th user is \( v_m = [v_{m1}, v_{m2}, \ldots, v_{mN_T}]^T \), where \( x^T \) represents the transpose of \( x \). The rotated received signal of the \( m \)-th user can be expressed as

\[
    r_m = e^{j\theta_m}H_m v_m s_m + e^{j\theta_m} \sum_{l=1,l\neq m}^K H_m v_l s_l + e^{j\theta_m} n_m
\]

where \( s_l, 1 \leq l \leq K \), is the transmitted one-dimensional modulation symbol for the \( l \)-th user, which is real and is assumed be i.i.d and with zero mean and unit variance, \( n_m \) is the complex additive white Gaussian noise with zero mean and variance of \( \sigma_n^2 \).

The real part of the rotated received signal is

\[
    \text{Re}(r_m e^{j\theta_m}) = H_m \Sigma_m v_m \Sigma_m s_m + \sum_{l=1,l\neq m}^K H_m \Sigma_m v_l \Sigma_l s_l + \text{Re}(e^{j\theta_m} n_m)
\]
where \( H_{m\Sigma} = \text{Re}(e^{i\theta_m} H_m) \) \( \text{Im}(e^{i\theta_m} H_m) \), and 
\[ v_{i\Sigma} = \begin{bmatrix} \text{Re}(v_i) \\ -i\text{Im}(v_i) \end{bmatrix}, \quad 1 \leq i \leq K. \] In (2), the first term is the useful signal for detection, the second is the interference from other data streams and the last is the noise part.

**A. Signal-to-Leakage Criteria:**

Since the precoding design to maximize the signal-to-interference and noise ratio (SINR) is very complex, an alternative solution is to maximize the signal-to-leakage ratio (SLR) [4], which is the ratio between the desired signal power and the power interfering to other users.

\[
SLR_m = \frac{\| H_{m\Sigma} v_{m\Sigma} \|^2}{\sum_{l=1, l \neq m}^K |H_{l\Sigma} v_{m\Sigma}|^2} = \frac{v_{m\Sigma}^H H_{m\Sigma}^H H_{m\Sigma} v_{m\Sigma}}{\sum_{l=1, l \neq m}^K |H_{l\Sigma} v_{m\Sigma}|^2} \tag{3}
\]

where \( x^H \) represents the conjugate transpose of \( x \). Let the singular value decomposition (SVD) of \( \sum_{l=1, l \neq m}^K H_{l\Sigma}^H H_{l\Sigma} \) be

\[
\sum_{l=1, l \neq m}^K H_{l\Sigma}^H H_{l\Sigma} = UQH^H,
\]

where \( U \) is a unitary matrix and \( Q \) is a diagonal matrix. Then the above equation can be further expressed as

\[
SLR_m = \frac{v_{m\Sigma}^H H_{m\Sigma}^H H_{m\Sigma} v_{m\Sigma}}{\sqrt{Q} U H^H \frac{v_{m\Sigma}}{\sqrt{Q} U H v_{m\Sigma}}} \tag{4}
\]

Let \( y_m = \sqrt{Q} U H^H v_{m\Sigma} \), then we have

\[
SLR_m = \frac{y_m^H \left[ H_{m\Sigma} \left( \sqrt{Q} U H^H \right)^{-1} \right] H_{m\Sigma} \left( \sqrt{Q} U H^H \right)^{-1} y_m}{y_m^H y_m} \tag{5}
\]

Let \( \lambda \) be the maximum eigenvector of

\[
\left[ H_{m\Sigma} \left( \sqrt{Q} U H^H \right)^{-1} \right] \left[ H_{m\Sigma} \left( \sqrt{Q} U H^H \right)^{-1} \right]^T,
\]

then the optimal \( v_{m\Sigma} \) to maximize the signal-to-leakage ratio satisfies that \( \sqrt{Q} U H^H v_{m\Sigma} = \lambda \), and therefore we have

\[
v_{m\Sigma} = \left( \sqrt{Q} U H^H \right)^{-1} \lambda .
\]

**B. Linear Precoding Schemes:**

The received signal for all \( K \) users can be expressed as

\[
R = HV + N \tag{6}
\]

where \( R = [r_1, r_2, \ldots, r_K]^T \), \( H = \begin{bmatrix} H_1 \\ H_2 \\ \vdots \\ H_K \end{bmatrix} \), \( V = [v_1, v_2, \ldots, v_K]^T \), \( S = [s_1, s_2, \ldots, s_K]^T \), and \( N = [n_1, n_2, \ldots, n_K]^T \).

The real part of rotated received signal is

\[
\text{Re}(M_\theta R) = \text{Hsup} V_\Sigma S + \text{Re}(N) \tag{7}
\]

where \( M_\theta = \begin{bmatrix} e^{i\theta_1} & 0 & 0 & 0 \\ 0 & e^{i\theta_2} & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & e^{i\theta_K} \end{bmatrix} \),

\[
\text{Hsup} = \begin{bmatrix} \text{Re}(e^{i\theta_1} H_1) & \text{Im}(e^{i\theta_1} H_1) \\ \text{Re}(e^{i\theta_2} H_2) & \text{Im}(e^{i\theta_2} H_2) \\ \vdots & \vdots \\ \text{Re}(e^{i\theta_K} H_K) & \text{Im}(e^{i\theta_K} H_K) \end{bmatrix}, \quad \text{and} \quad V_\Sigma = \begin{bmatrix} \text{Re}(V) \\ -\text{Im}(V) \end{bmatrix}
\]

The precoding matrix using zero-forcing (ZF) criteria can be obtained as \( V_\Sigma = \text{Hsup} (\text{Hsup}^H \text{Hsup})^{-1} \), and the precoding matrix using MMSE criteria can be obtained as

\[
V_\Sigma = \text{Hsup}^H (\text{Hsup}^H \text{Hsup} + \frac{\text{G}_n}{2} \text{I}_n)^{-1}, \quad \text{where} \quad \text{I}_n \text{ is an unit matrix.}
\]

The precoder structure of our IA system is shown in Fig. 2. Compared to the processing in complex number in the conventional precoding schemes, our precoding processing performs in real number. The input signal is real modulation:\n
- M-ary PAM, the channel matrix \( \text{Hsup} \) is real, and the precoded signal \( V_\Sigma S \) is real, which is further converted into complex signal \( VS \) for transmission, where \( V = [I_M \ 0 \cdots 0] \times V_\Sigma - j [0 \cdots 0 \ I_M] \times V_\Sigma \), \( I_M \) is an unit matrix of size \( M \) by \( M \), both \( [I_M \ 0 \cdots 0] \) and \( [0 \cdots 0 \ I_M] \) have the same of size \( M \) by \( 2N \).

**C. For Two Receive Antennas at Mobile Stations**

When the mobile station has two receive antennas, the received signal after combining can be expressed as

\[
r_m = H_{mc} v_m s_m + \sum_{l=1, l \neq m}^K H_{ml} v_l s_l + n_m \tag{8}
\]

where \( H_{mc} = H_m 1_W + H_m^2 W_m^2 \), \( H_m^1 \) and \( H_m^2 \) are the channel state gain vectors corresponding to the \( N_t \) transmit antennas and 1\textsuperscript{st} and 2\textsuperscript{nd} receive antenna of the \( m \)-th user, respectively. \( W_m^1 \) and \( W_m^2 \) are complex weighting factors for the two receive antennas of the \( m \)-th user. The real part of the above combined signal can be expressed as

\[
\text{Re}(r_m) = H_{mc} v_m s_m + \sum_{l=1, l \neq m}^K H_{mc} v_l s_l + \text{Re}(n_m) \tag{9}
\]
where $\mathbf{H}_{mc} = \begin{bmatrix} \text{Re}(\mathbf{H}_{mc}) \\ \text{Im}(\mathbf{H}_{mc}) \end{bmatrix}$.

It is very complex to obtain the precoding vectors and the receive weighting factors which achieve the optimal performance in terms of SINR or BER performance. Alternatively we propose an iterative algorithm to obtain the precoding vectors and the receive weighting factors as follows. The idea is that the weighting factors are obtained using the MRC (maximum ratio combining) to match the channel and to maximize SNR, i.e., $w_i^1 = \left( \mathbf{H}_i^H \mathbf{v}_i \right)^T$ and $w_i^2 = \left( \mathbf{H}_i^H \mathbf{v}_i \right)^T$, $1 \leq i \leq K$. After the weighting factors are obtained, the equivalent channel $\mathbf{H}_e$ can be calculated as $\mathbf{H}_e = \mathbf{H}_i^H w_i^1 + \mathbf{H}_i^H w_i^2$, $1 \leq i \leq K$, and then the precoding vectors can be obtained using SLR/ZF/MMSE criteria.

1) Initialize $w_i^1 = w_i^2 = \frac{1}{\sqrt{2}}$, $1 \leq i \leq K$.
2) Calculate all the equivalent channel gain and $\mathbf{H}_e = \mathbf{H}_i^H w_i^1 + \mathbf{H}_i^H w_i^2$, $1 \leq i \leq K$, for all users.
3) Use the combined channel $\mathbf{H}_e$ ($1 \leq i \leq K$) and the SLR/ZF/MMSE criteria to obtain the precoding vectors $\mathbf{v}_i$, $1 \leq i \leq K$.
4) Update $w_i^1 = \left( \mathbf{H}_i^H \mathbf{v}_i \right)^T / \sqrt{2 \left( \left| \mathbf{H}_i^H \mathbf{v}_i \right|^2 + \left| \mathbf{H}_i^H \mathbf{v}_i \right|^2 \right)}$, and $w_i^2 = \left( \mathbf{H}_i^H \mathbf{v}_i \right)^T / \sqrt{2 \left( \left| \mathbf{H}_i^H \mathbf{v}_i \right|^2 + \left| \mathbf{H}_i^H \mathbf{v}_i \right|^2 \right)}$, $1 \leq i \leq K$.
5) Go to step 2), otherwise stop after a pre-defined number of iterations.

III. SIMULATIONS AND COMPARISONS

The bit error rate (BER) performance of the proposed multiuser MIMO system is simulated. The performance of our proposed scheme is evaluated by simulations. The channel is a quasi-static flat Rayleigh fading channel and all channel coefficients are i.i.d. zero mean unit variance. More than 1000 independent channels are simulated. It is assumed that the channel state information is perfectly known for all precoding schemes. All mobile users have perfect channel gain information of the useful signals. For a given channel realization $\mathbf{H}_m$ ($1 \leq m \leq K$), different precoding schemes lead to different SINRs at receiver sides, in order to compare their performances, we use transmit SNR that is the total transmitted power over noise power

$$SNR = \frac{\sum_{m=1}^{K} \sum_{k=1}^{N_k} |v_{mk}|^2}{\sigma_n^2} = \frac{K}{\sigma_n^2}.$$ 

In order to achieve the same spectral efficiency for both systems, the new MIMO system using 2PAM and 4PAM modulation is compared with the conventional MIMO system uses BPSK and QPSK modulation, respectively. The number of transmit antennas $N_T = 8$.

Fig. 3 shows the BER performance of our proposed IA SLR precoding scheme and the conventional SLR precoding scheme. The number of users $K=8$. In order to achieve the same spectral efficiency, our new IA MU-MIMO uses 2PAM modulation signal and the conventional MU-MIMO uses BPSK modulation signal. For the IA-MU-MIMO, we simulate two different cases of phase rotation: 1) There is one phase rotation: $\{0\}$; 2) There are two possible phase rotations: $\{0, \pi/2\}$. The rotated phases for all users are jointly optimized to maximize the minimum SINR by exhaustive search. It is shown that the new IA precoding provides much better BER performance than the conventional precoding. There is more than 15.0dB SNR gain of the new IA precoding over the conventional precoding at BER of $10^{-2}$. Two-phase-rotation IA is more than 2.5dB better than one-phase-rotation IA at BER of $10^{-3}$.

Fig. 4 shows the BER performance of our proposed IA SLR precoding scheme and the conventional SLR precoding scheme with the number of users $K=10$. The IA scheme uses 2PAM while the conventional scheme uses BPSK for the same spectral efficiency. It is shown that the conventional precoding has an error floor which is above BER of $10^{-2}$. This is because that when the number of users is larger than the number of transmit antennas, the conventional scheme cannot fully eliminate interference. Since our new IA-MU-MIMO scheme uses one-dimensional precoding and one-dimensional modulation, it equivalently increases the number of transmit antennas and has larger degrees of freedom to mitigate interference. Therefore our new IA precoding does not have error floor and works well. Two-phase-rotation IA is about 3.0dB better than one-phase-rotation IA at BER of $10^{-3}$.

Figs. 5-6 show the BER performance of our proposed IA MMSE precoding scheme and the conventional SLR precoding scheme with $K=8$ and $K=10$, respectively. Our new IA-MU-MIMO uses 4PAM and the conventional MU-MIMO uses QPSK for the same spectral efficiency of 2bits/s/Hz per user. For $K=8$, the new IA precoding is more than 10.0dB better than the conventional precoding at BER of $10^{-2}$, and two-phase-rotation IA is about 2.5dB better than one-phase-rotation IA at BER of $10^{-3}$. For $K=10$, the conventional precoding has an error floor above BER of $10^{-2}$, and two-phase-rotation IA is about 2.5dB better than one-phase-rotation IA at BER of $10^{-3}$.

Figs. 7-8 show the BER performance of our proposed IA MMSE precoding scheme and the conventional MMSE precoding scheme with $K=8$ and $K=10$, respectively. Our new IA-MU-MIMO uses 4PAM and the conventional MU-MIMO uses QPSK for the same spectral efficiency of 2bits/s/Hz per user. For $K=8$, the new IA precoding is more than 5.0dB better than the conventional precoding at BER of $10^{-2}$, and two-phase-rotation IA is about 2.5dB better than one-phase-rotation IA at BER of $10^{-3}$. For $K=10$, the conventional precoding has an error floor above BER of $10^{-2}$, and two-phase-rotation IA is about 2.5dB better than one-phase-rotation IA at BER of $10^{-3}$. For $K=10$, the conventional precoding has an error floor above BER of $10^{-2}$, and two-phase-rotation IA is about 2.5dB better than one-phase-rotation IA at BER of $10^{-3}$. For $K=10$, the conventional precoding has an error floor above BER of $10^{-2}$, and two-phase-rotation IA is about 2.5dB better than one-phase-rotation IA at BER of $10^{-3}$. For $K=10$, the conventional precoding has an error floor above BER of $10^{-2}$, and two-phase-rotation IA is about 2.5dB better than one-phase-rotation IA at BER of $10^{-3}$. For $K=10$, the conventional precoding has an error floor above BER of $10^{-2}$, and two-phase-rotation IA is about 2.5dB better than one-phase-rotation IA at BER of $10^{-3}$. For $K=10$, the conventional precoding has an error floor above BER of $10^{-2}$, and two-phase-rotation IA is about 2.5dB better than one-phase-rotation IA at BER of $10^{-3}$. For $K=10$, the conventional precoding has an error floor above BER of $10^{-2}$, and two-phase-rotation IA is about 2.5dB better than one-phase-rotation IA at BER of $10^{-3}$. For $K=10$, the conventional precoding has an error floor above BER of $10^{-2}$, and two-phase-rotation IA is about 2.5dB better than one-phase-rotation IA at BER of $10^{-3}$. For $K=10$, the conventional precoding has an error floor above BER of $10^{-2}$, and two-phase-rotation IA is about 2.5dB better than one-phase-rotation IA at BER of $10^{-3}$. For $K=10$, the conventional precoding has an error floor above BER of $10^{-2}$, and two-phase-rotation IA is about 2.5dB better than one-phase-rotation IA at BER of $10^{-3}$. For $K=10$, the conventional precoding has an error floor above BER of $10^{-2}$, and two-phase-rotation IA is about 2.5dB better than one-phase-rotation IA at BER of $10^{-3}$. For $K=10$, the conventional precoding has an error floor above BER of $10^{-2}$, and two-phase-rotation IA is about 2.5dB better than one-phase-rotation IA at BER of $10^{-3}$. For $K=10$, the conventional precoding has an error floor above BER of $10^{-2}$, and two-phase-rotation IA is about 2.5dB better than one-phase-rotation IA at BER of $10^{-3}$. For $K=10$, the conventional precoding has an error floor above BER of $10^{-2}$, and two-phase-rotation IA is about 2.5dB better than one-phase-rotation IA at BER of $10^{-3}$. For $K=10$, the conventional precoding has an error floor above BER of $10^{-2}$, and two-phase-rotation IA is about 2.5dB better than one-phase-rotation IA at BER of $10^{-3}$. For $K=10$, the conventional precoding has an error floor above BER of $10^{-2}$, and two-phase-rotation IA is about 2.5dB better than one-phase-rotation IA at BER of $10^{-3}$.
Fig. 9 shows the BER performance of our IA scheme with two receive antennas used at each mobile station, where \( K=14 \) and 4PAM is used to achieve the spectral efficiency of 2bits/s/Hz per user. The number of iterations is 10 for the calculation of the precoding vectors and the receive weighting factors. Different precoding algorithms are compared for both one receive antenna and two receive antennas. It is shown that significant performance improvement can be obtained by using two receive antennas at the mobile stations.

IV. CONCLUSIONS

In this paper, we propose a new IA multi-user MIMO scheme for downlink transmission. The IA MIMO sacrifices half the signal space and equivalently increases the number of transmit antennas from \( N_T \) to \( 2N_T \), therefore it has larger degrees of freedom to maximize the useful signal power and also minimize interference. The simulations show that the IA scheme can provide much better BER performance than the previously known conventional multi-user MIMO schemes for the spectral efficiency of 1bit/s/Hz and 2bits/s/Hz per user. Note that this is the most likely SNR range for wireless communications. When higher order modulation is used, the IA MIMO has larger energy loss by using the real modulation \( M \)-ary PAM compared to the complex modulation \( M \)-ary QAM used in the conventional MIMO, and therefore less gain of our IA scheme over the conventional schemes is expected.

We also discuss two receive antennas at the mobile stations and propose an iterative algorithm to obtain the precoding vectors at the base station and the weighting factors at the mobile stations. It is found that the performance can be significantly improved by using two receive antennas.

REFERENCES

Fig. 4. The BER performance comparison of the conventional MU-MIMO and the new IA-MU-MIMO, SLR precoding, $K=10$, 1bit/symbol.

Fig. 5. The BER performance comparison of the conventional MU-MIMO and the new IA-MU-MIMO, SLR precoding, $K=8$, 2bits/symbol.

Fig. 6. The BER performance comparison of the conventional MU-MIMO and the new IA-MU-MIMO, SLR precoding, $K=10$, 2bits/symbol.

Fig. 7. The BER performance comparison of the conventional MU-MIMO and the new IA-MU-MIMO, MMSE precoding, $K=8$, 2bits/symbol.

Fig. 8. The BER performance comparison of the conventional MU-MIMO and the new IA-MU-MIMO, MMSE precoding, $K=10$, 2bits/symbol.

Fig. 9. The BER performance comparison of IA-MU-MIMO using one and two Rx antennas, different precoding, $K=14$, 2bits/symbol.