Semi-Blind Multiple-Frequency-Offset Estimation for Decode-and-Forward OFDM Cooperative Networks

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Abstract—Cooperative networks recently became attractive because they can achieve spatial diversity. Multiple decode-and-forward (DF) relay nodes result in multiple carrier frequency offsets (CFOs) because each relay node owns its local oscillator. This paper presents a novel semi-blind multiple-CFO estimator for DF OFDM cooperative networks. A procedure is designed to effectively derive the semi-blind multiple-CFO estimator for relay nodes. Each relay node occupies its own subchannels, and its CFO falls in a nonoverlapped spectrum. The CFO mapping can simply match the corresponding relay nodes. In addition, the proposed method can reduce hardware and computational complexity. Semi-blind random sequences are derived from the characteristics of the signal matrix in the multiple signal characterization (MUSIC) algorithm. Comprehensive simulations show that random sequences can replace periodic training sequences. Furthermore, some information can be conveyed by random sequences to increase spectral efficiency.

I. INTRODUCTION

Cooperative network systems using multiple relay nodes can achieve spatial diversity, increase capacity and enhance coverage [1] [2]. Cooperative networks create a virtual multiple antenna structure by combining multiple relay nodes. However, the process of synchronization might be very intricate due to the numerous nodes in this type of cooperative networks. Most past research on cooperative networks assumes perfect synchronization to simplify the model. Some research papers [3] [4] verify that the performance is degraded singularly if the synchronization errors are large. The presence of multiple carrier frequency offsets (CFOs) in cooperative networks arises from multiple distributed relay nodes. Each relay node owns its local oscillator and brings mismatch between with the destination respectively. Additionally, the OFDM communication is very sensitive to the CFO. The CFO could introduce intercarrier interference and thus dramatically degrade bit error rate performance. Therefore, handling the multiple frequency synchronization problem is a key issue for the successful deployment of a cooperative network.

Conventional CFO estimation methods in OFDM systems can be categorized into two types: cyclic-prefix-based methods [5] [6] and pilot-based methods [7]. These conventional CFO estimations only handle a single CFO. Multiple-CFO estimation has been studied in multiple-input-multiple-output (MIMO) systems [8]-[10]. A maximum-likelihood estimator is proposed in [8]. However, the computational complexity of the estimator is very large, and the accuracy is degraded if the CFO values are close to one another. A correlation-based estimator is proposed by using orthogonal training sequences with different antennas [9]. However, the correlation-based estimator suffers from an error floor. To eliminate the correlation estimator error floor, two iterative algorithms are proposed in [10]. However, the performance of the correlation-based estimator is degraded when large CFOs occur. These methods only handle point-to-point MIMO systems in [9] [10] and thus could not apply to cooperative systems directly. The use of blind synchronization and channel estimation by N antennas at the destination in decode-and-forward (DF) cooperative communication systems is proposed in [11]. The blind method requires a more complex system structure. Non-blind methods are more popular than blind methods. In [12], iterative estimators are proposed with the multiple signal characterization (MUSIC) algorithm for DF and amplify-and-forward (AF) relay protocols. Additionally, the proposed estimators can also extract both large and small CFO values. However, iterative processes with the MUSIC algorithm are too onerous.

In this paper, a semi-blind multiple-CFO estimator is proposed. It is assumed that the OFDM subcarriers are divided into a set of subchannels at relay nodes. The system is designed such that each relay node occupies one subchannel, and the subchannel cannot be used for another relay node. The design rule requires that each relay occupies a different subchannel and that each relay signal is orthogonal to the others. Thus, the range of effective CFOs with its corresponding relay node is derived and does not overlap. The estimated effective CFOs can easily match their corresponding relay by applying the MUSIC algorithm. By analyzing the characteristics of the designed system and the signal matrix in the MUSIC algorithm, a derivation can be carried out that replaces periodic training sequences in the MUSIC algorithm with linear independent random sequences. Furthermore, the transmitted signal from source terminal can carry several im-
important messages and synchronize the system simultaneously. The remainder of this paper is organized as follows. The signal modelling and system architecture for the DF OFDM cooperative networks are briefly introduced in Section II. The proposed semi-blind multiple-CFO estimator is described in Section III. The simulation results and discussion are provided in Section IV. Finally, conclusions are given in Section V.

II. SIGNAL MODELLING AND SYSTEM DESCRIPTION

A half-duplex OFDM cooperative network consisting of a source and destination pair and a cluster of $K$ relay nodes is considered. The DF relay nodes are introduced into the cooperative network. These nodes are assumed to be distributed throughout the cooperative network as shown in Fig. 1. The signal in each relay is down-converted to the baseband and decoded. Then, the new-coded signal is transmitted. Thus, different paths from different relays to the destination have distinct CFO values. The multiple-CFO estimation such as the estimation of CFOs corresponding to $K$ relay nodes using semi-blind sequences is analyzed in Section III.

In the previous study, the signal structure at the interleaved OFDMA uplink is proposed in [13]. First, consider the OFDMA uplink system consisting of $N$ subcarriers. Let the $N$ subcarriers be split into $Q$ subchannels and each subchannel have $P = \frac{N}{Q} = N/Q$ subcarriers. Thus, the subchannel $g$ is constructed of the subcarriers with index set $\{g, g+Q, \ldots, g+(P-1)Q\}$ and where $g = 0, 1, \ldots, Q-1$. Now, consider a DF OFDM cooperative system consisting of $N$ subcarriers and $K$ relay nodes, as studied in this paper. Each relay receives an OFDM symbol with $N$ subcarriers from the same source terminal. In other words, $K$ relay nodes own the same signal at the same time slot. Assume that $K$ relays have been synchronized in time and understand how the subchannels are distributed among the relay nodes. The $k$th relay node only transmits available subcarriers to the destination, and the unavailable subcarriers are masked. Each relay signal is orthogonal to the others, and the distributed subcarrier during an OFDM symbol of relay $k$ subchannel have $P$ subcarriers. Let $r^{(k)}$ the number of periods within the cooperative network as shown in Fig. 1. The effective CFO is defined in [13].

The effective CFO has an important property. Different relay nodes have distinct effective CFOs because each relay node occupies a different subchannel $g$. From this property, it can
be shown that if the relay occupies subchannel \( g \), its effective 
CFO will fall into the range \((-0.5 + g)/Q, (0.5 + g)/Q\). 
Because different relays occupy different subchannels, the 
ranges of the effective CFOs do not overlap.

III. SEMI-BLIND MULTIPLE CFOS ESTIMATOR

The proposed semi-blind multiple-CFO estimator is divided 
into two parts in this section. The estimator is composed of 
semi-blind sequences and the algorithm that can extract 
out multiple-CFO values. First, the extracted multiple-CFO 
method by the MUSIC algorithm is introduced. In typical 
situation, the method utilizes training sequences to perform 
the algorithm. But, according to the design of the subchannel 
rule at the relay nodes described in Section II, the traditional 
limitation that the MUSIC algorithm must use periodic training 
sequences could be broken. These proposed semi-blind 
sequences can be adopted in the MUSIC algorithm by deriving 
the matrix property and the setting at the relay nodes.

A. Multiple-CFO estimator with the modified MUSIC algo-

rithm

The received signal from the \( k \)th relay node in the destina-
tion is shown in equation (1). According to equations (1) to 
(3), the period sequence can be arranged into an \( R \times P \) matrix

\[
A^{(k)} = \begin{bmatrix}
    r^{(k)}(0) & \cdots & r^{(k)}(P-1) \\
    r^{(k)}(P) & \cdots & r^{(k)}(2P-1) \\
    \vdots & \ddots & \vdots \\
    r^{(k)}(N-P) & \cdots & r^{(k)}(N-1)
\end{bmatrix}_{R \times P}
\]

(5)

Thus, an OFDM symbol that includes all involved relay nodes 
can be expressed as a matrix form at the destination.

\[
Y = \sum_{k=1}^{K} A^{(k)} + W
\]

(6)

where \( W \) is an additive white Gaussian noise random variable 
matrix with zero mean and variance \( \sigma^2 \).

The multiple-CFO estimation can be executed by applying 
the MUSIC algorithm based on the above periodic signals. 
The main idea of the MUSIC algorithm is to find out the signal 
subspace and the noise subspace via eigenvalue decomposition 
of the autocorrelation matrix of the received signals at the 
destination. After these two subspaces are identified, a frequency 
estimation function is used to find multiple frequencies from 
to each other, a frequency estimation function \( \Theta \) is described:

\[
\Theta = \text{argmax} \frac{1}{|a(\theta)U_w \Lambda_w^H a(\theta)|^2},
\]

(9)

where \( a(\theta) = [1, e^{j2\pi \theta}, \ldots, e^{j2\pi (R-1)\theta}]^T \). For the MUSIC 
algorithm, the multiple-CFOs are estimated by searching for the 
K largest values of \( \Theta \). The CFOs \( \{\xi^{(k)}\}_{k=1}^{K} \) are calculated 
using \( \theta^{(k)} = \frac{\xi^{(k)}+\theta}{Q}, g = 1, \ldots, K \).

B. Semi-blind training sequence

The transmitting signals must satisfy two properties, mutual 
orthogonality and linear independence, to perform the MUSIC 
algorithm. The proposed relay transmitting rule could relax the 
restrictions in the MUSIC algorithm. The MUSIC algorithm 
can be performed if the transmitting signals only satisfy 
the linear independence criterion. The derivation is described 
below.

At first, we need to know the property of the original 
training sequence. According to the concept of linear algebra, 
equation (7) can be rewritten as

\[
\Gamma = \frac{1}{P} YY^H = QDQ^T
\]

(10)

To solve the K CFO values, the Rank(D) is equal to the 
number of relay K. In addition, matrix Q and Q^T are full 
rank. Thus, the condition of equation (10) can be presented as

\[
\text{Rank}(D) = K = \text{Rank}(\Gamma) = \text{Rank}(YY^H)
\]

(11)

The first target is to find the condition of \( \text{Rank}(Y) \). A simple 
derivation is given below.

\[
\exists \ a \ vector \ x \\
\Rightarrow x^H YY^H x = 0 \\
\Rightarrow (Y^H x)^H Y^H x = 0 \\
\Rightarrow \|Y^H x\| = 0 \\
\Rightarrow YY^H x = 0 \text{ if and only if } Y^H x = 0
\]

According to the result of the above derivation, equation (12) 
can be obtained:

\[
\text{Rank}(Y^H) = \text{Rank}(YY^H)
\]

(12)

Now, to obtain \( \text{Rank}(Y) \) by using \( \text{Rank}(Y^H) \),

\[
\text{Rank}(Y) \geq \text{Rank}(YY^H) = \text{Rank}(Y^H)
\]

let \( X = Y^H \),

\[
\text{Rank}(X) \geq \text{Rank}(XX^H) = \text{Rank}(X^H)
\]

\[
\Rightarrow \text{Rank}(Y^H) \geq \text{Rank}(Y)
\]

According to the above derivation, equation (13) can be obtained:

\[
\text{Rank}(Y) = \text{Rank}(Y^H)
\]

(13)
As a result, the condition $\text{Rank}(Y) = K$ needs to be satisfied. Without considering noise components, equation (6) is simplified

$$ Y = \sum_{k=1}^{K} A^{(k)} $$

We want $\text{Rank}(Y) = K$ and each $\text{Rank}(A^{(k)}) = 1$. Thus, the training sequences should be orthogonal and linearly independent.

Assume there are two relays $A$ and $B$.

$$ \text{Rank}(A + B) = \text{Rank}(A) + \text{Rank}(B) - \text{Rank}(A^T B) $$

We can obtain $\text{Rank}(A^T B) = 0$, and each $A^{(k)}$ should be orthogonal and linearly independent. According to the description in Section II, each relay owns its subchannel. Thus, equation (14) can be rewritten:

$$ Y = \sum_{k=1}^{K} A^{(k)} = V \{ U \odot (BZ) \} $$

$$ B = \begin{bmatrix}
H^{(1)}_1 X^{(1)}_1 & \cdots & H^{(1)}_P X^{(1)}_P \\
H^{(2)}_1 X^{(2)}_1 & \vdots & H^{(2)}_P X^{(2)}_P \\
\vdots & \ddots & \vdots \\
H^{(K)}_1 X^{(K)}_1 & \cdots & H^{(K)}_P X^{(K)}_P
\end{bmatrix} $$

- $V$ is a Vandermonde matrix.
- $U$ is the component of the CFO.
- $Z$ is an IFFT matrix

According to equations (13) and (15), $\text{Rank}(Y) = K = \text{Rank}(B)$. The data sequences only conform to the condition of linear independence. Thus, the proposed semi-blind estimation can use random sequences to replace the training sequence if and only if these random sequences are linearly independent from each other.

IV. COMPARISONS

Relays use the DF relay protocol. The length of a DF OFDM symbol and the FFT size are set to $N = 2048$. Quasi-static and flat-fading channels are considered, where the channel gains are assumed to be constant over the length of an OFDM symbol but to change from symbol to symbol. The CFO values are modeled as unknown non-random parameters. It is assumed that the relay nodes within the network are synchronized in time. The above simulation parameters are summarized in the Table 1.

<table>
<thead>
<tr>
<th>System Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relay type</td>
<td>decode-and-forward</td>
</tr>
<tr>
<td>Number of subcarrier</td>
<td>$N=2048$</td>
</tr>
<tr>
<td>Number of subchannel</td>
<td>$Q=8, 16$</td>
</tr>
<tr>
<td>Number of relay</td>
<td>$K=4, 7, 8, 12$</td>
</tr>
<tr>
<td>Noise</td>
<td>AWGN</td>
</tr>
<tr>
<td>Channel</td>
<td>quasi-static Rayleigh</td>
</tr>
<tr>
<td>S-R link error propagation at SNR</td>
<td>25 dB, 35 dB</td>
</tr>
</tbody>
</table>

When the DF relay node receives the signal from the source, the relay executes the decode-and-retransmit procedure. The CFO estimation errors between the source and the relay nodes will remain after the decode procedure. If the channel state is poor, the estimation error will lead to error propagation. The result shown in Fig. 3 are the mean squared errors (MSEs) when the error propagations occur at the relay nodes. The frequency offset compensation method between the source and the relay nodes uses a maximum-likelihood (ML) estimator [14]. The CFO estimation errors at the relay node assume with SNRs of 25 dB and 35 dB. The performance reduces in stages when the error propagation occurs. If the CFO error with SNR = 25 dB is assumed at the relay terminals, the SNR in the relay would dominate the SNR in the destination when the SNR is larger than $-20$ dB at the destination. The phenomenon is similar when the CFO error with SNR = 35 dB is assumed at the relay terminals.

The traditional MUSIC estimation method mostly utilizes a set of training sequences to perform the algorithm. In this paper, relay nodes are designed and transmit only to available subcarriers. According to the rule, relay signals are orthogonal to each other in the frequency domain and generate the periodic property in the time domain. The property of these sequences can fit the requirements of the MUSIC algorithm. The MSEs of CFO estimation with conventional Hadamard sequences and random sequences that fit the proposed rule are presented in Fig. 4. These two types of sequences have almost the same performance. Thus, random sequences can be replaced by the conventional training sequences according to the derivation in Section III and the evidence in this simulation.

According to the design of the rule in the Section II, different relay nodes occupy different subchannels. Intuitively, the number of subchannels $Q$ must be larger than the number of relay nodes $K$. However, what is the trade-off between $Q$ and $K$? Several experiments are shown in Fig. 5 and Fig. 6. In Fig. 5, the number of relay nodes $K$ is fixed and the number of subchannels $Q$ is set to 8 and 16. From this experiment, the performance is better when the value of $Q$ is bigger. In Fig. 6, the number of subchannel $Q$ is fixed and the number of relay nodes $K$ is set to 4, 8 and 12. According to this test, the performance is better when the value of $K$ is smaller.

A ratio value $W$ is defined to summarize the results from Fig. 5 and 6. $T = \frac{Q-K}{Q}$ is the ratio of the number of subchannels not occupied by the relay nodes to the total number of subchannels. When $T$ is bigger, performance is better because the MUSIC algorithm uses the noise subspace to find the frequency sinusoid compositions. If the number of relay nodes $K$ is fixed, a larger value for $T$ indicates that $Q$ is bigger. The available number of subcarriers $P = \frac{N}{Q}$ at each relay node can be deduced. In other words, the number of transmitting data samples will be decreased and the transmitting efficiency will be degraded. Thus, the trade-off between transmitting efficiency and performance is an important concern.
V. CONCLUDING REMARKS

A novel semi-blind multiple CFOs estimation for the DF OFDM cooperative network has been proposed and studied. In this paper, available subcarriers of each relay node are designed by the proposed rule. By using the design rule of the relay node and theoretical analysis of the signal matrix, the cooperative network can use semi-blind random sequences to perform the MUSIC algorithm and extract multiple CFOs corresponding to each relay node. If the synchronization signal can use random sequences, the signal can carry several important messages and be used to synchronize the system simultaneously. Because each relay occupies a different sub-channels, the CFO ranges of the relay nodes will not overlap. The CFOs mapping can easily match the estimated effective CFOs, which corresponds to the relay nodes and reduces hardware and computational complexity.

REFERENCES


