Spectrally-Temporally Adapted SMSE Waveform Design Using Imperfect Channel Estimates

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Abstract—The impact of channel estimation error is investigated for Spectrally Modulated, Spectrally Encoded (SMSE) waveform designs in a coexistent environment containing multiple 802.11 Primary User (PU) systems. As previously demonstrated, the SMSE waveform design process can exploit statistical knowledge of PU spectral and temporal behavior to maximize SMSE system throughput (bits/second). This can be done by enforcing SMSE and PU bit error rate constraints while limiting mutual coexistent interference limited to manageable levels. Since maximum system performance requires accurate channel state knowledge at the SMSE transmitter, the presence of channel estimation error decreases the ability to design spectrally agile signals that optimally exploit coexistent spectral regions. Relative to a spectrally-only adapted system, the spectrally-temporally adapted SMSE system provides significant performance improvement by leveraging knowledge of PU temporal statistics to design temporally agile signals while maintaining desired performance levels for each system. Superiority of spectrally-temporally adapted signals is demonstrated here in terms of increased SMSE throughput (bits/symbol) and greater tolerance to increased channel estimation error.

I. INTRODUCTION

This work addresses the demand for greater communication system performance using the 70% to 95% of estimated spectrum that is under and/or inefficiently utilized at any point in time \cite{1}–[3]. As with many previous works, the goal is to use existing resources more efficiently in support of Cognitive Radio (CR) and Software Defined Radio (SDR) technologies which have considerable potential to increase spectrum usage \cite{4}, [5]. Spectrally efficient design approaches range from simple spectral notching/avoidance \cite{6} to more complex methods such as spectral water-filling \cite{7}, [8]. However, these methods generally either provide limited performance improvement or impose unrealistic design constraints. More practical techniques employ traditional data modulation with parameters varied on either an inter-symbol (symbol-to-symbol) basis \cite{9} or intra-symbol (within a symbol) basis \cite{10}–[15].

Spectrally Modulated, Spectrally Encoded (SMSE) waveforms have been exploited to design spectrally adaptive OFDM-based waveforms via intra-symbol variation of subcarrier power and modulation order \cite{16}–[18]. Using a coexistent environment containing multiple PU signals, SMSE system throughput (bits/second) was maximized while enforcing bit error rate (BER) constraints on both the SMSE and in-band PU systems. Results demonstrated that additional improvement in SMSE system throughput can be obtained by exploiting knowledge of PU transmission statistics to employ a spectrally-temporally based waveform design as opposed to a spectrally-only design. When the PU system exhibits some degree of temporal structure, SMSE performance was shown to increase further by employing a temporally predictive waveform as compared to merely reactive design process.

The impact of varying the temporal responsiveness of the SMSE system to PU state changes was also investigated by varying the latency ($\tau$) and update interval ($K$) at which the SMSE can estimate and respond to PU temporal changes \cite{11}, [19]. The result was a diminished ability of the SMSE to efficiently design a temporally-agile signal. As the ability of the SMSE system to accurately characterize the PU temporal state continued to decrease, the SMSE performance eventually reduces to that of a spectrally-only based design.

Similarly, if the SMSE has a decreased degree of spectral awareness, it will have a decreased ability to design a spectrally agile signal. A common assumption made in the literature on adaptive modulation is that the transmitter designs its waveform based on knowledge of the wireless channel observed at the receiver. However, in a realistic scenario there will be some error in the channel estimates available at the transmitter. In \cite{20}, [21], the authors demonstrate that for various sources of channel estimation error, the transmitter can compensate by taking into account the mean squared error (MSE) of the estimates. However, the practical utility was limited by using continuous modulation orders and not considering power adaptation.

Demonstration of SMSE waveform adaptability is continued here using a coexistent environment containing multiple OFDM-based 802.11 PU signals to investigate the impact of channel estimation error on achievable performance using both spectrally-only and spectrally-temporally based SMSE waveform designs. As expected, results show that each system design experiences a performance degradation when imperfect channel estimation occurs. As the channel estimation error increases, both the spectrally-only and spectrally-temporally designed systems produce waveforms that are similar to those produced by a system having no channel state knowledge, and the resultant SMSE spectral response is based entirely on the PU spectrum shape. However, by exploiting knowledge of PU temporal statistics the spectrally-temporally based SMSE system is shown to achieve a significantly higher throughput and is more tolerant of channel estimation error.
II. BACKGROUND

A. SMSE Analytic Framework

The SMSE framework remains unchanged from previous work [16]–[19] and is summarized here for completeness. SMSE waveform design parameters include: coding, \( c = [c_1, c_2, \ldots, c_{N_f}] \), data modulation, \( d = [d_1, d_2, \ldots, d_{N_f}] \), windowing, \( w = [w_1, w_2, \ldots, w_{N_f}] \), and a phase-only orthogonality term, \( \phi = [\phi_1, \phi_2, \ldots, \phi_{N_f}] \), \( \Theta \equiv \{1 \} \). Collectively, these terms functionally incorporate various waveform design features that are commonly employed in communications. The intra-symbol frequency components used to generate each SMSE symbol are controlled by the assignment, \( a = [a_1, a_2, \ldots, a_{N_f}] \), \( a_i \in \{0, 1\} \), and use, \( u = [u_1, u_2, \ldots, u_{N_f}] \), \( u_i \in \{0, 1\} \) parameters, where zeros indicate there is no transmission at that particular frequency.

The spectral representation of the \( k \)th SMSE symbol is given by [22], [23]

\[
\mathbf{s}_k = \mathbf{a} \odot \mathbf{u}_k \odot \mathbf{c} \odot \mathbf{d}_k \odot \mathbf{w} \odot \mathbf{o}_k ,
\]

where \( \odot \) denotes a Hadamard product. The \( m \)th subcarrier component of \( \mathbf{s}_k \) is given as

\[
\mathbf{s}_k[m] = o_m u_m k c_m d_m k w_m e^{j(\theta_m k + \theta_m + \theta_w + \theta_{o_m k})}.
\]

As shown in [16]–[19], when the constraints stated above are interpreted with respect to the current subchannel response estimate, they can be expressed as:

\[
M_{m} = \{1, 4, 16\}
\]

where \( M_m \) is the \( m \)th subcarrier modulation order, \( P_m(M_m, H_m) \) is the power transmitted on the \( m \)th subcarrier using modulation order \( M_m \), \( H_m \) is the channel gain on the \( m \)th subcarrier, \( \Lambda_P \) is the total average SMSE symbol power, and \( I_m^v(M_m, H_m) \) is the resultant effective interference power observed by the \( v \)th PU due to the \( m \)th SMSE subcarrier (after passing through the PU receive filter), \( \varepsilon \equiv \{0, 0, \ldots, N_{PU} - 1\} \), \( \Lambda_{I_v} \) is the maximum effective interference power (after passing through the PU receive filter) that the \( v \)th PU can tolerate from the SMSE and still maintain its BER limit, and \( E[\cdot] \) denotes the expectation operator. Note that \( M_{m} = 1 \) is introduced in (3) to account for unused subcarriers, as identified by zero entries in SMSE variable \( u \) in (1), with \( P_m(M_m, H_m) = 0 \) when \( M_m = 1 \).

B. SMSE Waveform Design

The SMSE framework allows intelligent selection of parameters, the values of which are assigned subject to the following design constraints:

1. Fixed total average SMSE power (summed across all subcarriers).
2. Fixed maximum SMSE BER limit (constant across all subcarriers).
3. Fixed maximum BER limit for each independent 802.11 PU signal.

The design is further constrained to operate with a predetermined set of \( N_f \) contiguous assigned frequencies with coding, windowing, and orthogonality terms in (1) set to unity. The data modulations are selected independently from a set pool containing 4-QAM, 16-QAM, 64-QAM, or 256-QAM. Thus, the final design process involves optimal selection of data modulation \( d \) and frequency use \( u \) parameters. Specifically, within the overall goal of maximizing its average data rate (bits/sec), the SMSE system first selects which subcarriers will be used and which will go unused. For each subcarrier selected, the SMSE system then selects 1) the modulation order \( 4, 16, 64 \) or 256) 2) the power to be allocated.

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\]

III. CHANNEL ESTIMATION ERROR

With no channel estimation error, the BER observed by the \( m \)th SMSE subcarrier can be approximated by [20], [21]:

\[
\begin{align*}
P_{b_m}(H_m) & \approx c_1 e^\left\{-\frac{c_2 |H_m|^2 E_s}{\Upsilon_m(M_m - 1)}\right\} \\
& \quad (6)
\end{align*}
\]

where \( c_1 = 0.2, c_2 = 1.6, E_s \) denotes the symbol energy employed by the SMSE on the \( m \)th subcarrier, and \( \Upsilon_m \) denotes the interference observed on the \( m \)th SMSE subcarrier due to additive Gaussian noise plus PU signal presence.

When there is some degree of channel estimation error, the SMSE system performance will need to account for the presence of the estimation errors in order to meet BER requirements. The new BER equation can be computed by evaluating the expected value of (6) over the error probability distribution. While the amount of the error and exact form that the error probability distribution takes depends highly upon the various sources of error and the method used to estimate the channel, typical values for the MSE of the channel estimates have been shown to be below -5 dB [24]–[26]. For simplicity and proof of concept demonstration, the channel
estimation error is modeled such that the channel estimate for the \(m\)th subcarrier \(\hat{H}_m \equiv H_m + e_m\), where \(e_m\) is a zero mean circularly-symmetric complex Gaussian random variable with variance \(\sigma_e^2\) equal to the MSE of the channel estimate. Furthermore, the estimation error \(e_m\) is assumed independent between subcarriers.

The resulting BER can be shown to be [21]

\[
P_{b_m} = E \left[ P_{b_m}(\hat{H}_m) \right] \\
\approx c_1 \frac{\Upsilon_m(M_m - 1)}{c_2\sigma_m^2 E_{s_m} + \Upsilon_m(M - 1)} \\
\times \exp \left\{ - \frac{c_2\mu_m^2 E_{s_m}}{c_2\sigma_m^2 E_{s_m} + \Upsilon_m(M - 1)} \right\} \tag{7}
\]

where

\[
\mu_m = E \left[ H_m | \hat{H}_m \right] \approx \frac{\hat{H}_m}{1 + \sigma_e^2}
\]

\[
\sigma_m^2 = E \left[ |H_m - \mu|^2 | \hat{H}_m \right] = \frac{\sigma_e^2}{1 + \sigma_e^2}
\]

are the mean and the variance, respectively, of the actual subcarrier response given the estimated subcarrier response.

IV. TIME-FREQUENCY POWER DISTRIBUTION

The time-frequency PSD responses of the PU signals and adapted SMSE signals are shown in Fig. 1 for a representative scenario. In this case the coexistent optimization scenario includes two non-overlapping 802.11 PU systems that are spectrally located at \(F_{c1}\) and \(F_{c2}\) and an adaptive SMSE system operating in a multipath fading environment. In response to the PU signals shown in Fig. 1(a), the adapted SMSE signal is shown in Fig. 1(b) for the case with no channel estimation error \((\sigma_e^2 = -\infty \text{ dB})\) and in Fig. 1(c) for the case with severe estimation error \((\sigma_e^2 = 20 \text{ dB})\).

The SMSE system designs a waveform that avoids interference to the PU while accounting for channel estimation error. In Fig. 1(b) the SMSE system has spectrally and temporally adapted its waveform to the PU systems with no channel estimation error \((\sigma_e^2 = -\infty \text{ dB})\). The resultant SMSE response fully exploits perfect channel state knowledge and 1) avoids spectral regions occupied by the PU signals only when they are actually present, and 2) avoids spectral regions having poor channel responses. In Fig 1(c), the SMSE system has adapted its waveform to the PU systems with severe channel estimation error \((\sigma_e^2 = 20 \text{ dB})\). While the resultant SMSE response still clearly avoids spectral regions occupied by the PU signals when they are actually present, the SMSE system is no longer able to effectively exploit spectral regions with high gain and avoid spectral regions with low gain. As a result, spectral shape of the SMSE signal is based entirely on the PU spectrum.

V. SIMULATION RESULTS

Simulation results are presented for a scenario containing the SMSE system designed to adaptively coexist with two in-band OFDM-based 802.11 PU networks. The 802.11 networks span two adjacent 20 MHz channels centered at 5.0 GHz.
and 5.02 GHz. Consistent with 802.11 specifications [9], the PUs are operated according to: 1) average transmit power fixed at 100 mW per user, 2) a pre-encoded data rate of 24 Mbits/sec with a variable length packet structure, 3) rate 1/2 forward error correction, 4) 16-QAM modulation on the 48 data subcarriers, and 5) pilot tones are present. An Additive White Gaussian Noise (AWGN) channel model is used for the PU systems with the noise power set to achieve an in-band $SNR = 16.7$ dB. The SMSE signal employs a maximum of $N_f = 128$ possible subcarriers, each spanning 344.5 KHz. The resultant maximum SMSE bandwidth is 44.096 MHz centered at 5.01 GHz (spectrally centered between 802.11 bands). The SMSE signal uses a 32 length cyclic prefix and propagates through multipath Rayleigh faded channel with an exponential power delay profile having rms and maximum delay spreads of 0.1 $\mu$s and 0.8 $\mu$s, respectively.

For design purposes, both the PU and SMSE systems are constrained to a minimum channel BER of $P_B = 10^{-2}$. It is assumed that the SMSE system is able to update its subcarrier selection, power, and modulation order assignment every 50 SMSE symbols. One additional constraint is imposed on the SMSE power distribution such that its interfering power within a given 20 MHz band does not degrade 802.11 preamble detection performance, i.e., all 802.11 users can reliably detect greater than 90% of received preambles per the 802.11 specification.

When performing the waveform design process outlined in (3)–(5), the SMSE systems use the BER expression in (7) to determine the amount of power required for each subcarrier used. For larger values of estimation error (larger $\sigma^2_e$), the SMSE system will require more power to transmit on any given subcarrier. As a result, the overall capacity achieved by the SMSE system at a given maximum transmission power limit decreases as $\sigma^2_e$ is increased.

Simulated results in Fig. 2 show SMSE throughput as a function of the SMSE transmission power limit for various levels of channel estimation error $\sigma^2_e$, where the given level of $\sigma^2_e$ is the same for all subcarriers. As expected, the presence of channel estimation error degrades performance by approximately 5–10% at $\sigma^2_e = -10$ dB and by up to approximately 20% at $\sigma^2_e = -5$ dB. For all cases considered, the observed SMSE throughput is shown to increase steadily as the SMSE transmit power limit is increased up to a value of approximately 300 mW. Prior to reaching this point, the SMSE system operates in a purely power-constrained mode, i.e., it expends its entire power budget without being impacted by the BER constraint for the PU system. As the power limit approaches approximately 300 mW, the SMSE system begins to limit its actual transmitted power to something less than its power limit (constraint) such that the PU BER constraint is maintained.

As in previous work with no channel estimation error [18], the spectrally-temporally based SMSE system provides superior performance over the spectrally-only system for the range of SMSE transmit power and channel estimation error considered. Superiority of the spectrally-temporally adapted SMSE system is evident in two ways: 1) The worst case performance of the spectrally-temporally based system with severe channel estimation error ($\sigma^2_e = -5$ dB) is better than the best case performance of the spectrally-only system with no channel estimation error ($\sigma^2_e = -\infty$ dB); and 2) The spectrally-temporally adapted system provides approximately 34% higher throughput for all $\sigma^2_e$ considered at SMSE transmit powers above 200 mW.

Advantages of combined spectral and temporal adaptivity are further illustrated using Fig. 3 which shows performance of spectrally-only and spectrally-temporally adapted systems over a wider range of channel estimation error $\sigma^2_e$ using a maximum SMSE transmit power limit of 650 mW. As shown, performance of each system degrades steadily for $-20$ dB < $\sigma^2_e$ < $-10$ dB. For $\sigma^2_e > 10$ dB each system effectively loses all useful information about the channel response and the spectral design of each is based solely on the PU PSD. However, by exploiting the additional dimension of temporal adaptivity the spectrally-temporally based SMSE system is once again superior and significantly outperforms the spectrally-only system at all $\sigma^2_e$ values.

Several additional observations can be made with regard to Fig. 3 results. First, the percentage of total degradation in performance across the range of $\sigma^2_e$ considered is nearly identical for both systems. This is intuitively pleasing given that the SMSE waveforms are designed such that the BER constraint is individually satisfied for each subcarrier. So while the temporal agility of the spectrally-temporally adapted system provides more spectral regions for the SMSE to use, each subcarrier is still degraded by the same amount of channel estimation error.

The two final observations from Fig. 3 are based on throughput performances indicated by the horizontal dashed lines. The
line at approximately 140 bits/symbol represents best case performance for the spectrally-only adapted system and intersects the spectrally-temporally adapted curve at \( \sigma^2 \approx 0 \) dB. Thus, the spectrally-temporally adapted system can tolerate as much as \( \sigma^2 \approx 0 \) dB estimation error and still outperform a spectrally-only adapted system operating with minimal estimation error. The line at approximately 118 bits/symbol is the asymptotic limit for worst case spectrally-temporally adapted system performance with very poor channel estimation (severe estimation error). Even under these worst case conditions, the spectrally-temporally adapted system provides nearly 82% of best case throughput achieved by a spectrally-only adapted system with very good channel estimation (minimal estimation error). Thus, the final decision as to whether or not combine both spectral and temporal adaptivity is driven by channel estimation capability.

VI. SUMMARY AND CONCLUSION

Expanding upon previous SMSE waveform design work that considered perfect channel estimation conditions, this work characterizes coexistent performance of spectrally-temporally adapted SMSE waveforms under imperfect channel estimation conditions. This is done for a coexistent environment containing multiple OFDM-based 802.11 PU networks with SMSE performance evaluated using both spectrally-only and spectrally-temporally design procedures. As would be expected, increasing levels of channel estimation error is shown to steadily reduce the SMSE system’s ability to optimally exploit unused regions of the PU channel. With increasing channel estimation error the SMSE systems reach a point where their resultant waveform designs are based entirely on the spectral shape of the PU systems.

The added diversity in spectrally-temporally SMSE waveform designs is shown to provide clear advantages and significant performance improvement relative to spectrally-only SMSE waveform designs. The spectrally-temporally adapted waveform advantages are especially evident under imperfect channel estimation conditions and include: 1) worst case performance under very high estimation error conditions being better than best case performance of spectrally-only adapted waveforms with no channel estimation error present, and 2) 34% higher throughput relative to spectrally-only adapted waveforms for all combinations of estimation error and SMSE transmit powers considered. Furthermore, the maximally degraded spectrally-temporally adapted SMSE system achieved nearly 82% of the best case throughput realized by the spectrally-only adapted system with no channel estimation error present.

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