Low Complexity Maximum-likelihood Decoder for VBLAST-STBC scheme in MIMO Wireless Communication Systems

Van-Su Pham, Minh-Tuan Le, Linh Mai, and Giwan Yoon

Abstract—In this work, we propose a low-complexity maximum-likelihood decoding approach based on QR decomposition (QRD) for signal detection in VBLAST-STBC systems, which employ less receive antennas than transmit antennas. With the example illustration of (6,3) system using rate 3/4 STBC for three transmit antennas, computer simulations are implemented to verify the performance and complexity of the proposed decoder.

I. INTRODUCTION

Recently, research on application of multiple transmit and receive antennas, as known as multiple-input multiple-output (MIMO) systems, to wireless communication systems has attracted a lot of attention because it is theoretically proven that MIMO systems can significantly improve spectral efficiencies [1], [2]. By additionally exploiting space-time codes [2]-[5], the use of multiple antennas offers not only enormous spectral efficiencies but also a remarkable improvement in system performance.

Orthogonal space-time block codes (OSTBCs or STBC) are the most attractive schemes in space-time codes. OSTBCs can significantly improve reception quality in quasi-static MIMO channels since they have full diversity property. In addition, OSTBC can result in very simple receiver implementation as simple optimal maximum-likelihood (ML) decoders [3]. Unfortunately, complex OSTBCs with full rate do not exist for more than 2 transmit antennas [4]. In recent works [6], [7], the quasi-orthogonal space-time block codes (QSTBCs) can provide full rate and full diversity at the expense of higher detection complexity due to the requirement of joint detection. Another space-time codes (STCs) that are able to achieve both coding gain and diversity are space-time trellis codes (STTCs) [2]. However, STTCs suffer from a critical drawback that their decoding complexity increases exponentially with the transmission rate, thus limiting maximum achievable data rates.

Another space-time code, which provides high spectral efficiency, is the Vertical Bell Laboratories Layered Space Time (VBLAST) code [9]. Similar to STBCs or QSTBCs, the optimal decoder for the VBLAST as well as other high-rate STCs is obviously ML decoder. However, unlike the STBCs whose ML decoding scheme is very simple based on only linear processing of the received signal, the complexity of ML decoder for the VBLAST and other high-rate STCs grows exponentially with the number of transmit antennas, making it impractical when large number of transmit antennas and/or high-order modulation schemes are employed.

To avoid the complexity problem associated with ML detection of VBLAST and other high-rate STCs, bunches of sub-optimal decoding schemes have been proposed such as: zero forcing (ZF) [2], minimum mean square error (MMSE) [2], interference nulling and cancellation using QR decomposition (ZF-QRD) [2], interference nulling with ordered successive interference cancellation, namely ZF-VBLAST [1], sorted QR decomposition (SQRD) [10], etc. For the VBLAST using either ZF, MMSE or QRD interference suppression, the diversity order of the first decoded layer, so-called the lowest layer, is $G = n_R - n_T + 1$. Thus, if VBLAST systems employ equal transmit and receive antennas, this diversity is reduced to 1, leading to a very poor system performance. Although, MMSE-BLAST and MMSE-SQRD [11], [12] show remarkable system performance improvement in comparison with other sub-optimal decoders, the slope of the bit-error-rate (BER) curves indicate that they are able to improve diversity of the system only in low signal-to-noise power ratio (SNR) region. To increase the diversity order of the first decoded symbol, and thus of the system, a combination of ML and QRD decoding algorithm was proposed [15]. Because of using ML, the system could hardly achieve high diversity order under the constraint of reasonable complexity. One another way to increase the diversity order of the lower layer in VBLAST is the use of combination of VBLAST and STBC [8], that diversity order is considerably increased compared to VBLAST system, yet at the cost of spectral lost.

The scheme in [8] and other QRD-based decoder for VBLAST system can only be applicable and obtain advantage when the number of receive antennas is greater or equal to the number of received antennas. In practical, it is still infeasible to implement many receive antennas on mobile device such as hand-phone due to the difficulty of RF components. In this work, we propose a new decoder for MIMO system employing VBLAST-STBC scheme with the number of receive antennas less than or equal to the number of receive antennas. In addition, by employing the operation principle of VLCMLDec1 and VLCMLDec2 decoders [14], the proposed decoder can significantly reduce the computational load while still obtain the ML performance-like.

The remaining part of the paper is organized as follows. In section II, we describe the system model using in our consideration. The detail of the proposed decoding scheme is
presented in section III. The simulation results and discussion are given in section IV. Finally, the conclusion is given in section V.

II. SYSTEM MODEL

Let us consider the MIMO system as depicted in Figure 1, which is an uncoded VBLAST-STBC configuration with \( n_T \) transmit and \( n_R \) receive antennas, denoted as \((n_T, n_R)\) system. At the transmitter, the input data sequence is partitioned into \( N \) sub-streams - so called layers, each of which is then modulated by an M-level modulation scheme such as M-PSK or M-QAM. After that, the modulated signals are divided in to blocks of length \( P \), then modulated by given space-time block code (STBC) resulting the signal matrix \( G_i(s) \) \( (i = 1, \ldots, N) \). Each signal matrix is transmitted over \( L \) transmit antennas.

![Block diagram of VBLAST-STBC system](image)

In our system, the number of receiver antennas satisfies the condition \( n_R \geq N \) and can be less than the number of transmit antennas. The transmit signals is then written by stacking all signal matrices resulting from space-time encoding of all layers as:

\[
G(s) = \begin{pmatrix}
G_1(s) \\
G_2(s) \\
\vdots \\
G_N(s)
\end{pmatrix}
\]  

(1)

At the receiver, we have the received signal given as:

\[
y = HG(s) + n
\]

(2)

where, \( n \) is the \( n_R \times T \) noise matrix of additive Gaussian noise with variance 0.5 per real dimension, \( T \) is the number of symbol duration to transmit one STBC. \( H \) denotes the \( n_R \times n_T \) complex channel matrix containing independent identical distribution complex fading gains \( h_{i,j} \) from the \( i^{th} \) transmit antenna to the \( j^{th} \) receive antenna. We assume that the channel is Rayleigh flat fading, i.e. the magnitude of the elements of \( H \) have a Rayleigh distribution and the channel is constant over one data frame and independently changed from one to another.

With the assumption that the channel matrix is perfectly known at the receiver, the transmitted signals can be ML decoded as follows:

\[
\hat{s} = \arg \min_{s \in S} \| y - HG(s) \|_2
\]

(3)

where \( S \) is set of signal constellation corresponding to the given modulation scheme.

III. PROPOSED DECODER

First of all, we see that in our system the number of receive antennas is less than the number of transmit antennas. Thus, the decoder [8] is not be able to directly apply to our system. In addition, the QRD-based decoders [10]-[12] can not also be directly used. It is shown in [13] that the ML decoder with less receive antennas than transmit antennas could still provide sufficient increase in data rate and performance. However, the ML decoder deals with detection problem in equation (3), which has a complexity order of \( O(S^M \times M \times N) \), where \( S \) is the number of signal points in the given modulation scheme. Therefore, the computational load of ML decoder can become infeasible for systems employing large number of transmit antennas and/or high-level modulation scheme.

The aforementioned issue motivates us to propose the decoding scheme with much lower computational load that still achieves ML-like performance. From the equation (2), let us defined the transmitted signal vector as:

\[
s = \begin{pmatrix}
s_1 \\
s_2 \\
\vdots \\
s_N
\end{pmatrix}
\]

(4)

With the definition of \( s \), the receive signal can be rewritten as:

\[
z = Hs + \bar{n}
\]

(5)

where \( H \) is the equivalent channel matrix of dimension \( n_R T \times n_T \) containing \( h_{i,j} \)'s and their complex conjugates, \( \bar{n} \) denotes the equivalent noise vector. With the equivalent channel matrix, we can directly apply the QRD-based detection [10]-[12] or ZF-VBLAST decoder [9]. However, it is worth emphasizing that those approach are sub-optimal ones, that obtain low computational load at the expense of remarkable degradation in bit-error-rate (BER) performance compared to the optimal ML decoder.

We start with the QR decomposition of the equivalent channel matrix \( H \) as \( H = QR \), where the \( n_R T \times n_T \) matrix \( Q \) has orthogonal columns with unit norm and the \( n_T \times n_T \) matrix \( R \) is an upper triangular matrix. By pre-multiplying the equation (5) with \( Q^H \), where \( (.)^H \) denotes the Hermitian transform, we obtain the following equation:

\[
\tilde{z} = Q^H z = Rs + \eta
\]

(6)

In (6), since \( Q \) is an unitary matrix, the noise term \( \eta = Q^H \bar{n} \) has the same statistical properties as \( n \). Consequently, the ML solution of vector \( s \) in (4), \( \hat{s} \), can be obtained by utilizing either the decision rule in (3) or the following rule:

\[
\hat{s} = \arg \min_{s \in S} \| \tilde{z} - Rs \|^2
\]

(7)

Due to the upper triangular structure of matrix \( R \), we can express the \( k^{th} \) element of \( \tilde{z} \) as follows:

\[
\tilde{z}_k = r_{k,k} s_k + \sum_{i=k+1}^{n_T} r_{k,i} s_i + \eta_k
\]

(8)
where $r_{i,j}$ is the element in the $i^{th}$ row and $j^{th}$ column of matrix $R$. From the equation (8), we can see that the $k^{th}$ element is free of interference from the layer $1, 2, \ldots, k - 1$. Therefore, it is easy to conclude that the $n^{th}$ element is totally free of interference and can be decoded directly. With the assumption that the $n^{th}$ element is correctly decoded, the interference of the $n^{th}$ element can be recovered without difficulty. By doing the same way, we are able to detect the remaining transmitted symbols.

A. Proposed decoder for (6,2) system using STBC code rate 3/4

In this work, we first concentrate on the (6,2) system using STBC code rate 3/4 for three transmit antennas given in equation (3.49) of [2]. The generalized case is a part of our future work.

The space time signal matrix is rewritten as follows:

$$G_i(s) = \begin{pmatrix} s_{11} & s_{12} & 0 & 0 & 0 & 0 \\ -s_{12} & s_{13} & 0 & -s_{33} & 0 & 0 \\ -s_{13} & 0 & s_{31} & s_{32} & 0 & 0 \\ \end{pmatrix}$$

Thus, the signal matrix from the transmitter is stacked of two

$$G_i(s) \quad (i = 1, 2)$$

$$G(s) = \begin{pmatrix} G_1(s) \\ G_2(s) \end{pmatrix}$$

The equation (5) can be rewritten as:

$$z = Hs + \tilde{n}$$

where $y^{(j)}_i$ is the received signal at the $i^{th}$ receive antenna on the $j^{th}$ time slot of the space-time block, $x^*$ is complex conjugate of $x$, $s = [s_1, s_2, s_3, s_4, s_5, s_6]^T$ and $\tilde{n} = [\tilde{n}_1, \tilde{n}_2, \tilde{n}_3, \tilde{n}_4, \tilde{n}_5, \tilde{n}_6]^T$. By taking the QR decomposition of the equivalent channel matrix $H$ in the equation (11) $H = QR$ and pre-multiplying with $Q^H$, the equation (11) can be rewritten as follows:

$$\tilde{z} = Rs + \eta$$

In (12), the upper triangular matrix $R$ has the following format:

$$R = \begin{pmatrix} r_{1,1} & 0 & 0 & 0 & 0 & 0 \\ r_{2,2} & 0 & 0 & 0 & 0 & 0 \\ r_{3,3} & 0 & 0 & 0 & 0 & 0 \\ r_{4,4} & 0 & 0 & 0 & 0 & 0 \\ r_{5,5} & 0 & 0 & 0 & 0 & 0 \\ r_{6,6} & 0 & 0 & 0 & 0 & 0 \\ \end{pmatrix}$$

Let us define the Euclidean distance of the $k^{th}$ element as:

$$d_k = |\tilde{z}_k - \sum_{i=k}^{nT} r_{k,i}s_i|^2$$

From (7) and (14) we have:

$$\hat{s} = \arg \min_{s \in S} \sum_{k=1}^{nT} d_k$$

Based on the working principle of VLCMLDec1 approach [14], the detail of the proposed decoder for this case is given below. For simplicity in describing the proposed decoder, the QPSK modulation scheme is employed. First, our approach decodes the lowest element, $s_6$. The decoder lets $s_6$ be signal points in the QPSK constellation $S$. By applying the equation (14), it computes $d_6$ for all 4 possible signal points of $s_6$. Each signal point is considered as a node. After this steps, we obtain 4 values of $d_6$, denoted as $D_6(i), \quad (i = 1, 2, 3, 4)$, corresponding to 4 nodes within the QPSK constellation. Then, the proposed decoder arranges the nodes in such a way that $D_6(i), \quad (i = 1, 2, 3, 4)$ follows and ascending order. Second, the proposed decoder chooses $s_6(1)$ corresponding to $D_6(1)$, the smallest among 4 possible values of $D_6(i)$, as a possible ML solution for $s_6$. The chosen $s_6(1)$ is used together with 4 signal points in $S$ for possible solution of $s_5$ to continue computing 4 possible values of $d_5$ as given in the equation (14). Then each value of $d_5$ and $D_5(1)$ are added together to yield $D_5(2)$, $(i = 1, 2, 3, 4)$. Similar to the lowest element, the 4 nodes representing 4 possible values of $s_4$ are arranged in ascending order of $D_5(i), \quad (i = 1, 2, 3, 4)$. The proposed decoder chooses $s_4(1)$ corresponding to $D_4(1)$, the smallest among all 4 possible values of $D_5(i)$ $(i = 1, 2, 3, 4)$, as a possible ML solution for $s_6$. Now, $s_5(1), s_5(2)$ and 4 signal points of $S$ are substituted into the equation (14) to compute $d_4$. Then, each value of $d_4$, together with $D_4(1)$ and $D_4(2)$, are summed to give $D_4(3)$, $(i = 1, 2, 3, 4)$. And nodes representing the values of $s_4$ are sorted in an ascending order of $D_4(i), \quad (i = 1, 2, 3, 4)$. Similar to previous element, the proposed decoder selects $s_4(1)$ corresponding to $D_4(1)$, the smallest among $D_4(i)$, as a possible ML solution for $s_4$. At this point, thanks to special format of the upper triangular matrix $R$, the $s_i, \quad (i = 1, 2, 3)$ are independent to one others. Thus, with the assumption of possible ML solution for previous element as $s_i(1), \quad (i = 4, 5, 6)$, the proposed decoder can simultaneously compute $d_k, \quad (k = 1, 2, 3)$. The nodes representing possible values of $s_i, \quad (i = 1, 2, 3)$ are listed in an ascending order of $d_k(i), \quad (k = 1, 2, 3)$ $(i = 1, 2, 3, 4)$. After that, the proposed decoder chooses $s_i(1),(i = 1, 2, 3)$ as possible ML solutions.
for $s_i$ ($i = 1, 2, 3$), $D_k(1)$ and $d_k(1)$ ($k = 1, 2, 3$) are summed together to get $D_1$. The proposed decoder marks $\tilde{s} = [s_1(1), s_2(1), s_3(1), s_4(1), s_5(1), s_6(1)]^T$ as the candidate ML solution for the transmitted vector and set the minimum Euclidean distance as $D_{\text{min}} = D_1$. If the decoding process stops here, we will obtain a performance which is equivalent to that of ZF-QRD decoder. To achieve ML performance-like, more searches are probably required. The proposed decoder continues by comparing $D_1(2)$ with $D_{\text{min}}$. If $D_1(2) \geq D_{\text{min}}$, the proposed decoder stops looking up the possible solution for $s_4$, and jumps up to $5^{th}$ layer, and compares $D_2(2)$ with $D_{\text{min}}$. If $D_2(2) \geq D_{\text{min}}$, the proposed decoder continues jumping up to $6^{th}$ layer, compares $D_3(2)$ with $D_{\text{min}}$. If $D_3(2) \geq D_{\text{min}}$, the proposed decoder terminates and reports the found $\tilde{s}$ as the ML solution for transmitted signal. Otherwise, it selects $s_6(2)$ as a new possible ML solution for $s_6$ which has Euclidean distance $D_3(2)$, to perform the same fashion as it did with $s_6(1)$. In case $D_3(2) < D_{\text{min}}$, the proposed decoder continues re-computing $D_4$ and $D_3(i)$ ($i = 1, 2, 3, 4$) by using $s_6(1)$ and $s_6(2)$, where $s_6(2)$ is chosen as a new possible ML solution for $s_6$. Then, resulted nodes representing all possible values for $s_4$ are arranged as ascending order of new resulted $D_4(i)$ ($i = 1, 2, 3, 4$). After that, the proposed decoder selects new $s_4(1)$ as a possible ML solution for $s_4$, and now $(s_4(1), s_5(2), s_6(1))$ are used to simultaneously find new nodes representing $s_i$ ($i = 1, 2, 3$). These nodes are sorted in ascending order of $d_4(i)$ ($k = 1, 2, 3$) ($i = 1, 2, 3, 4$). Then $D_3(1)$ and $d_3(1)$ ($k = 1, 2, 3$) are summed up to get new $D_3$. And then, the proposed decoder compares new $D_4$ with $D_{\text{min}}$. If $D_3 \geq D_{\text{min}}$, the values of $D_{\text{min}}$ and the found vector $\tilde{s}$ remain unchanged, the proposed decoder continues searching by jumping up to compare $D_{\text{min}}$ with $D_3(2)$. Otherwise, the decoder will update $D_{\text{min}}$ and solution vector $\tilde{s}$ as $D_{\text{min}} = D_3$, $\tilde{s} = [s_1(1), s_2(1), s_3(1), s_4(1), s_5(2), s_6(1)]^T$. The searching process for detection ML solution will be finished if one of the values of $D_k(i)$ ($i = 2, 3, 4$) is greater than $D_{\text{min}}$ or if all the possible signal points at this layer were examined. The Figure 2 illustrates for the initialization of the decoding process.

The decoder can be summarized as follows. In this summary, $s$ is the vector containing all the signal points of the transmission constellation, whose size is $S$. $\text{DECSORT}()$ is a function that uses $s$, the equation (14) to compute $d_k$ and to sort the values of $d_k$ as well as the corresponding signal points in $s$ in an ascending order of $d_k$. The output of $\text{DECSORT}()$ are vectors $d_k$ and $s_k$, which contain the sorted values of $d_k$ and $s$ respectively.

1) (Initialization) Set $k := n_T$, $D_{\text{min}} := 0$, $D_k := 0$, $D_{n_T + 1} = 0$.
2) Set $l_k := 1$, $m := k + 1$, compute $\xi_k := \tilde{z}_k - \sum_{j=1}^{n_T} r_{k,j} x_{j}(l_j)$, and find $[d_k, x_k] := \text{DECSORT}([\xi_k, s])$.
3) Find $[d_k, x_k] := \text{DECSORT}([\xi_k, s])$.
4) If $k = n_T$ then $D_k = d_k(l_k)$ else $D_k := d_k(l_k) + D_m$.
5) Set $x_k(l_k)$ as a solution for $s_k$, if $k > 4$ then $k = k - 1$ and goto Step 2; else compute

$$\xi_k := z_k - \sum_{j=1}^{n_T} r_{k,j} x_{j}(l_j)$$

and find $[d_k, x_k] := \text{DECSORT}([\xi_k, s])$ (k = 1, 2, 3) and $D_k := d_k(1) + d_k(2) + D_4$.
6) (Searching) Set $D_{\text{min}} := D_k$, $k := 4$, $l_k := l_k + 1$, and goto Step 8.
7) If $k > 3$ find $[d_k, x_k] := \text{DECSORT}([\xi_k, s])$, $l_k := 1$ else find $[d_k, x_k] := \text{DECSORT}([\xi_k, s])$, $l_k := 1$ for $k = 3, 2, 1$.
8) If ($k > 3$) then $D_k = d_k(l_k) + D_{k+1}$ else $D_k = d_k(1) + d_k(2) + D_4$.
9) If ($D_k \geq D_{\text{min}}$) or ($l_k > S$) then: if $k = n_T$ then terminate and report result; else set $k := k + 1$, $l_k := l_k + 1$ and goto Step 8.
10) If ($k > 3$) then let $m := k$, $k := k - 1$, compute $\xi_k$, and goto Step 7, else set $D_{\text{min}} := D_k$, save $x_{j,l_j} (j = 1, ..., n_T)$, let $k := k + 1$, $l_k := l_k + 1$ and goto Step 8.

IV. COMPUTER SIMULATION RESULTS AND DISCUSSION

To verify performance and complexity of the proposed decoder, we apply it to a VBLAST-STBC scheme in a (6,2) system. In addition, the modulation scheme is BPSK. The frame length is set to equal to 100 STBC symbol periods, i.e. 400 symbol periods.

Figure 3 shows the BER performances versus average bit-energy-to-noise ratio (Eb/N0) per receive antenna of the proposed decoder, the ZF-ORD decoder and the optimal ML decoder. It can be seen from the Figure 3 that the proposed decoding scheme obtains almost the same BER performance as the optimal ML decoder does. In other words, the proposed decoder is able to provide ML-like performance to a MIMO system applying VBLAST-STBC scheme. On the other hands, the proposed decoder significantly outperforms the ZF-ORD decoder in term of BER performance. For instance, at BER=$10^{-3}$ the improvement is approximated to 5dB.

\[
\begin{align*}
\text{Fig. 2. Illustration of decoding processing of the proposed approach for} \\
\text{VBLAST-STBC system using STBC code rate 3/4 with 6 transmit antennas,} \\
\text{2 receive antennas, QPSK modulation.}
\end{align*}
\]
The average computational loads per symbol period of our proposed decoder, ZF-QRD decoder and optimal ML decoder in term of complexity addition and multiplication operations are given in the Table I. In the Table I, we only count all the complex additions and multiplications in implementation of the searching process without taking the complexity of the preprocessing stage into consideration. The parameters for the simulation is the same as those used for Figure 3. In addition, the EbN0 is kept constant at 10dB, the number of iterations is set to $5 \times 10^3$.  

![Plot of BER vs EbN0 for VBLAST-STBC scheme](image)

**Fig. 3.** BER performance of the proposed decoder, ML decoder and ZF-QRD for VBLAST-STBC scheme in a (6,2) system using STBC code rate 3/4 and QPSK modulation scheme.

<table>
<thead>
<tr>
<th>Decoder Name</th>
<th>Number of additions</th>
<th>Number of multiplications</th>
</tr>
</thead>
<tbody>
<tr>
<td>ZF-QRD decoder</td>
<td>20</td>
<td>26</td>
</tr>
<tr>
<td>Proposed decoder</td>
<td>59</td>
<td>51</td>
</tr>
<tr>
<td>ML decoder</td>
<td>3520</td>
<td>3584</td>
</tr>
</tbody>
</table>

It can be seen from the Table I that the complexity of the proposed decoder is comparable to that of ZF-QRD decoder while its BER performance is dramatically lower than BER performance of the ZF-QRD decoder. Furthermore, the computational load of the proposed decoder is remarkably smaller than that of the optimal ML decoder while they both obtain almost the same BER performance.  

**V. CONCLUSIONS**

In this work, we present the new decoder for MIMO system employing VBLAST-STBC while the number of receive antennas is less than or equal to number of transmit antennas. The proposed decoder is shown to be capable of not only providing the systems with ML performance-like but also obtaining extremely low complexity. Although it suffers from spectral efficiencies loss by nature of VBLAST-STBC, the significant low complexity and the capacity of using in case number of receive antennas less than number of transmit antennas make it to be a very promising decoder for practical applications.  

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