Spatial and Temporal Modeling of Few-Cycle Ti:Sapphire Lasers

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Abstract

In few-cycle Kerr-lens mode-locked Ti:sapphire lasers, the laser crystal is exposed to extremely high intensities which can result in crystal damage and performance degradation with time. Modeling of the intracavity pulse dynamics can deepen the understanding of the different phenomena contributing to possible damage and of how to best optimize the laser performance. We present a one-dimensional laser model based on dispersion managed mode-locking that accurately captures the temporal and spectral intracavity dynamics and reproduces the output characteristics of these lasers in great detail. In addition, a spatiotemporal model is demonstrated. We find that the ultra-high intensities in the crystal induce plasma formation that shapes the spatial beam propagation in agreement with experimental observations.

1. Introduction

Few-cycle Ti:sapphire lasers with ultra broadband dispersion compensating mirrors (DCMs) generating octave-spanning spectra directly from their laser cavity have been widely successful. Stable and reliable long-term operation is achieved by experimentally optimizing the laser cavities. However, it can be observed that lasers in the few-cycle regime frequently develop crystal damage since the Ti:sapphire crystal is exposed to high intracavity intensities due to the short pulse durations. Therefore, modeling of few-cycle Ti:sapphire lasers is crucial not only to optimize the intracavity elements and the laser performance but also to understand the different phenomena contributing to possible damage of the components. Though several approaches to modeling the intracavity pulse dynamics of mode-locked Ti:sapphire lasers have been pursued, the question remains of how to best capture the pulse evolution in the few-cycle regime. In particular, it remains a challenge how to best translate the complex spatial and temporal dynamics of the Kerr-lens mode-locking in the Ti:sapphire crystal into a generic model.

A basic temporal model to capture the pulse dynamics is the master equation [1] that describes the average pulse dynamics in the cavity. In this context, the Kerr-lens mode-locking impact is approximated by a fast saturable absorber. Dispersion managed mode-locking (DMM) [2] has further extended this temporal modeling to include the localized dispersion impact. Thus, pulses undergo a breathing behavior depending on the encountered group delay dispersion in each cavity element and the optical spectrum is broadened and recompressed due to self-phase modulation in the crystal. Such modeling was successfully applied to standard non-octave-spanning laser cavities [3, 4]. However, to predict all the features in the output spectra for few-cycle pulses, more detailed implementation of the intracavity elements, including not only second but also higher order dispersion and exact reflectivities of the mirrors and output coupler is important. In the following, we present a one-dimensional model that analyzes the pulse dynamics in great detail. We demonstrate that with such a temporal model, the intracavity spectral breathing is captured and we can directly compare simulated and experimental output spectra as well as theoretically obtained pulse durations and experimental measurements. Thus, the laser dynamics, temporal stability, and steady-state spectral output can be predicted and optimized based on realistic parameters with this model. However, for details about the stability region and wavelength dependent beam distortions, spatial effects have to be incorporated, resulting in a higher numerical complexity. First attempts at space-time focusing were conducted [5] and the spatiotemporal pulse dynamics were numerically studied for Gaussian beam propagation combined with DMM [6, 7]. With high intensities and short pulse durations in few-cycle lasers, gain-guiding and a tight focus, the plasma effect becomes non-negligible for the simulations. Plasma is generated in the Ti:sapphire crystal and plays an important role in linking the temporal with the spatial dynamics of the laser. Adding these pulse-plasma interactions to spatial simulations explains unique features of these lasers that have been experimentally observed, such as the limited laser efficiency and strongly wavelength-dependent beam profile. Therefore, to learn how to best align the cavity, identify stable operating regimes and understand the resulting beam profiles within the laser cavity, a second numerical model is presented. This model has the potential to capture the full dynamics of dispersion-managed, few-cycle Ti:sapphire...
lasers by focusing on additional spatial effects. With the two different models we have a comprehensive toolset to optimize the laser output by means of the temporal model and we can obtain information about the actual beam profile and stability region based on the spatial numerical simulation.

2. Theoretical models

During each round-trip in the laser cavity, the pulse encounters nonlinear effects within the crystal that reshape its temporal and spatial profile. For the remaining linear propagation outside the crystal, dispersion and spectral shaping due to the wavelength dependent reflectivity of the dispersion compensating mirrors and wedges determine the pulse evolution. In the following, a standard four mirror ring laser cavity with custom designed DCMs is studied that generates octave-spanning spectra directly from the laser cavity as shown in Fig. 1(a). To account for the discrete impact, spatial location and order of each intracavity element, the temporal evolution of the pulse envelope \( A(z, t) \) in time and frequency domain is followed in a retarded time frame \( t \) for the intracavity position \( z \) as seen in the schematic in Fig. 1(b) for the temporal model. The crystal itself is modeled by the nonlinear Schroedinger equation and solved numerically by the split-step Fourier method. Higher order dispersion in form of a dispersive phase \( \Phi(\omega) \), Lorentzian gain filtering \( \Delta \omega = 240 \) rad THz) and gain saturation \( g(z, t) \) with \( W_{\text{Sat}} \), self-phase modulation and self-steepening with a nonlinear coefficient \( \delta \) as well as the Kerr-lens mode-locking by means of a fast saturable absorber \( q(z, t) \) is taken into account (for details in parameter choice see [8]).

For the spatial model, one has to consider that the pulse diverges very quickly after exiting the crystal. Therefore, an equivalent optical system is implemented, using high radius of curvature lenses attached to the front and back of the crystal and separated by an effective distance \( D' \), as shown in Fig. 1(c). This ensures that the maximum spot size outside the crystal does not exceed that inside the crystal. Instead of using a specific spatial profile (e.g. Gaussian) with some characteristic parameters, a radial symmetry \( r \) is assumed for the beam (to reduce the numerical complexity) and a simplified transformation is applied for propagating the pulse (e.g. ABCD matrix).

![Fig. 1: (a) Schematic of a Ti:sapphire ring cavity with dispersion compensating mirrors and wedges for fine-tuning of the dispersion. (b) Schematic for the pulse evolution in the temporal model which takes the impact of each discrete elements into account to predict the output pulse parameters. (c) Equivalent optical system of the unfolded ring laser cavity: the mirrors are substituted by lenses for the spatial modeling of the intracavity pulse propagation in the crystal.](image)

\[
\frac{\partial A(z,t)}{\partial z} = \left[ \hat{N}_{\text{laser}} + \hat{I}_{\text{cavity}} \right] \cdot A(z,t)
\]

\[
\frac{\partial A(z,r,t)}{\partial z} = \left[ \hat{N}_{z} + \hat{N}_{r} + \hat{I}_{\text{cavity}} \right] \cdot A(z,r,t)
\]

\[
\hat{N}_{\text{laser}} = g - q - j \delta \left[ |A|^2 + \frac{j}{\omega_c} \frac{\partial}{\partial t} |A|^2 \right] - j \phi(\frac{j}{\partial t})
\]

\[
\hat{N}_{z} + \hat{N}_{r} = \frac{j}{k} \nabla_z - jD - jy |A|^2 + g - \frac{\beta_{(x)}}{2} |t|^{\nu-1} - \frac{\sigma}{2} (1 - j \omega r) \rho
\]

\[
g(z,t) = F^{-1} \left[ \frac{g_0}{1 + 2 |A|^2 \omega^2 + \Delta \omega_c^2} \right]
\]

\[
g(r,z,t) = F^{-1} \left[ \eta r \omega \omega_0 \frac{W_{\text{pump}}(r,z) / W_{\text{sat}}}{1 + (2W_{\text{pump}}(r,z) / W_{\text{sat}}) + \Delta \omega_c^2 \omega^2} \right]
\]

Eq. (1) compares the modeling equations for both approaches; in both instances the propagation through the Ti:sapphire crystal is accounted for by a nonlinear operator \( \hat{N}_{\text{laser}} \) or \( \hat{N}_{z} + \hat{N}_{r} \) (crystal with plasma effects). The linear operator \( \hat{I}_{\text{cavity}} \) describes the encountered dispersion in wedges for fine-tuning of the overall cavity dispersion and the mirror reflectivities in the case of the temporal model; for the spatial model dispersion and diffraction are considered. In the spatial model for the gain, the overlap of the pump with the laser beam is implemented. To speed up the very computation-intensive simulation of the spatiotemporal model, a personal supercomputer with 96 GB memory, two Intel Xeon 5680 CPUs, and two Nvidia C2070 GPU-computing cards is used for GPU-based parallel computing.
3. Dispersion managed temporal pulse dynamics in sub-two cycle Ti:sapphire lasers

The governing pulse dynamics in a few-cycle Kerr-lens mode-locked laser cavity are given by the nonlinearities in form of self-phase modulation combined with self-steepening and dispersion in the laser crystal. Fig. 2(a) displays how this affects the pulse duration within one cavity round-trip: Starting in the geometric middle of the crystal, the pulse is compressed and features a FWHM pulse duration of 12 fs. Due to gain filtering and higher order dispersion effects, the minimum pulse duration in the crystal is close to 9 fs and is reached slightly offset from the geometrical middle. Upon further propagation, the pulse gets chirped and stretches to a FWHM of ~47 fs due to the encountered normal material dispersion and the maximum peak power of the pulses is reduced by at least a factor of 3. The spectrum breathes as self-phase modulation generates new frequency components, filling and even exceeding the gain bandwidth. In the linear part of the cavity, the pulse stretches and compresses according to the dispersion encountered from each cavity element, whereas the spectrum hardly changes, except for little amplitude shaping in the wings from the wavelength dependent mirror reflectivity. Thus, after only one DCM bounce the pulse gets further suppressed to a FWHM pulse duration of ~6 fs, resulting in an overall pulse breathing within the laser by a factor of ~8. Over a certain parameter range (comparable to conditions in the laser cavity), the numerical analysis converges to steady state and supports stable pulse operation. To obtain few-cycle pulses, strong self-phase modulation is required as well as good dispersion compensation over a large bandwidth. In this case, the best performance was found for a slight net positive cavity dispersion. With a realistic parameter choice and by matching the output pulse energy to the experiment, the numerical simulation allows us thus to predict the laser output characteristics. In this case, the numerical evaluation was compared to the measured output spectrum from a 500 MHz ring laser as shown in Fig. 2(c). Even spectral features in the wings of the output spectrum are accurately reproduced and power levels in different wavelength regions can be predicted. In addition, the transform limited pulse and compressed pulse duration obtainable from the analysis matches the experimental results and supports sub-5 fs output pulses. This analysis therefore enables us to optimize intracavity elements (e.g. test different broadband mirror designs), the intracavity pulse formation and output spectrum shaping and to obtain performance predictions even before implementation of the actual laser cavity.

![Fig. 2](image-url)

**Fig. 2:** (a) Pulse stretching and recompression for different positions within one cavity round trip. (b) Spectral breathing with generation of new frequencies within the crystal. (c) Simulated output spectrum reproduces measured results.

4. Plasma formation in a spatial model for few-cycle Ti:sapphire lasers

As we have seen in the temporal model, the pulses undergo a strong self-amplitude modulation action (SAM) in the crystal. The SAM is dominated by the spatial overlap between the intracavity pulse and the gain profile defined by the pump beam. The nonlinear spatial effects alter the diffraction of the beam depending on its intensity. In addition, preferential gain is provided to the higher intensity portion of the beam and shortens the pulses. In addition to the self-focusing Kerr effect and gain-guiding, we found that in this few-cycle regime the plasma-induced effects play a key role in balancing the pulse transients. Since these transients can reach high instantaneous powers close to the critical power of the material, serious pulse collapsing can occur and generate misleading results if the plasma interaction is neglected as in existing models. In reality, a significant amount of plasma is generated before the pulse collapses and provides a negative feedback by defocusing and absorbing these high intensity transients. This also suggests that a non-negligible loss caused by the plasma exists and this can explain the poor efficiency of these lasers.

In our model, we perform a full-wave simulation that allows us to clearly trace the highly wavelength-dependent beam profile that has been observed experimentally (see Fig. 3(b)-(i)). In general, the mode is less structured in a wavelength range where the cavity quality factor is high. For the wavelength region where the cavity has low loss and a nearly zero round-trip phase (λ ~ 700-900 nm), a fundamental Gaussian mode is found, as shown in Fig. 3(d)-(f). For the wavelength region with a higher output coupling/loss or a larger round-trip phase (λ ~ 600-700 nm and λ ~ 900-100 nm), higher-order spatial modes appear (see Fig. 3(c) and (h)). In regions where the cavity is extremely...
lossy ($\lambda < 600 \text{ nm}$ and $\lambda > 1100 \text{ nm}$), the mode structure is strongly determined by the nonlinear effects occurring inside the crystal in one pass (see Fig. 3(b)). In these cases, a typical ring structure appears, caused by the interplay between the Kerr self-focusing and plasma defocusing effects, which can also be found in filamentation experiments. We believe that the full spatiotemporal model derived in this paper can be used to fully understand the spatial behavior of few-cycle Ti:sapphire lasers and facilitate the cavity optimization.

Fig. 3: (a) Simulated wavelength-dependent spatial beam profile of the laser beam at the end of the crystal for different wavelengths. (b)-(h) Measured beam profile at various wavelengths for a 2 GHz Kerr-lens mode-locked ring laser. (i) Continuous-wave beam profile before the mode-locking is initiated.

5. Conclusion

In this paper, we present two different modeling approaches to study the pulse dynamics and spatial evolution in octave-spanning Kerr-lens mode-locked Ti:sapphire lasers. A temporal model gives us insight into the breathing dynamics within the laser cavity with pulse durations changing by a factor of ~8 and the pulse peak power varying by a factor of ~6. The numerically evaluated output spectrum was compared to experimental data and reproduced even specific spectral features. With the spatial model, the impact of plasma formation on the pulse shaping and spatial beam profile was analyzed. The simulated wavelength dependent beam profile from the simulations deepened the understanding of observed beam profiles in the actual laser cavity. Therefore, with these two models we demonstrate an important toolset to better optimize the cavity construction in terms of laser performance and stability for octave-spanning Ti:sapphire lasers.

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7. References