Time-Resolved Ultrasonic Body Wave Measurements of Material Anisotropy Using A Lensless Line-focus Transducer

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Abstract - For plate-like sample geometries, a line-focus transducer can be used to detect back-reflected echoes through the thickness of the sample. The interaction of the convergent cylindrically focused probing wave with the material anisotropy produces multiple echoes which can be interpreted as the reflected and mode converted waves. These echoes are time-resolved and their arrival times are polarization dependent. A simple polar display of the rotationally scanned time waveforms reveals intriguing details that resemble slowness curves. We present both experimental and theoretical results for body wave measurements using our line-focus transducer on various crystals.

INTRODUCTION

Mechanical anisotropy is common in crystals, composites and many structural materials. Among the many ultrasonic techniques for measuring anisotropy, acoustic-signature V(z) measurements with a line-focus transducer are versatile, precise and capable of high spatial resolution. The technique relies on the measurement of the reflected tone burst echo amplitude, V, as a function of amount of defocus, z, and analysis of the interference minima in the V(z) curve to obtain various surface wave velocities. A line-focus transducer launches surface waves in a direction controllable by the alignment of the transducer. Consequently the combination of rotation about the z axis and translation in the z direction is used to study material anisotropy. Numerous anisotropic elastic property measurements in crystals, thin films on crystal substrates, and samples under stress have been reported [1-4].

We have developed a lensless line-focus transducer made of a low cost piezoelectric polymeric film [5]. The transducer has wide bandwidth and is suitable for time-resolved transient-pulse (rather than interference) measurements. With 25.4mm focal length and 28.2mm aperture, the f-number of the transducer is 0.9. The relatively low f-number of the transducer produces clearly detectable leaky surface waves in most structural materials when the specimen is located in the near field (defocused condition). We have also developed graphical data presentation techniques allowing the scanned data to be interpreted as velocity plots [6]. The application of this transducer to time-resolved leaky surface wave velocity measurements on isotropic and anisotropic solids has been reported elsewhere [6-8]. Here we report a new use of the transducer for body-wave measurements. Because the transducer does not have a lens, the development of the theory to predict and interpret the results is relatively simple. We show some sample results of the comparison between theory and experiments for both fused quartz and crystal quartz plates.

TEST CONFIGURATION AND DATA PRESENTATION

In addition to the surface wave velocity measurement mentioned above, a line-focus transducer can be used to detect echoes reflected from the back surface of a plate-like sample. Shown in Fig. 1 is a schematic drawing of a plate of thickness d, placed below a transducer of focal length F and aperture half-angle, α. As the cylindrically focused pulsed waves converge and interact with the interface between the liquid and solid, multiple echoes are produced. They are separately visible in the time waveform shown in the Fig. 2. The sample is a fused quartz plate 3.14 mm thick. The sampling interval is 0.01 microseconds and the trace spans 5.00 microseconds. Though resolvable in time, these echoes are sometimes difficult to identify in a single recorded waveform.

Figure 1. Experimental setup. The line-focus transducer of focal length F is probing a plate sample of thickness d. Both the transducer and the sample are submerged in water. The plate is placed inside the focal point of the transducer with the front surface of the plate located at a distance z above the focal point. Arrows show some of the possible ray paths.

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Figure 2. A typical waveform acquired with the line-focus transducer.

We have developed a graphical display technique to ease this difficulty. A sequence of waveforms is recorded for a number of positions on the z axis. When the reference time is set to zero with respect to the arrival of the first echo (the reflection of the center ray from the front surface), and by setting the first echo as the trigger signal for the transient recorder, all the later arrivals have a simpler z dependency. The sequence of waveforms can then be represented as a color 2-D plot with time and defocus distance as the two axes. Shown in Fig. 3 is an example of such a 2-D plot. The sequence consists of 51 scans for z = 0-10 mm. On this plot, the leftmost vertical line, C, represents the center reflection from the front surface of the plate. The four vertical lines, L2, L4, L6, and L8, represent the reflections from the back surface going through the plate twice, four times, six time and eight times, at the longitudinal wave speed of the fused quartz. The inclined line, R, represents the arrival of the leaky surface wave which is a linear function of z; the slope of the line is the basis for computation of the surface wave velocity [5]. Between time interval 1.5 and 2.5 μs, two additional arrivals can be seen. They become more visible at relatively larger z, and they are slightly curved to the right, consistent with the delay due to increased path length due to larger refracting angle at larger z. They are mode converted reflections from the back surface, going through the plate twice; the first one, LS, once at longitudinal wave speed and once at shear wave speed and the second one, S2, both at shear wave speed.

A rotational scan at a fixed z over an anisotropic plate can also be plotted as a 2-D color plot, this time in polar coordinates to show the angular dependency of the data. One example of a polar plot of data obtained on a rotationally scanned Z-cut quartz single crystal plate is shown in Fig. 4. The defocus distance, z, is fixed at 11.5 mm. The radial coordinate corresponds to time while the angular coordinate represents the transducer alignment angle. The center ring shows the first arrival of the front surface center reflection. The six-fold symmetric figure represents the Rayleigh wave and pseudo-Rayleigh wave arrivals, as identified and theoretically predicted [8]. The outer circles show the reflections from the back surface of the crystal plate. Some reveal intriguing details that resemble slowness curves. Similar plots, but with totally different patterns, have been obtained for some other cuts of quartz crystals.

Figure 3. Experimentally scanned waveform v(t) as function of defocus distance z, plotted as two-dimensional v(t,z) plot. C, R, L2, LS, and S2 mark the arrivals of various waves as explained in the text.

Figure 4. Polar plot of experimentally scanned waveform V(t) as function of the transducer alignment meridian angle φ, for a z-cut quartz plate at a defocus distance 11.5 mm. The radial coordinate is time in microseconds. C, R, and R, are center reflected, surface, and pseudo-surface waves.

THEORY

Time domain waveforms of a line-focus transducer probing a liquid/solid interface can be computed by integration of the time-domain Green's function, if the Green's function is known. The integration must be carried out twice, once over the curved transducer as the pressure source, and once as the detector. We have previously developed a computer program to simulate the time domain echo waveform of a line-focus transducer probing, through a liquid,
a thick isotropic solid modeled as semi-infinite half-space [6].

The agreement between theory and experiment is excellent, but the computation is rather time consuming even though we have a closed form formula of the Green’s function. For anisotropic samples of finite dimension, the Green’s function is complicated and must be computed numerically. Further integration of the computed Green’s function over two-dimensional space would require too much time to be practical.

Cylindrical transducers are highly symmetric. The transducer-sample system has a simple frequency domain representation that may be derived from angular spectra. This representation has served as the theoretical basis for predicting and considering it anisotropic of the transducer system.

We start with the \( V(z) \) for a fixed frequency being represented as an integration over the aperture angle as [12]:

\[
V(\phi, z) = \int_0^{\phi_a} R(\phi, \theta) P(\theta)e^{i2\pi k z \cos \theta} \cos \theta d\theta
\]

(1)

Here \( \phi \) is the angle between alignment of the line-focus transducer and some reference \( x \)-axis on the surface of the sample, \( R \) is the reflection coefficient, \( P \) is the pupil function of the transducer, \( k \) is the wave number in the fluid which has a value of \( 2 \pi f c \), \( f \) is the frequency, \( c \) is the wave speed in the fluid, \( \theta \) is the azimuthal angle which is also the integration variable, and \( \phi_a \) is the half aperture angle of the transducer.

Since our transducer does not have a lens, the pupil function can be idealized as being of unit value inside, and zero outside, the aperture. On the other hand, the reflection coefficient function usually is dispersive, and therefore frequency dependent, for plate of finite thickness. By taking into account the implicit frequency dependence of equation (1) and considering it as the transfer function, \( H \), for our line-focus transducer coupled to a sample through a liquid, we can write

\[
H(\phi, \phi, z) = \int_0^{\phi_a} R(\phi, \theta) P(\theta)e^{i2\pi f t \cos \theta} \cos \theta d\theta
\]

(2)

The time domain waveform can then be obtained from the inverse Fourier transform of the convolution of this response function with the complex spectrum of the source waveform of the transducer [8,13]. In equation form, the transducer output voltage, \( v(t, \phi, z) \), can be represented by the following expression:

\[
v(t, \phi, z) = \text{FFT}^{-1}[S(f)H(f, \phi, z)]
\]

(3)

where \( S(f) \) represents the complex spectrum of the source waveform. The spectrum \( S(f) \) can be obtained experimentally by forward Fourier transform of the reflected echo waveform of a thick isotropic material such as glass placed at the focal point of the transducer. For this test configuration, the function \( H \) can be shown to be a constant, characteristic only of the transducer and the electronic pulser/receiver settings.

### NUMERICAL COMPUTATIONS

Once the spectrum \( S(f) \) is determined, the numeric computation of the time domain waveform is rather simple in concept. First the reflectance function \( R \) is computed. The frequency response function \( H \) is then computed by numerical integration of Equation 2. Then a convolution of the source and the transfer functions in the frequency domain, followed by an inverse Fourier transform yields the time waveform \( v(t) \) as shown by Equation 3. The whole numerical computation process is therefore hinged on finding an efficient way to compute the reflectance function \( R \).

For materials modeled as isotropic, a half-space, a plate, or a layered half-space, formulas to compute \( R \) have been given by Brekhovskikh [14]. We give in the appendix the explicit form of the exact reflection and transmission coefficients for a submerged plate. The numerical computation is rather efficient, even though the computation of the reflectance must be carried out frequency by frequency first, then the integration using Equation 2, and inverse Fourier transform by Equation 3. One example of the computed results is shown in Figure 5. Compared with Figure 3, the agreement between theory and experiment is obvious.

For anisotropic materials such as crystals, formulas to compute \( R \) have been derived in many of the references cited above for the \( V(z) \) computation. Additional derivations have been reported [15-17]. In our work, we use the algorithm derived by Tsukahara [17] because it is general and simple. However, because the reflectance is a function of three variables, \( f, \phi, \) and \( \theta \), even for a fixed \( z \), the computation is much more elaborate. Work is still in progress to develop a program using multi-processor parallel computing algorithms.

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**Figure 5.** Numerically computed \( v(t,z) \) two dimensional plot for a fused quartz plate and the experimental conditions which produced the results shown in Figure 3.
CONCLUSIONS

We have developed a line-focus transducer capable of rotational scans and time resolved measurements. In addition to leaky surface waves, reflected and refracted body wave arrivals can be resolved in the time waveforms. Orientation dependent body wave arrivals have been observed in different cuts of quartz crystals. Their interpretation is complex. But since the transducer design is lensless, the theory for predicting the time waveforms can be greatly simplified. For the case of the isotropic plate, agreement between experiment and theory is very good.

APPENDIX

For an isotropic plate of thickness d submerged in a liquid, the reflection coefficient \( R \) and transmission coefficient \( T \) for a plane incident wave of frequency \( f \) and incidence angle \( \theta \), can be computed using formulas derived by many authors. We restate Brekhovskikh's [14] formulation which can be coded easily.

Given \( c \), the wave speed in the fluid, \( c_0 \) the longitudinal wave speed, \( c_s \), the shear wave speed in plate, and \( \rho \), the density of the plate to that of the fluid, we first compute some intermediate parameters, defined by the input parameters, as follows:

\[
\theta_p = \arcsin \left( \frac{c_0 \sin \theta}{c} \right), \\
z = c / \cos \theta, z_p = \rho c / \cos \theta_p, z_s = \rho c_s / \cos \theta_s, \\
k_p = 2 \pi fd / c_p, k_s = 2 \pi fd / c_s, \\
M = \frac{z_p \cos^2(2 \theta_p)}{z} \cot(k_p \cos \theta_p) + \frac{z_s \sin^2(2 \theta_s)}{z} \cot(k_s \cos \theta_s), \\
N = \frac{z_p \cos^2(2 \theta_p)}{z \sin(k_p \cos \theta_p)} + \frac{z_s \sin^2(2 \theta_s)}{z \sin(k_s \cos \theta_s)}.
\]

Finally,

\[
R = \frac{i(N^2 - M^2 - 1)}{2M + i(N^2 - M^2 + 1)}, \quad T = \frac{2N}{2M + i(N^2 - M^2 + 1)}.
\]

Note that when the incident angle is larger than the critical angle for refraction in the solid plate, the argument in the arcsine function is larger than unity. However, the computation requires no special treatment if we use complex arithmetic for all variables.

REFERENCES