Characterization of Loss Mechanisms in Piezoelectric Ceramic Microresonators

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Abstract—Piezoelectric ceramic microresonators possess the very attractive attributes of low cost and high piezoelectric coupling; these make them potentially suitable for high-volume telecommunications applications such as cellular radio/comm-on-the-move. Actuators for microelectromechanical structures (MEMS) applications using bimorph cantilevers are also envisioned using piezoceramics for the same reasons. One problem impeding development is an adequate treatment of loss; it is often the case that devices are designed that do not fulfill their promise because insufficient attention has been addressed to the various dissipation mechanisms involved. For piezoceramics especially, because of the high values attained by the coupling coefficient, it is frequently unjustified to make simplistic assumptions concerning the incorporation and treatment of loss. We explore the incorporation of complex material coefficients into a complex electromechanical coupling coefficient, $k' = k' - jk''$. Resulting from this is a complex resonance frequency of the resonator. We also propose a method for more accurately characterizing lossy ceramic resonators.

INTRODUCTION

Loss (energy dissipation) in piezoelectric ceramic resonators is a topic which, until recently, was only mentioned in brief instances. The reason for this is that high Q (low loss) materials have solely been used in the resonator industry for many years. Recently however, because of their low cost and high electromechanical coupling, ceramics are being utilized. In bulk form, piezoelectric ceramics have been used in ultrasonic transducer applications, while ceramic thin films have very recently become useful in microelectromechanical structures (MEMS).

Fig. 1 shows various piezoelectric materials and how they are presently rated in terms of Q and $k$. High Q materials such as quartz have very low, almost non-existent loss. High $k$ materials such as ceramics are very effective electromechanical transducer materials, however, ceramics intrinsically have high loss. Refractory oxides such as lithium niobate offer both of these qualities, however, like quartz, the processing costs are relatively high compared to those of ceramics. In today's economy, cost is often the determining factor in choosing materials, hence this is the driving force for understanding losses in low-cost ceramic materials.

In this study we will work with complex material coefficients (piezoelectric coefficient, $e'$, permittivity, $\varepsilon^*$, and stiffness, $c'$) of a piezoelectric material (representing the physical losses). The complex material constants will then be incorporated into the electromechanical coupling coefficient and, subsequently, into the input admittance equation for a thickness mode oscillator and the effects on the resonance frequency will be determined.

ORIGINS OF LOSS

The primary contributors to loss in a piezoelectrically vibrating plate are: DC conductivity, $\sigma$; dielectric loss, $\varepsilon''$; acoustic viscosity, $\eta$; and piezoelectric loss, $e''$ [1-4]. All of these loss mechanisms affect the electromechanical coupling coefficient, $k$, hence they are all lumped into the expression for $k$ ($e'$ is a function of $\sigma$ and $c'$ is a function of $\eta$ and $\sigma$, when piezoelectric stiffening is
that alone determines the resonance frequency of the piezoelectric material. It follows, therefore, to extend the definition of \( k \) to the generalized case incorporating loss. In this case, \( k \) is redefined as \( k' = \frac{e^*}{\sqrt{e^* c^*}} \), where the asterisks denote complex quantities and correspond to \( e, E, \) and \( c \) in the lossless case with the addition of imaginary parts.

**Complex Electromechanical Coupling**

The complex electromechanical coupling coefficient, \( k' \), is physically comprised of \( e^* \), \( e^* \), and \( c^* \). Mathematically, it is simply a complex number consisting of real and imaginary parts, \( k' = k' - j k'' \) (\( j = \sqrt{-1} \)) with the imaginary term being a relative measure of loss. To ensure that the material observes energy conservation laws, that is, at most, all of the energy that is input to the material is converted, it is necessary that both \( k' \) and \( k'' \) be positive or zero. This follows from work done by Holland in 1967 on complex material coefficients of piezoelectric ceramics [1].

Fig. 2 represents the values of \( k' \) for all piezoelectric materials. Materials with low loss and low electromechanical conversion effectiveness such as quartz reside in the lower left of the diagram. Ceramics, because of the multitude of materials and compositions available cover a strip slightly off the lossless \( (k''=0) \) axis, with values of \( k' \) ranging from 0 to about 0.65 (even higher with extensive processing such as hot-pressed relaxor-PT systems). Extremely lossy piezoelectric polymers or poorly processed ceramics have values of \( k'' \) that are even higher.

![Fig. 2](image)

Three representative values of \( k' \) are shown in Fig. 2. These are for moderately effective electromechanical energy converters \( (k'=0.50) \) with varying degrees of loss ranging from lossless \( (k'=0.50) \) to very lossy \( (k'=0.50-j0.80) \). These values will be used as examples in the mathematical analysis.

**Mathematical Analysis**

For thickness excitation (TE) of thickness modes in a piezoelectric plate resonator of infinite extent (diameter much greater than thickness dimension), where the driving electric field is parallel to the thickness coordinate, Ballato et al. and Kelly et al. [5-8] have derived the input admittance, \( Y_m \), including loss, as follows:

\[
\frac{Y_m}{\omega_0 C_0} = j \frac{X'}{1 - k'^2 T(X')} \]  \( \text{(1)} \)

where \( T(X') = \tan(X')/X' \), and \( X' = \omega_0 h / v' \). Frequency, \( \omega_0 \), is complex, \( h \) is the plate half-thickness (thickness of the resonator is \( h \)), and \( v' \) is the complex velocity of the acoustic wave in the resonator. The velocity is complex because it is related to the stiffness of the material, which is a lossy material property as discussed above. This equation shows how admittance is dependent on frequency and the electromechanical coupling coefficient, which is a property characteristic of the material.

In this mathematical analysis, we choose particular values for \( k' \) and determine the resonance frequency. The resonance frequency, by definition, is obtained from Eq. (1) when \( X' \) makes the value of admittance infinite. This occurs when the denominator of Eq. (1) is equal to zero. Hence, the following equation is solved for the location of the resonance frequency for a material with a given value of \( k' \):

\[
1 - k'^2 \tan(X')/X' = 0. \]  \( \text{(2)} \)

Because of the transcendental nature of Eq. (2), it can not be solved analytically, rather an iteration method must be utilized to arrive at the proper value of \( X' \). Upon obtaining the correct value of \( X' \) in Eq. (2), determining the resonance frequency is complicated because the variable \( X' \) also contains \( v' \). However, the antiresonance frequency is dependent on geometry and \( v' \) in such a way that multiplying \( X' \) by the antiresonance frequency gives the resonance frequency.
Antiresonance is the frequency at which the input admittance is zero. This is obtained when the tangent function in Eq. (2) takes on infinite values. This is periodic, however we are only concerned with the fundamental antiresonance frequency in this study. Also note that the argument of the tangent function is complex.

Values of \( k' \) were put into Eq. (2) and the resonance frequencies were determined and are shown in a normalized representation in Fig. 3. \( \Omega \) and \( \chi \) are the real and imaginary parts of complex frequency, respectively. Fig. 3 specifically shows the loci of resonance and antiresonance frequencies for all piezoelectric materials, as determined by mapping their electromechanical coupling coefficients to the frequency plane via Eq. (2). The mapping of the three representative values of \( k' \) from Fig. 2 are highlighted.

For lossless materials, represented by the case with \( k'=0.50 \), the resonance and antiresonance frequencies lie along the real frequency axis (\( \chi=0 \)). As loss is increased represented by the magnitude of \( k'' \), the corresponding resonance and antiresonance frequencies move into the complex frequency plane.

The importance of these results is paramount. This shows that in order to obtain a lossy material's true resonance and antiresonance frequencies, the material must be excited with a complex frequency source.

Computer simulations of the admittance and frequency response of lossy piezoelectric ceramics were performed. In Fig. 4 the admittance loci is plotted in three dimensions for a ceramic resonator with \( k'=0.50-j0.10 \). The peak corresponds to the true resonance frequency. It is shown set back into the complex frequency plane as required by the complex nature of the electromechanical coupling coefficient.

Since commercially available impedance analyzers supply only pure sinusoidal excitation, they will not allow one to observe the true resonance frequency of lossy materials. However, we have computer-synthesized some results to demonstrate a possible characterization technique. A lossy ceramic resonator was represented by the admittance in Eq. (1) having \( k'=0.50-j0.10 \). We then applied a complex excitation signal having a constant imaginary component and while sweeping real frequency component, analogous to sweeping the pure sinusoid on commercial analyzers. Fig. 5a shows the admittance when a pure sinusoidal signal is used for excitation. A small, blunt, peak is observed. Presently available characterization techniques would suggest taking the frequency of this maximum as the resonance frequency. However, if we use a complex sinusoidal signal with constant imaginary part (0.17 in the case of this material) and sweep the real frequency component, a sharp resonance peak is observed in the admittance spectrum. This is shown in Fig. 5b.

CONCLUSIONS

Losses in ceramic resonators are represented by complex material coefficients such as \( e', e'', \) and \( c' \). Since these material coefficients comprise the electromechanical coupling coefficient, \( k \), it follows that \( k \) should be a complex material coefficient as well, redefined as \( k' \). The convention is \( k'=k'-jk'' \), where it is necessary that \( k' \) and \( k'' \) are both positive or zero or the material to remain
Fig. 5b. Computer simulation of the admittance spectra of a lossy ceramic resonator, represented by the admittance in Eq. (1), having $k'=0.50-j0.10$. The excitation is with a damped sinusoidal (complex) source.

passive. All piezoelectric materials can be represented by a value of $k'$ where $k'$ corresponds to how well the material converts electrical energy to mechanical energy or vice versa and $k''$ is a measure of loss.

Complex resonance and antiresonance frequencies arise due to the lossy nature of ceramics and other piezoelectric materials. The input admittance is the tool which maps $k'$ to its corresponding resonance and antiresonance frequencies in the complex frequency plane.

Lossy resonators must be stimulated with complex frequency excitation signals to obtain their true resonance and antiresonance frequencies, which are complex. We propose sweeping the complex frequency plane in a manner analogous to the method in which presently available commercial impedance analyzers sweep purely real frequencies. By observing the admittance spectra, the true resonance and antiresonance frequencies could be determined.

Complex resonance and antiresonance frequencies provide insight to new domains in which lossy materials can be utilized.

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