Experimental Comparison of Ultrasonic Techniques to Determine the Nonlinearity Parameter $\beta$

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Abstract. — We describe experiments to evaluate the use of an infrared Michelson interferometer as a quantitative probe of nonlinear ultrasonic propagation. This technique exhibits several distinct advantages over previous methods using capacitive or piezoelectric detectors, including high spatial resolution (typically 10 μm), flat broadband response (up to 300 MHz), and simple self-calibration. The method is evaluated by determining the nonlinearity parameter $|\beta|$ in a fused silica specimen, and comparing the results to those obtained for the same specimen using a capacitive receiver and a piezoelectric transducer. When ultrasonic diffraction effects are included, values for $|\beta|$ are in good agreement with the literature values of $|\beta| = 11-14$. These results indicate that optical methods can be used for practical and accurate detection of nonlinear ultrasound.

I. INTRODUCTION

In this paper, we describe our experimental evaluation of a Michelson interferometer as a method for quantitative nonlinear ultrasonic measurements. Other detection methods for nonlinear ultrasonics, most notably capacitive and piezoelectric receivers, have been in use for several years. However, interferometric detection possesses advantages not available with these methods. The interferometer provides a direct means of absolute amplitude calibration, is noncontacting, possesses a wide bandwidth, requires less extensive sample preparation than the capacitive method, and affords excellent spatial resolution (typically 10 μm).

A common technique for measuring nonlinear ultrasonic behavior uses the phenomenon of harmonic generation. A finite-amplitude toneburst at frequency $\omega_0$ is launched in the specimen. The detected wave contains a component of amplitude $A_1$ at the fundamental frequency $\omega_0$, a component of amplitude $A_2$ at the second-harmonic frequency $2\omega_0$, and so on. It is standard to define the nonlinearity parameter $\beta$ comprising a combination of the second- and third-order elastic constants $C_{ij}$ and $C_{klm}$. For instance, $\beta = 3 + (C_{111}/C_{11})$ for longitudinal waves in an isotropic solid. This implies [1]

$$|\beta| = \frac{8\pi^2 A_2}{\omega_0^2 \varepsilon A_1^2},$$

where $v$ is the ultrasonic phase velocity and $\varepsilon$ is the specimen thickness. Therefore, $|\beta|$ can be experimentally determined by measuring the absolute displacement amplitudes $A_1$ and $A_2$ of the fundamental and second-harmonic displacements in a harmonic generation experiment.

We have determined $|\beta|$ for an isotropic fused silica (SiO$_2$) specimen using three detection methods: the Michelson interferometer, a capacitive receiver, and a piezoelectric transducer. To obtain quantitative agreement between methods, we have found it necessary to include corrections for ultrasonic diffraction effects. When these effects are included, all three methods gave values for $|\beta|$ in good agreement with previously cited values of $|\beta| = 11-14$.

II. INTERFEROMETRIC DETECTION OF NONLINEAR ULTRASOUND

A. Experimental Apparatus and Procedure

A path-stabilized infrared Michelson interferometer [2] was used to measure the ultrasonic displacements. For a voltage waveform $V_{det}(t)$ detected by the differential photodiode circuit, the fundamental and second-harmonic amplitudes $A_1$ and $A_2$ in the bulk are determined by

$$A_1 = \frac{\sqrt{2}\lambda}{8\pi} \left[ \arcsin \left( \frac{2V_{det}(\omega_0)}{V_{pp}} \right) \right]_{\text{rms}},$$

and

$$A_2 = \frac{\sqrt{2}\lambda}{4\pi} \left[ \frac{V_{det}(2\omega_0)}{V_{pp}} \right]_{\text{rms}} \left[ 1 - \left( \frac{4\pi A_1}{\lambda} \right)^2 \right]^{-1}.$$

Here $\lambda = 1064$ nm is the laser wavelength, and $V_{pp}$ is the full-scale interferometer response. The equations indicate that $A_1$ and $A_2$ are determined from the root-mean-squared value of the toneburst amplitude for the portion of $V_{det}$ at $\omega_0$ or $2\omega_0$. The equations are derived from the standard expression for the detected voltage due to a surface displacement $\delta$ [3], using the fact that $\delta = 2A$ and without the usual assumption that $4\pi\delta/\lambda \ll 1$. 

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A harmonic-generation experiment is usually performed in the following manner. First, the calibration is achieved by driving the reference mirror’s piezoelectric control with a low-frequency sine wave. The interferometer consequently passes through multiple fringes, and $V_{\text{det}}(t)$ can be measured from the photodiode output. Next, a relatively large-amplitude toneburst wave (typical displacement 1-10 nm) is launched into the specimen, and the transmitted waveform $V_{\text{det}}(t)$ detected by the interferometer is recorded. Digital notch filtering is used to convert $V_{\text{det}}(t)$ to its components $V_{\text{det}}(\omega_0)$ and $V_{\text{det}}(2\omega_0)$. Then (2) and (3) are used to calculate the displacement amplitudes $A_1$ and $A_2$. The measurements are repeated for a series of amplitudes using stepped attenuators on the toneburst output.

As (1) indicates, $|\beta|$ may be determined by measuring several $(A_1, A_2)$ pairs and then finding the slope of the line $A_2$ vs. $A_1^2$. This "slope-determination method" offers more precision than a single $(A_1, A_2)$ pair. Moreover, the measurement uncertainties $\delta A_1$ and $\delta A_2$ can be included in the calculation of $\delta \beta$ by using a linear least-squares fit which accounts for errors in both $x$ ($A_1^2$) and $y$ ($A_2$) [4].

B. Experimental Results

Figure 1 shows data obtained with the interferometer at a fundamental frequency $f_0=\omega_0/2\pi=10.0$ MHz. The data are for a high-purity fused silica disk $19.15 \pm 0.05$ mm thick. The estimated experimental uncertainty is $\delta A/A = \pm 0.035$.

\[ |A_1(z)| = 2A_1^0 \left| \sin \left( \frac{x}{4} \right) \right|, \quad (4) \]

\[ |A_2(z)| = \frac{A_2^0}{z} \left| \int_0^z \left[ A_1(z-\sigma/2) \right]^2 d\sigma \right|, \quad (5) \]

where $x = ka^2/z$ is determined by the ultrasonic wavevector $k = \omega/v$, the transmit transducer radius $a$ (in this case, $a = 2.42$ mm), and the specimen thickness $z$. $A_1^0$ and $A_2^0$ indicate the plane-wave amplitudes of the fundamental and second-harmonic layers.
second harmonic, respectively. Applying these equations to the experimental results, we obtain a diffraction-corrected value $|\beta_{\text{net}}|' = 11.4 \pm 0.4$.

How does this experimental value compare to other results? Over the last 30 years, several authors have determined $|\beta|$ in fused silica using a variety of techniques [2]; values range from approximately 11 to 14. Therefore, our value of $|\beta_{\text{net}}|' = 11.4 \pm 0.4$ is in good agreement with previously reported values.

III. COMPARISON WITH OTHER METHODS

A. Capacitive Detection Method

As a corroboration of the interferometer results, we also determined $|\beta|$ for the same specimen using capacitive detection methods. In this well-established technique [6], [7], the specimen acts as one plate of a capacitor. Ultrasonic vibrations of the surface create variations in the capacitor gap spacing and hence in the voltage $V_{\text{out}}(t)$ across the capacitor. The displacement amplitude is given by

$$|A(t)| = \frac{d V_{\text{out}}(t)}{2 V_0},$$

where $d$ is the equilibrium gap spacing (usually 1-10 $\mu$m) and $V_0$ is the capacitor bias voltage. The amplitudes $A_1$ and $A_2$ are measured directly (rather than after digital analysis) by inserting analog bandpass filters and amplifiers in the detection electronics.

Calibration involves replacing the capacitive detector with an equivalent circuit and using a high-accuracy function generator to obtain the scaling relation between the capacitor voltage $V_{\text{out}}$ and the voltage $V_{\text{meas}}$ measured at the oscilloscope. The spacing $d$ is determined using a capacitance meter to infer the gap spacing. With this approach, the specimen must be conductive (or have a conductive film applied), and should be optically flat and parallel across the receiver area (typically 1.3 cm$^2$). The estimated uncertainty in amplitude for this technique is $\delta A/A = \pm 0.03$.

Using the same fused silica specimen and a fundamental frequency $f_0 = 5.0$ MHz, the capacitive experiments yield a value of $|\beta_{\text{capac}}| = 12.6 \pm 0.4$. This value falls in the middle of the literature values of $|\beta|$, and is in fair agreement with the interferometric value.

B. Piezoelectric Detection Method

In a third experiment, we determined $|\beta|$ for fused silica using a detection method which incorporates a piezoelectric transducer [8], [9]. In this case, the displacement as a function of frequency $A(\omega)$ is determined from the current $I(\omega)$ across the receiver transducer through the relation

$$|A(\omega)| = |I(\omega)||H(\omega)|,$$

where $H(\omega)$ is a calibration function. The calibration requires an independent pulse-echo experiment in which the current and voltage $I_{\text{in}}(t)$, $I_{\text{out}}(t)$, $V_{\text{in}}(t)$, and $V_{\text{out}}(t)$ into and out of the receiving transducer are measured. Then $|H(\omega)|$ can be determined by [9]

$$|H(\omega)|^2 = \frac{|D(z, \omega)| |V_{\text{in}}(\omega) + I_{\text{in}}(\omega) \left[ \frac{V_{\text{out}}(\omega)}{I_{\text{out}}(\omega)} \right]|}{2 \omega^2 \rho \pi b^2 |I_{\text{out}}(\omega)|},$$

where $\rho$ is the specimen density and $b$ is receiver transducer radius. Diffraction effects $|D(z, \omega)|$ [10] for the round-trip calibration are also included. The estimated measurement uncertainty is $\delta A/A = \pm 0.04$. An experimental value $|\beta_{\text{piezo}}| = 9.4 \pm 0.4 (f_0 = 10.0$ MHz) was obtained using this technique. The result is lower than that obtained with other methods and is somewhat less than the literature values.

C. Diffraction Corrections – Other Methods

Diffraction effects are typically not included in the calculation of $\beta$, because experiments use large transducers and work in the near field. However, since diffraction played a significant role in the interpreting the laser interferometer data, we have investigated possible diffraction corrections for the other detection methods. An expression $|D_\beta|$ which is the total diffraction correction to $\beta$ due to both the fundamental and second harmonic waves has been developed [2]. The correction makes use of theoretical expressions developed by Ingenito and Williams [11] for the velocity potential of the fundamental and second harmonic in the near-field region. Calculation of $|D_\beta|$ involves a numerical integration over the path length $z$ of $D^2(z, \omega)$, in which previously developed expressions are used [10], [12].

For the configurations used here, corrections were less than 10% ($0.9 < |D_\beta| < 1.1$). However, this is comparable in size to other measurement errors. Therefore we have calculated $|D_\beta|$ for the capacitive and piezoelectric experiments, using the following dimensions for the transmitting transducer (radius $a$) and the receiver (radius $b$): capacitive experiment, $a = b = 6.35$ mm; piezoelectric experiment, $a = 6.31$ mm and $b = 3.77$ mm.

IV. DISCUSSION

Table I summarizes our experimental results. The table gives the original or “raw” experimental values $|\beta|$ as well as the values $|\beta|'$ obtained after applying diffraction corrections. With these corrections, $|\beta|'$ determined by the interferometer and the capacitive detector are in excellent agreement with each other as well as with literature values. However, the piezoelectric value for $|\beta|'$ is distinctly lower than the other experimental values, and falls outside the range of previously published values.

Figure 3 shows another way to compare results from all three methods. Whereas the values for $|\beta|'$ in Table I were determined from a least-squares-fit slope to all data points in each experiment, it is also possible to calculate $|\beta|'$ for
TABLE I

<table>
<thead>
<tr>
<th>detection method</th>
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<tbody>
<tr>
<td>interferometer</td>
<td>$</td>
<td>\beta'</td>
<td>$ (raw)</td>
</tr>
<tr>
<td>capacitive</td>
<td>15.0 ± 0.5</td>
<td>0.761</td>
<td>11.4 ± 0.4</td>
</tr>
<tr>
<td>piezoelectric</td>
<td>12.6 ± 0.4</td>
<td>0.950</td>
<td>12.0 ± 0.4</td>
</tr>
<tr>
<td></td>
<td>9.4 ± 0.4</td>
<td>1.080</td>
<td>10.2 ± 0.5</td>
</tr>
</tbody>
</table>

Fig. 3. Comparison of diffraction-corrected experimental results for all three detection methods. Values for $|\beta'|$ are plotted vs. the measured fundamental amplitude $A_1$ for data using capacitive (circles), piezoelectric (triangles), and interferometric (squares) detection methods.

each data point individually. Figure 3 contains the individual values of $|\beta'|$ for data in all three experiments as a function of the measured fundamental amplitude $A_1$. For an ideal material and no experimental uncertainty, the points should fall on a horizontal line with a y-intercept equal to $|\beta'|$. When error bars of the sizes indicated are considered, it is clear that results from the interferometric and capacitive methods actually agree quite well. The main discrepancy occurs at large values of $A_1$, and is responsible for the difference in $|\beta'|_{\text{interf}}$ and $|\beta'|_{\text{capac}}$. Figure 3 also indicates that the piezoelectric results are smaller for most values of $A_1$.

V. SUMMARY

In this paper, we have demonstrated the use of a Michelson interferometer to determine the nonlinearity parameter $\beta$. We validated the method using a fused silica specimen, and compared the results to those obtained with a capacitive receiver and a piezoelectric transducer. With the experimental geometry used for the interferometric experiments, diffraction is a significant effect. When ultrasonic diffraction corrections are applied to the experimental results, $|\beta'|$ is in good agreement with literature values. Our comparison gives us confidence in using the Michelson interferometer as a reliable method to study nonlinear ultrasonic wave propagation.

REFERENCES