A Self-Similar Model for Conduction in the Plasma Erosion Opening Switch

D. MOSHER, J. M. GROSSMANN, P. F. OTTINGER, AND D. G. COLOMBANT

Abstract—The conduction phase of the plasma erosion opening switch (PEOS) is characterized by combining a 1-D fluid model for plasma hydrodynamics, Maxwell’s equations, and a 2-D electron-orbit analysis. A self-similar approximation for the plasma and field variables permits analytic expressions for their space and time variations to be derived. It is shown that a combination of axial MHD compression and magnetic insulation of high-energy electrons emitted from the switch cathode can control the character of switch conduction. The analysis highlights the need to include additional phenomena for accurate fluid modeling of PEOS conduction.

I. INTRODUCTION

URING the past several years, the plasma erosion opening switch (PEOS) has become an important means to compress and condition pulsed power [1]–[9]. A low-density plasma, filling a region of vacuum, the cathode, which is self-similar, is created by the generator and load, shunts the generator current away from the load during switch conduction. At a current level determined by plasma conditions and the coax geometry [10], the plasma ceases to conduct (the switch opens) in a time faster than the current rise time.

The load then experiences a rapid rate of current rise and, through inductive energy stored on the generator side of the switch, multiplication of voltage and power over those deliverable by the generator alone [5].

PEOS experiments in coaxial geometry have been carried out on generators with currents ranging from 100 kA to several megamperes and with switch conduction times ranging from about 30 ns to nearly 1 μs [9], [11]. The detailed results of these experiments are consistent with a switch model [10] in which switch conduction is controlled by electron emission processes at the cathode (inner conductor) and subsequent opening is due to erosion of the cathode sheath when the emission becomes self-magnetically insulated. A circuit model for the switch employing these processes and coupled to a transmission-line code is successful in predicting the details of load waveforms in PEOS experiments [10], [11]. Switch operation is therefore well characterized phenomenologically. However, the details of the coupled plasma and electron dynamics during the conduction phase are still a major theoretical concern.

In this and the next paper [12], the conduction phase is characterized by combining a 1-D fluid model with a 2-D electron-orbit analysis. The fluid equations provide simple axial variations of plasma and field variables by neglecting radial variations. The radial cathode sheath is included in the analysis by imposing a space-charge limit [13] to the radial electron current and by matching the sheath drop to the local plasma potential determined from fluid hydrodynamics. In this paper, a self-similar approximation for the hydrodynamics allows analytic expressions characterizing conduction to be derived. Validity of the self-similar approach and other approximations is tested by the more complete fluid code modeling of [12]. Results of the simple analysis are used to identify the mechanisms for electron conduction in different experimental regimes. Important neglected phenomena are described for future inclusion in more sophisticated analyses.

II. CONDUCTION PHASE PHENOMENOLOGY

The configuration used in most Naval Research Laboratory (NRL) experiments is shown schematically in Fig. 1. Plasma sources outside the coax inject plasma into the interelectrode gap with velocities $v_D$ of about $1 \times 10^7$ cm/s and doubly ionized carbon densities $n_0$ in the $10^{12}$–$10^{13}$ cm$^{-3}$ regime [14]. The best studied configurations have $R_c = 2.5$ cm, $R_a = 5$ cm, and $l$ (as measured by the length of the transmitting region of the anode) varying from about 5 to 30 cm in Gamble I and POP experiments [11]. Larger diameter coaxial lines have been used in higher power Gamble II experiments at NRL [15], and Blackjack 5 experiments at Maxwell Laboratories [7]. PEOS experiments are conducted in a biconic feed on PBFA II at Sandia National Laboratories [6] and in a linear strip line at Cornell [16]. However, only the Gamble I [17] and POP [18], [19] experiments have a large database of magnetic field measurements during conduction. In these experiments, arrays of magnetic probes are used to map the spatial and temporal distribution of magnetic field in the plasma.

A representative history of magnetic field penetration axially through the plasma at a point midway between the electrodes is shown in Fig. 2. Measurements show that current is conducted radially, and is distributed in an axially broad channel of width $u_c(t)$ between the $B$-field front at $z_f(t)$ and a rapid decrease of current density at $z_c(t)$. The front penetrates the plasma with a character-

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istic velocity \(v_f\) [17]. The penetration velocity is nearly constant when the generator current rises linearly and slows as the current approaches maximum. The front has also been observed to stop (\(v_f = 0\)) during the decreasing portion of the current pulse in switches configured for long conduction time. A most important observation is that the switch begins to open (current and/or voltage appear at the load) only after the \(B\)-field front reaches the load-side plasma–vacuum boundary at \(z = l\) and \(t = \tau\). Before this time, the switch therefore acts like a perfect short circuit. The time \(\tau\) is called the conduction time, and \(I(\tau) = I_s\) is the current flowing in the switch just prior to opening.

The above behavior can be explained by postulating that the magnetic field penetrates at a rate determined by space-charge-limited (scl) emission of electrons [13] from the plasma-covered switch cathode. As the current increases, a larger area of the cathode emits to accommodate the scl limit, so that

\[
I(t) = 2\pi R_c v_f(t) \cdot j_{bp0} \tag{1}
\]

where

\[
j_{bp0} = Z n_0 v_D (M/Zm)^{1/2} \tag{2}
\]

In these equations, \(j_{bp0}\) is the bipolar scl current density corresponding to the ion flux injected into the cathode sheath, \(Z\) and \(M\) are the ion charge state and mass, and \(m\) is the electron mass. The ratio \(M/Zm\) is \(1 \times 10^4\) for the doubly ionized carbon produced in experiments. For the linearly rising portion of the current pulse, the constant characteristic front velocity is therefore determined from

\[
\dot{j} = 2\pi R_c v_f j_{bp0} \tag{3}
\]

and the peak switch current is

\[
I_t = 2\pi R_c j_{bp0}. \tag{4}
\]

These basic relations have been confirmed in experiments with systematic \(n_0\) and \(l\) variations [11]. Some parameters for PEOS experiments are shown in Table I.

Although these scaling relations assume that the entire cathode region \(0 \leq z \leq z_f\) emits electrons that contribute to the current flow, the \(B\)-field measurements show that the current channel width \(u_c = z_f - z_c\) is narrower than \(z_f\) and that current densities in the plasma are correspondingly higher than \(j_{bp0}\). Typically, \(u_c = l/2\) and \(j = 2j_{bp0}\) late in the conduction phase, though both larger and smaller compressions of the current channel have been observed.

A partial explanation for current-channel narrowing that preserves (1) is provided by axial hydrodynamic compression of the plasma due to \(j \times B\) forces. As the generator-side vacuum–plasma interface at \(z_b\) accelerates from zero, the portion of the cathode with \(z < z_b\) no longer emits since it faces a vacuum. For the emitting region \(z_b \leq z \leq z_f\)

\[
I(t) \sim \int_{z_b}^{z_f} j_{bp} dz - \int_{z_b}^{z_f} n(z) dz = n_0 z_f (t) \tag{5}
\]

where \(j_{bp}\) is the generalization of (2) for variable ion density \(n(z)\). The equality holds by conservation of particle number: all ions swept by the front (those in \(0 \leq z \leq z_f\)) must appear in the density profile behind the front. The total current is therefore the same as if the plasma did not move, although current densities are higher for the hydrodynamically compressed plasma.

This mechanism is believed to narrow the current channel width in experiments such as POP where the long conduction times permit strong hydrodynamic compression to evolve. However, it cannot explain channel narrowing in long-switch Gamble I experiments where no significant hydromotion develops during the short conduction times. A major result of this paper and the next [12] is to show that in such cases, electrons emitted in the region \(z < z_c\) are magnetically insulated in the plasma above the cathode. Insulated electrons flow axially within the plasma.

<table>
<thead>
<tr>
<th>Table I</th>
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<tbody>
<tr>
<td>(R_c (\text{cm}))</td>
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</tr>
<tr>
<td>Gamble I</td>
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towards the $B$-field front until they enter the conduction channel so that the current scaling of (1) is again preserved.

For most experiments, both the hydrocompression and magnetic-insulation mechanisms play a role in determining the width of the current channel. Here, it is desired to calculate most simply how these two mechanisms determine conduction characteristics in different regimes of switch operation. To do this, the following approximations are made:

1) A 1-D slab geometry is employed for the plasma hydrodynamics.
2) The front velocity $v_f$ is constant.
3) Fluid and field variables evolve self-similarly in the channel; they depend on position and time only through $u = v_f t - z$, the distance behind the front.
4) The plasma is quasi-neutral.
5) Ion and electron fluids are cold.

Assumption 1 is supported by the similar evolutions of particle-in-cell (PIC) simulations [20] in slab and coaxial geometries. Assumptions 2–4 are relaxed, and their validity is examined in the next paper. Assumption 2 is shown to be correct when the generator current rises linearly. Assumption 3 yields good representations of fluid-code evolution in all cases studied. Assumption 4 is correct for experimental conditions, but not for the parameter regime of PIC simulations. Assumption 5 will be shown below to be usually incorrect for the electron fluid.

III. A SELF-SIMILAR SOLUTION

In electromagnetic CGS units, the experimental magnetic field evolution for a rising current can be approximated by

\[
B_y = 0, \quad u = v_f t - z \leq 0 \\
B_y = 4\pi j u, \quad 0 \leq u \leq u_c(t) \\
B_y = 4\pi j u(t) = \frac{2I(t)}{R}, \quad u \geq u_c(t) \tag{6}
\]

where $R$ is the average radius within the coaxial gap, $j$ is in the $x$ direction and is, for the present, treated as constant, and $v_f$ is constant. The $x$-$y$ plane approximates the experimental $r$-$\theta$ geometry. All field and plasma variables in the current channel are assumed to depend only on $u$. Within the current channel, Maxwell’s induction equation applied to (6) yields the induced “radial” electric field

\[
E_z(u) = v_f B_y \tag{7}
\]

so that electrons in the channel $E \times B$ drift axially with the front velocity $v_f$. This axial drift can create a space-charge separation from the slower moving ions that produces an axial electrostatic field

\[
E_z = \frac{-\partial \phi}{\partial z} = \frac{\partial \phi}{\partial u}. \tag{8}
\]

The “radial” $E \times B$ drift produced by this axial electrostatic field drives the conduction current in the channel.

Experiments show that, within limits of detectability, no voltage appears at the load during switch conduction: The switch cathode and anode electrodes are both at ground potential. The plasma potential $\phi(u)$ must therefore be dropped across both radial electrode sheaths. In the present simple treatment, the sheaths that bound the plasma are not resolved in $x$, but they do provide boundary conditions for electron emission. Emitted electrons are accelerated across the sheath potential $\phi(u)$ and are injected into the plasma with high kinetic energy. The plasma potential variation is determined from the requirement that the conduction current derives from the radial $E \times B$ drift:

\[
j = Z e n(u) E_z/B_y. \tag{9}
\]

For current in the positive sense, $j$ and $B_y$ are negative, $E_z$ is positive, and potential increases with $u$ in the current channel. The analysis permits $j$ to be a function of $u$. Since $j = 0$ for $u > u_c$, $\phi = \phi(u_c)$ satisfies (9) behind the channel. A potential structure fitting the above arguments is shown schematically in Fig. 3 with the slab-derived picture imposed on the experimental coaxial geometry.

The axial plasma density distribution during conduction differs from the initial uniform distribution because ions accelerate towards the load in the axial electric field. The ion density and axial flow velocity $V_i$ follow from mass and momentum conservation:

\[
\frac{\partial n}{\partial t} + \frac{\partial}{\partial z} (nV_z) = 0 \tag{10}
\]

\[
M n (\frac{\partial V_z}{\partial t} + V_z \frac{\partial V_z}{\partial z}) = Z e n E_z = j B_y. \tag{11}
\]

The $V_i B_y$ term is ignored in (11) since the ion gyroradius associated with the drift velocity is large compared to the interelectrode gap. This is equivalent to ignoring the ion contribution to the radial current during conduction. Pressure is ignored. The right-hand side of (11) follows from (9).

For $j$ a function of $u$, (6) is replaced by

\[
\frac{\partial B_y}{\partial z} = \frac{dB_y}{du} = 4\pi j(u). \tag{12}
\]

The displacement current, down by a factor of $(v_f/c)^2$ from the $B$-field term, has been ignored.

With (12), (9)–(11) can be solved exactly by noting that $\partial/\partial t = v_f (d/du)$ for self-similar variations, and requiring $\phi = 0$, $V_z = 0$, $n = n_0$, and $B_y = 0$ for $u \leq 0$. The solution set is

\[
n = \frac{v_f}{v_f - V_z} = \frac{1}{1 - (B_y/B_0)^2} \tag{13}
\]

\[
V_z/v_f = 1 - \sqrt{1 - \frac{2Z e\phi}{Me v_f^2}} = (B_y/B_0)^2 \tag{14}
\]
Hydrodynamic compression becomes strong as $B_y$ approaches $B_0$. In such cases, the plasma potential approaches $MV_f^2/2Ze$. For the largest $v_f$ values encountered in experiments, the plasma potential can be several megavolts. Electrons emitted from the cathode and accelerated across the sheath are then relativistic in the plasma.

The solution fails for $B_y > B_0$ because there is no mechanism to limit the density gradient for strongly developed hydrodynamics. In reality, a number of physical effects will keep $n(u)$ finite. For example, $n$ should not vary greatly over an electron gyroradius $\rho_e$, which can be a substantial fraction of the channel width. Given such a mechanism to limit density, the solution can, in principle, be extended to $B_y > B_0$. Adding an electron pressure term to (11) can provide finite gyroradius corrections to the density gradient.

Explicit variations with $u$ require specification of $j(u)$. Here, following the experimental approximation of (6), $j = \text{const}$ is assumed. In that case, the variation with $u$ in (13)–(15) is given by

$$U = \frac{u}{u_H} = \frac{B_y}{B_0}$$

where

$$u_H = \frac{Mn_0 v_f^2}{2\pi j^2}$$

and $j$ is determined from experiment. The hydrodynamic compression length $u_H$ varies from about 1 cm in POP to about 50 cm in long-switch Gamble I experiments. For $j = j_{Bo}$, $u_H \sim n^{1/2} v_F / v_D$. For most experiments, $n^{1/2}$ and $v_F$ vary by about a factor of three while $v_D$ varies by nearly two orders of magnitude. The penetration velocity is therefore the major indicator of hydroevolution; larger $v_f$ implies larger $u_H$ so that hydrodynamic motion is reduced.

If magnetic insulation is assumed to play no role, $j(u)$ will be given by $j_{Bo}$. The self-similar development of hydrodynamics for that case is presented in the next paper [12]. Since the backs of the plasma and current channel are coincident if magnetic insulation is ignored, $u_c$ is defined by the hydrodynamics. The $j = \text{const}$ solution, however, gives no information on $u_c$. Its value must be determined by experiment or analysis of magnetically insulated electron flow (Section IV).

Once $u_c$ is determined, conservation of mass can be used to specify the motion of the plasma back. Behind the $B$-field front, this can be written for $j \text{const}$ as

$$\int_0^{u_c} n_0 \frac{du}{1 - (u/u_H)^3} + \frac{n_0 (u_b - u_c)}{1 - (u_c/u_H)^3} = n_0 v_f t$$

where (13) and (17) determine the density distribution in the current channel and the constant density $n(u_c)$ follows from $\phi = \phi(u_c)$ in the region behind the channel. Solving for $u_b$ leads to

$$U_b = \frac{u_b}{u_H} = U_c + (1 - U_c^2)$$

$$\frac{v_f t}{u_H} \cdot \frac{1}{2} \ln \left( \frac{1 + U_c}{1 - U_c} \right)$$

where $U_c = u_c / u_H < 1$ for finite density.

When $U_b$ is close to $U_c$, the analysis of the next paper [12] is preferred over (20). For the interesting case of $U_c$ small, either because $u_H$ is much larger than the switch length or magnetic insulation produces a narrow current channel, (20) reduces to

$$z_b = z_c \cdot U_c^2$$

so that the plasma back moves only slightly as the magnetic field penetrates the plasma.

IV. TEST ELECTRON ORBITS

Here, it is desired to calculate two-dimensional electron orbits in the field environment determined in the last section to estimate how magnetic insulation reduces the current channel width. Electron orbits in the $x-z$ plane can be calculated for the electric and magnetic field variations determined above. The plasma fields do not vary in $x$ so that canonical momentum is conserved in that direction. The fields depend on time only through $u$ so that energy is also conserved in the moving $x-u$ frame. Therefore,

$$\left( 1 + P_x^2 + P_z^2 \right)^{1/2} - \Phi = \text{constant}$$

$$P_x = eA_x / mc = \text{constant}$$

where $P = p / mc$, $\Phi = e\phi / mc^2$, and $B_y = -dA_x / du$.

The constants of the motion are evaluated by considering electrons emitted from the cathode at $u = u_0$ with zero rest frame velocity to be accelerated across the sheath po-
tential $\Phi(u_0)$ and injected normally into the plasma. Then, for nonrelativistic $v_f$, constant $j$, and $u \leq u_c$, 

$$P_x^2 + P_z^2 = 2\Phi(U) + \Phi^2(U) + (v_f/c)^2$$  \hspace{1cm} (24)  

$$P_x = \left[2\Phi(U_0) + \Phi^2(U_0)\right]^{1/2}$$  

$$+ \beta(M/Zm)^{1/2}(v_f/c)(U - U_0^2)$$  \hspace{1cm} (25)  

where $\beta = v_f/jv_D$, and from (15) and (17) 

$$\Phi(U) = (M/Zm)(v_f/c)^2(U^2 - \frac{1}{4}U^4).$$  \hspace{1cm} (26)  

The quantity $J$, the ratio of $j$ to $j_{ph}$, is the current multiplication factor due to hydrocompression and magnetic insulation. In the constant field region behind the channel, $U \geq U_c$, the corresponding equations are 

$$P_x^2 + P_z^2 = 2\Phi(U_c) + \Phi^2(U_c) + (v_f/c)^2$$  \hspace{1cm} (27)  

$$P_x = \left[2\Phi(U_c) + \Phi^2(U_c)\right]^{1/2}$$  

$$+ 2U_c\beta(M/Zm)^{1/2}(v_f/c)(U - U_0).$$  \hspace{1cm} (28)  

The orbit equations are characterized by the two dimensionless parameters $v_f/c$ and $\beta$. Relativistic corrections are important only when $\Phi \approx 1$, i.e., when $U_c$ is not small and $v_f/c \geq (2m/M)^{1/2} = 0.01$. The ratio of the electron gyroradius to $u_0$ is of order $\beta^{-1}$. Measured values of $\beta$ vary from about 3 in PEP experiments to about 50 in long-switch Gamble I experiments. Equations (24)–(28) are integrated over time for electrons emitted from the cathode sheath into the plasma.

Magnetically insulated electrons emitted from the cathode sheath return to it during the next cyclotron orbit. Such electrons are assumed to be specularly reflected within the repulsive sheath. In other words, $P_x$ changes sign and $P_z$ is unchanged when the orbit crosses $x = 0$. Fig. 4(a) and (b) shows electron orbits in both the lab $(x - z)$ and moving $(x - u)$ frames for $v_f/c = 0.01$, $\beta = 8$, and the assumption that the current fills the plasma ($U_c = U_0$). Electrons are all emitted at the time the magnetic field has penetrated to the axial location shown. As time progresses and the front moves through the plasma, the emitting region $0 \leq U \leq U_0$ grows. Thus, at earlier times when $U_c = 0.2$, the electron with orbit 3 is emitted at the back of the plasma and orbits starting at larger $U$ values do not yet exist. 

Orbit 1 is emitted just as the $B$-field front passes. Electrons emitted at the front are accelerated into the channel by the $E_x$ field before reflection towards the front by the magnetic field. In the lab frame, these electrons are thrown forward with velocity $2v_f$ and do not contribute to radial current conduction. Orbit 2 is emitted into a nearly force-free orbit similar to those observed in PIC simulations. Orbits 3 and 4 drift “radially” while executing cyclotron motion. Orbits 2–4 maintain fixed excursions in $u$ over many periods by drifting axially with the front velocity $v_f$. Orbits 5 and 6 correspond to magnetically insulated electrons that “bounce” forward off the cathode sheath until their radial drift is sufficient to clear it. Once clear, they contribute to the radial flow in the current channel. Orbits for the extreme experimental $\beta$ values of 32 and 3 are shown in Fig. 5 for $v_f/c = 0.01$. 

Fig. 6 summarizes results of many orbit calculations with a range of $\beta$ values under the conditions described above. The back of the plasma is determined from $U_b = B_y/B_0$ with $B_0 = 2I(t)/R$. Fig. 6(a) holds for $v_f/c \leq 0.01$. When $v_f/c = 0.03$, the value of $U_c$ is shown to increase because of relativistic corrections. The dashed curves represent a lower limit to $U_c$ described below. The reduction of channel width suggested by the illustrated magnetically insulated orbits is inconsistent with the assumption that the radial current fills the channel behind the front. Regions that carry no radial current should have a constant magnetic field in the plasma and no axial electric field. The channel widths predicted by the solid curves in Fig. 6 should therefore be considered upper limits since the data assume a radial $E_x \times B_y$ drift in the magnetically insulated region.

To remedy the inconsistency between the assumed fields and the insulated electron orbits, $U_c$ is reduced until it coincides with the channel width determined from orbits. In the region $U_c < U < 1$, orbits are calculated using (27) and (28). Results are shown in Fig. 7 for the conditions of Fig. 4 except $U_c = 0.31$. It is found that the value of $U_c$ consistent with orbits corresponds to the initial position of an electron which grazes the cathode surface at $x = 0$ during the first cyclotron orbit. This orbit (close to orbit 4 in Fig. 4) has been determined numerically for a range of $v_f/c$ and $\beta$ values, and results are summarized by the dashed curves in Fig. 6. The values of $U_c$ determined from grazing orbits represent lower limits to the
Fig. 5. Electron orbits in the moving frame for \(v_f/c = 0.01\): (a) \(\beta = 32\), and (b) \(\beta = 3\).

Fig. 6. Estimates of the maximum and minimum channel widths versus \(\beta\) for (a) \(v_f/c = 0.01\) and (b) \(v_f/c = 0.03\).

channel width since they are defined by electrons that are never insulated. In the absence of other important physical effects, one expects the true channel width to be close to this lower limit.

Within the two limits, it can be generally stated that magnetic insulation is important (unimportant) for \(\beta\) large (small). The value of \(v_f\) primarily determines the magnitude of \(\beta\). Since large (small) \(v_f\) also indicates that \(U_H\) is large (small), magnetic insulation can contract the current channel when hydrodynamics cannot, and vice versa.

In two cases, large values of \(v_D\) with moderate \(v_f\) values lead to small values of \(\beta\). This occurs in the IBOS experiments at Cornell [16], where a high-energy ion beam replaces the drifting plasma in the switch, and in PIC simulations [10], where parameter rescaling is required for numerical reasons. The analysis predicts hydrocompression to a narrow channel and weakly magnetized electron orbits (orbits 2 and 3 of Fig. 4(b)) for these cases.

V. DISCUSSION

Evolution of the penetrating magnetic field during PEOS conduction has been measured in numerous Gamble I and POP experiments. In order to conform with approximations made in the analysis, data for which \(v_f\) was measured to be constant for the full conduction period are examined. These data were obtained on low-density shots with short conduction times, so that current rose linearly. The experimental channel widths for this subset of Gamble I and POP data are narrower than observed with longer conduction times. Three such examples are summarized in Table II.

In Table II, the first five entries were measured in the experiment with \(v_D\) determined from plasma source timing and \(v_f\), \(B_0\), and \(u_e\), determined from probe measurements to within about 20-percent accuracy. The plasma length is larger than the anode mask to account for expansion in the interelectrode gap. The quantity \(J_{b,0}\) derives from (3) with \(n_0\) then coming from (2). Equations (16) and (17) are used to calculate \(B_0\) and \(U_H\). With these numbers in hand, the measured channel width can be compared to the two limits derived in the analysis.

The analysis leading to Fig. 6 is used to estimate the theoretical limits to \(U_e\). Note that the values of \(U_e\) (max) are not consistent with \(B_0/B_0\) for the reasons discussed in the last section. Upper limits compare reasonably well with the experimental value for the two Gamble I shots. The lower limits are always much less than the experimental values. The experimental value of \(U_e\) is used in (20) to determine the location of the plasma back for the
two Gamble I shots. Entries are missing for POP since the analysis fails for $B_s/B_0 > 1$ or $u_c > 1$.

The value of $z_b$ for the long-switch Gamble I shot indicates only a 10-percent hydrodynamic compression of plasma length during conduction. The observed width of the conduction channel is therefore associated with magnetic insulation of electron flow. Though the short-switch Gamble I shot shows compression of one third of the original length, hydromotion may still be insufficient to account for the measured channel width.

The POP example represents an opposite extreme. There, the measured channel width is double the calculated value of $u_p$. This is a problem characteristic of POP experiments: The observed channel widths are usually larger than the calculated plasma width behind the front. In large part, this difficulty occurs because the conduction time exceeds the transit time of an ion between electrodes. Axial ion momentum gained in the $E_z$ field is lost to the cathode and new ions entering the plasma must be accelerated from zero axial velocity. This momentum loss slows the hydromotion acceleration.

Only the upper limits of $U_c$ compare reasonably well with the measured channel widths. In addition, data for longer conduction times indicate even wider conduction channels. We therefore seek neglected phenomena which can increase the predicted widths. Several limitations of the above analysis are relaxed in the next paper and somewhat larger upper limits to channel width are calculated. Also, electron scattering and energy loss due to instabilities near the cathode are shown there to broaden channels limited by magnetic insulation. Anomalous resistivity in the plasma body has a similar effect. Other important effects are missing in both analyses but should be included in future modeling of the PEOS conduction phase. A few of these neglected phenomena are now briefly discussed.

### A. Electron Pressure

The orbit plots show that much of the electron kinetic energy is associated with cyclotron, rather than drift, motion. Since the kinetic energy of cathode-emitted electrons is $\phi$ in the plasma and orbit crossing is common, the electron pressure is of order $n\phi$ and is never negligible in an electron fluid treatment. Denoting the relevant pressure tensor component as $\Pi_{uu}$ leads to a modification of (13)–(15) in which

$$\left( \frac{B_y}{B_0} \right)^2 \left( \frac{B_y}{B_0} \right)^2 + \Pi_{uu} = \frac{B_0^2}{8\pi}. \quad (29)$$

It is difficult to determine $\Pi_{uu}$ for the coupled finite-gyroradius dynamics of cathode-emitted electrons and the plasma. However, the electric potential and pressure terms are of comparable magnitude in the equations of motion so that the density, velocity, and field distributions, along with electron orbits can be very different from those calculated in the cold fluid approximation. Including electron pressure prevents nonphysical density compression, provides smoothing over the electron gyroradius scale length, and allows for diamagnetic currents driven by the density gradient.

Since electron pressure should not be ignored in a 1-D fluid treatment of the plasma body for which the gyroradius is typically a fraction of the channel width, it must also be included in 2-D fluid treatments which resolve the smaller scale radial sheath. Determining reasonable forms for the electron pressure tensor in such cases is a challenging task but is necessary for physically correct fluid modeling. The ion contribution to pressure can usually be ignored since their gyroradii exceed system dimensions and their initial temperature is low.

### B. Magnetic Field in the Insulated Region

In the insulated region, a portion of the current is carried axially by electron flow in the plasma above the cathode rather than in the metal. The average magnetic field experienced by an insulated electron is therefore less than that in the plasma above the flow. The thickness of the insulated electron layer is therefore larger than that in the presented orbits. Thus, electrons can enter the current channel at larger $x$ values than shown. This may lead to larger values of $u_c$.

To estimate the size of the correction, consider the flow of current at $u_c$ in the case of small hydrocompression. The current flowing in the metal at $u_c$ is that which is field emitted in the current channel ($-j_{hp0}u_c$). The total current flowing is proportional to $j_{hp0} = v_t \tau_f$ so that the ratio of magnetic field in the plasma body to that at the cathode is about $v_t \tau_f / u_c$. Factor-of-2 increases in insulated layer thickness may then be expected near the end of the conduction phase.

### C. Sheath Effects

Our analysis assumes that the current density emitted by the cathode is given by $j_{hp}$. For a 1-D, space-charge-
limited sheath with incident ion current density $Zen_{dp}$, a more correct expression is

$$j_{sc1} = j_{bp} \left( \sqrt{1 + F - 1} \right)$$

$$F = \frac{2Ze\phi}{MV^2}.$$ (30)

The emitted current density is $j_{bp}$ when $F$ is large but deviates significantly from this value when $F \leq 100$. From (15) for $B_1 = B_0$, $F \approx \left( \frac{v_f}{v_d} \right)^2$ so that a reduction in emission is indicated for long-conduction-time experiments such as POP.

Fig. 8 shows evidence for this reduced emission [19]. The delay time $t_d$ specifies the plasma conditions in the gap before current begins to flow: As $t_d$ increases, $n_0$ increases and $v_d$ decreases. The solid line is a fit to the Gamble I data for which $j_{sc1} = j_{bp}$ is assumed to hold since $F \gg 100$. Gamble I points are labeled according to the length of the anode mask. The dashed line is a reduction of this fit by the bracketed factor in (30) with $F = \left( \frac{v_f}{v_d} \right)^2$ calculated for each of the POP data points. These values of $F$ are the maximum possible since the maximum potential is assumed. The POP data should therefore be, and is, near or below the dashed line. Note that reduced emission is expected when hydrodynamic bunching is extreme and magnetic insulation is weak.

VI. SUMMARY

A simple description of PEOS conduction based on 1-D axial hydrodynamics and 2-D electron orbit analysis has been developed. In this model, the magnetic field penetrates the plasma towards the load at a rate determined by space-charge-limited electron emission from the switch cathode. The radial electric field induced by the time-varying magnetic field causes electrons to $E \times B$ drift axially. This drift creates a space charge which produces an axial electrostatic field in the plasma body. For some experiments, plasma potentials in the megavolt regime are calculated. Since no voltage appears at the load during switch conduction, the plasma potential is dropped across both electrodes in radial sheaths. Electrons emitted from the cathode are accelerated to high energy in the sheath and $E \times B$ drift to the anode in the axial electrostatic field. This high-energy electron flow carries the switch current and provides charge neutralization in the plasma. The width of the conduction channel is determined by a combination of hydrodynamic compression of the plasma and magnetic insulation of electron flow on the generator side of the cathode. Thus, the magnetic insulation opening mechanism may be well established in a portion of the switch before the switch begins to open.

The relative importance of magnetic insulation and hydrodynamics to conduction is determined by $v_f/v_d$. In short-conduction-time long-switch Gamble I experiments ($v_f/v_d$ large), hydrodynamics is calculated to play a negligible role for the observed channel widths. In the long-conduction-time POP experiments ($v_f/v_d$ small), hydro-compression alone is more than adequate to explain channel narrowing. Both mechanisms should be important in the higher power pulse line experiments.

Phenomena important to PEOS conduction but neglected in this analysis may be important for accurate fluid modeling. Several, including anomalous collisions due to instabilities, are described in the next paper. It is argued that electron pressure is important in fluid treatments though difficult to include correctly. The high-energy electron orbits predicted by the analysis indicate that the electron pressure terms in the fluid equations of motion are comparable in magnitude to the electric field terms and should not be neglected. It is hoped to include appropriate electron pressure terms in future fluid modeling of PEOS conduction.

REFERENCES

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