DOUBLE BEAT-WAVE PARTICLE ACCELERATION

Paul L. Csonka*
Brookhaven National Laboratory
Upton, N.Y. 11973

Abstract

Described are two mechanisms which make it possible for an accelerated charged particle to stay in phase indefinitely with a moving accelerating potential well set up by electromagnetic radiation in a plasma beat-wave accelerator. 1) Double beat-wave mechanism, in which interference between two beat-waves cancel a) the fields in the interfering electromagnetic waves, b) the electron oscillations which would otherwise be set up by those waves. 2) Alternating density mechanism, in which one or each of two beat-waves set up resonant electron oscillations in alternate plasma regions, but not in others.

In a plasma beat-wave accelerator, charged particles are accelerated by an electrostatic potential well set up in the plasma by a ponderomotive force generated by laser light. As seen from the laboratory, the potential well moves with a speed

\[ v = c (1 - \omega_p^2 / \omega_i^2)^{1/2}, \]

where \( \omega_p \) is the plasma frequency, and \( \omega_i \) is the frequency of the laser. Particles which acquire a speed \( v > v_g \), will sooner or later overtake the potential well, and subsequently those particles will no longer be accelerated, or, indeed may even be decelerated by the potential well. To acquire higher energy, the particles would have to be accelerated not by a single, but by a sequence of several appropriately phased plasma beat-wave accelerators.

This limitation can be overcome by the proposed surfatron mechanism, in which the particle moves along a line which makes an angle \( \theta \neq 0 \) with the direction of motion of the potential well. Choosing \( \theta \) appropriately, one can insure that for any given \( v_g \) and \( v > v_g \), the particle and potential well will stay in phase indefinitely. Two difficulties arise in connection with the surfatron mechanism. First, when \( \theta \neq 0 \), the potential well will exert not only a longitudinal force on the particle, but a transverse force as well. That force will tend to curve the particle trajectory. To overcome such curving, one has to impose a magnetic field perpendicular to both \( \vec{V} \) and \( \vec{v} \). Second, since \( \vec{V} \) has a component (\( \text{vain} \)) parallel to the wave front of the potential well, the transverse dimension of that well has to be correspondingly larger (which in turn requires more space, and, in addition, also causes some of the laser energy to be wasted). This second difficulty can be significantly alleviated by the ingenious device of "transverse optical mixing.,"

In the present paper two alternatives to the surfatron mechanism will be outlined. Both of these insure that the particle will continue to be accelerated indefinitely, in principle to arbitrary high energies. In both mechanisms the particles and potential wall may move parallel to each other without dephasing, and in neither mechanism is an externally imposed magnetic field required.

With the mechanisms here proposed it is possible to avoid multi-staging, which would result from dephasing between accelerated particles and the accelerating wave. Multi-staging may also be required because of pump depletion; the latter is a consequence of insufficient radiated energy, and that difficulty cannot, of course, be cured by the methods described in this paper.

Double Beat-Wave Mechanism

The underlying idea here is to create not one, but two beat-waves which will tend to interfere constructively whenever the particle is in phase with the potential well generated by both of them. On the other hand, in those regions where the particle would be out of phase with the potential well created by one of the beat-waves, the second beat-wave should tend to cancel the effect of the first one, thereby decreasing any decelerating potential. The particle would then simply drift through these regions with little energy loss, until it reaches the next region where the two beat-waves are again in phase, an accelerating potential well is again present, and further acceleration of the particle will take place, etc.

To realize the idea just outlined, let four electromagnetic plane waves (e.g. generated by a laser or lasers) have circular frequencies \( \omega_1, \omega_2, \omega_3, \omega_4 \) respectively, and equal amplitudes: \( A_1 = A_2 = A_3 = A_4 \). Let all four waves travel in a plasma along the \( z \) direction. The electric field in all waves is polarized along the \( x \) axis, and will be written as

\[ E_i(t,z) = \Re(A_i \exp(\text{i} \omega_i t - k_i z + \phi_i)) \]

\( i = 1, 2, 3, 4 \).

The electron density in the plasma is \( n_e \), and the plasma frequency is \( \omega_p \). The \( \omega_i \) \( i = 1, 2, 3, 4 \) are chosen so that

\[ \omega_1 - \omega_2 = \omega_p, \]

\[ \omega_3 - \omega_4 = \omega_p, \]

where \( \Delta \omega \) is to be specified later. As a result of this choice, the first and second waves together will then generate a beat-wave with beat frequency \( \omega_b \). The third and fourth waves will generate another beat-wave, also with beat frequency \( \omega_b \), but with different phase velocities and wave numbers.

Set all \( \phi_i = 0, \) \( i = 1, 2, 3, 4 \). Then at time \( t = 0, \) at \( z = 0 \) the electric field in all four waves reaches its maximum value, all four waves are in phase, and the beat-wave amplitudes of both beat-waves also go through their respective maximum.

To investigate the time and space dependence of the resultant electric field generated by the four waves, note that the group velocity, \( v_g \) of the \( i \)th electromagnetic wave approaches \( c \), if \( \omega_i^2 / \omega_p^2 \rightarrow 0, \) \( i = 1, 2, 3, 4 \). Since the phase velocity of the electrostatic potential well created by an electromagnetic wave pulse in a plasma is \( c l - \omega_p \) the group velocity of the electromagnetic wave, and since the beat-wave accelerator is designed to accelerate high energy particles, which travel with a velocity close

*Permanent address: Institute of Theoretical Science, University of Oregon, Eugene, Oregon 97403.

to c, it is desirable to choose \( v_{gi} \) close to c, i.e., to choose

\[
\omega_p / \omega_i \ll 1 \quad i = 1, 2, 3, 4 . \tag{4}
\]

If inequality (4) holds, then to a good approximation

\[
\begin{align*}
\nu_g,1 \quad & \sim \nu_g,2 \quad \nu_g \left( 1 - \frac{1}{2} \left\{ \frac{\omega}{\omega_1} \right\} \right), \tag{5a} \\
\nu_g,3 \quad & \sim \nu_g,4 \quad \nu_g \left( 1 - \frac{1}{2} \left( \frac{\omega}{\omega_1 + \Delta \omega} \right) \right). \tag{5b}
\end{align*}
\]

The electrostatic potential well oscillates with the plasma frequency. Therefore the wave vector for the potential wave generated by the first and second beat-wave respectively are

\[
k_{p1} = \frac{\omega_p}{\nu_g,1}, \tag{6a}
\]

and

\[
k_{p3} = \frac{\omega_p}{\nu_g,3}. \tag{6b}
\]

We will refer to the beat-wave between the two beat-waves as "super beat-wave." The amplitude (related to the envelope) of the super beat-wave, as a function of \( t \) and \( z \), is

\[
|A_s| = 4|A| \cos \frac{1}{2} \left( \omega_1 t - (k_1 - k_3) z \right) = \\
\sim 4 |A| \cos \frac{1}{2} \left( (\Delta \omega) t - \frac{\omega}{c} \left( 1 + \frac{1}{2} \left\{ \frac{\omega}{\omega_1} \right\} \right) \right) \\
- (1 + \frac{1}{2} \left( \frac{\omega}{\omega_1 + \Delta \omega} \right) z \right). \tag{7}
\]

If

\[
\Delta \omega \ll \omega_1, \tag{8}
\]

then

\[
A_s \sim 4 |A| \cos \frac{1}{2} (\Delta \omega) t | \tag{9}
\]

Consider now an accelerated relativistic particle travelling with speed \( v \), and \( \gamma = \sqrt{1 - (v/c)^2} \), moving towards the right along the \( z \) axis. At \( t = 0 \) the particle is located at the left boundary of a potential well which accelerates the particle towards the right. As the particle is accelerated, it moves across the potential well picking up energy all the while, until it reaches the right boundary of the well. There the force acting on the particle is zero. But the particle is relativistic, assumed to be moving faster than the potential well. Therefore, it will penetrate beyond the right boundary, into a region in which decelerating forces act on the particle. Unless something is done to prevent the decelerating forces to act on it, the particle will not be able to acquire additional energy. For the double beat-wave acceleration mechanism one chooses \( \Delta \omega \) so that when the particle approaches the right boundary of the accelerating well, the two beat-waves tend to cancel, reduce the decelerating force, and thus allow the particle to drift with little energy loss until it reaches the left boundary of the next accelerating potential well.

If the accelerating potential well (as seen from the laboratory) is \( L \) long and moves with a speed \( v_g \), then the distance, \( L \), which the particle will travel in the plasma while moving across the accelerating potential well can be expressed as

\[
L = L + \frac{v}{v - v_g} f_g \tag{10}
\]

and the time it takes for the particle to travel across the accelerating potential is

\[
t_L = \frac{L}{v} = \frac{L}{v - v_g} f_g \tag{11}
\]

In Eqs. (10) and (11) the multiplicative factors \( f_g \) and \( f_v \) equal unity when \( v \) does not change significantly while the particle moves across the well. Such is the case when \( \gamma \sim 1 \). If the change in \( v \) is significant, then in Eqs. (10) and (11) the \( v \) is understood to mean the initial value of the particle speed while it moves across an accelerating potential well, and then in general \( f_v \neq 1 \). We consider two distinct methods to realize the double beat-wave mechanism.

a. Choose \( \Delta \omega \) so that while an accelerated particle travels a distance \( L \), one half of a complete cycle of the super beat-wave passes over it (e.g., first the particle experiences complete constructive interference between the two beat-waves, and then complete destructive interference). That requires

\[
\pi = (\omega_1 - \omega_2) L - (k_1 - k_3) L \tag{12}
\]

Denote the wavelength of the plasma wave generated by the first electromagnetic beat-wave by \( \lambda_{b1} \). Assuming

\[
L >> \lambda_{b1} \tag{13}
\]

condition (8) must hold, and then Eq. (12) can be written as

\[
\Delta \omega = \pi \frac{v-v_g}{2L \sqrt{\frac{c}{c}} \left( 1 - \frac{\omega}{c} \right)^{-1}} \frac{v-v_g}{2L \sqrt{\frac{c}{c}} \left( 1 - \frac{\omega}{c} \right)^{-1}}
\]

Ideally one would like the particle to experience the full effect of any accelerating potential well, while experiencing none of the effects of any decelerating potential. To realise this case, one would need the two beat-waves to add completely constructively whenever the particle is in an accelerating well, while interfering completely destructively when the particle would experience deceleration. In fact, the transition between constructive and destructive interference of the two beat-waves is not discontinuous, but gradual. As a result, the accelerating efficiency of the device is less than unity which it would be in the idealized case, and depends on the shape of the potential. For example, if the accelerating force acting on the particle within the potential well varies as the cosine of position, and if the super beat-wave is arranged to reach its maximum when the particle is at the point of maximum acceleration, then the efficiency is more than 97%. This minimum value is obtained by assuming that the background electrons in the plasma oscillate with a speed \( \sqrt{c} \), so that the ponderomotive force is proportional to the (laser field) \( \alpha \), where the exponent \( \alpha \) is unity. A more realistic minimum value is obtained by
assuming that the electron oscillation speed is < c. That is expected at least in regions of destructive interference between the beat-waves. Then $\alpha = 2$, and the efficiency is >79%. If $\alpha \neq 2$ were to hold in regions of predominantly constructive interference, while $\alpha \neq 2$ in regions of predominantly destructive interference, then the efficiency would be 84%. In practice one would expect efficiencies of about 80%.

In choosing the parameters, including $\Delta \omega$, care must be taken that while the wanted beat frequencies resonate in the plasma, no unwanted ones should do so.

The method just described requires that the laser frequencies $\omega_i$ (i = 1,2,3,4) have bandwidths which do not exceed $\Delta \omega$. At any rate, in the contrary case the length of the electromagnetic beatwave in the plasma would be $\omega_i/\Delta \omega$.

As an illustration, consider the case when $\omega_1 = 1.88 \times 10^{15}$ sec$^{-1}$, (corresponding to a vacuum wavelength $\lambda_1 = 10^{-6}$ cm), $\omega_p = 5.64 \times 10^{13}$ sec$^{-1}$, (implying $\rho_p = 1018$ cm$^{-2}$) and the accelerated particle is highly relativistic, so that $v = \gamma c$. Then $L = 5.5 \times 10^{-10}$ sec, $L = 7.8$ cm, and one needs $\Delta \omega = 1.27 \times 10^{-10}$ sec$^{-1}$, i.e., $\Delta \omega/\omega_1 = 6.8 \times 10^{-6}$.

b. The method described under a) above will in general tend to lead to a $\Delta \omega$ value which is small compared to $\omega_1$. In certain cases one may prefer $\Delta \omega$ to be closer in magnitude to $\omega_1$. The method to be described next will accomplish that.

Let us choose $\Delta \omega$ so that while the accelerated particle travels a distance $2L$, the second beat-wave (which travels with a phase velocity as given by Eq. (5b)) moves ahead of the first beat-wave (which travels with a phase velocity as given by Eq. (5a)) by a distance $\lambda_1 L$. If the two beat-waves are in phase when the particle is accelerated by a potential well, then the two beat-waves will again be in phase when the particle is similarly accelerated by the next potential well. On the other hand, halfway between these two points, the particle will experience no decelerating force, because there essentially no oscillating charge separation is allowed to develop in the plasma. That is so, because in those regions the maxiima of the first beat-wave are located halfway between the maxima of the second one. The result is a ponderomotive force which is essentially constant in time (except for high frequency components with which the plasma cannot resonate). For this to occur, $\Delta \omega$ has to be chosen so that

$$\pi = (k_p^1 + k_p^2) L .$$

(15)

The expected efficiencies are, of course, again generally less than unity. Again, $\Delta \omega$ has to be chosen so that no unwanted beat frequencies should be resonant in the plasma.

To illustrate, choose $\omega_1$, $\omega_p$ and $v$ as in the example given at the end of a) above. Then Eq. (15) will be satisfied by $\Delta \omega = 0.779 \times 10^{15}$ sec$^{-1}$, corresponding to $\omega_3 = \sqrt{2} \omega_1$.

In the present case, contrary to case a) discussed above, he waves with circular frequency $\omega_1$ and $\omega_2$ may have a polarization different from that of the waves with circular frequency $\omega_3$ and $\omega_4$.

Alternating Density (Double Beat-Wave) Mechanism

Let the accelerated particle travel along the z axis in a plasma. The plasma consists of two types of regions. In regions of the first type the electron density is $\rho_A$ and the plasma frequency is $\omega_A$, while in regions of the second type the corresponding quantities are $\rho_B$ and $\omega_B$. We will refer to regions of the first and second type as A-regions and B-regions respectively. Choose two electromagnetic plane waves with circular frequencies $\omega_1$ and $\omega_2$, and with equal amplitudes: $A_1 = A_2 = A$, and equal polarization, travelling parallel to the accelerated particle. Set

$$\omega_1 - \omega_2 = \omega_A ,$$

(16)

These two waves set up a beat-wave which will generate resonant electron oscillations in all A-regions. Select the length of these regions to be $L$. Then an accelerated particle can traverse one of these regions while being continuously accelerated. As the particle is about to leave the accelerating potential well, it enters a B-region in which the beat-wave cannot set up resonant electron oscillations, so that no potential well exists in these regions to decelerate the particle. Upon entering the next A-region, the particle is accelerated further, etc.

The above scheme can be improved by selecting two more electromagnetic plane waves, each with amplitude $A$, in equal polarization states, both moving parallel to the accelerated particle. The circular frequency of these two waves are $\omega_3$ and $\omega_4$ respectively. Choose

$$\omega_3 - \omega_4 = \omega_B ,$$

$$\omega_3 - \omega_4 = \Delta \omega ,$$

(17)

and select $\Delta \omega$ so that except for the wanted frequencies, no beat frequency will overlap with any resonant frequency region in the plasma. These two waves will generate a second beat-wave which will set up resonant electron oscillations in all B-regions. If $\rho_A$ and $\rho_B$ are sufficiently different, this second beat-wave cannot set up significant oscillations in any of the A-regions, while, similarly, the first beat-wave cannot set up resonant electron oscillations in any B-regions. Therefore, if correctly phased relative to the particle, each beat-wave will accelerate the particle in regions of the plasma where it cannot set up resonant oscillations. In the case where the particle is not decelerated by particle acceleration in other regions. Thus, the particle can be continuously accelerated while it travels in the plasma.

Clearly, the polarization states of the two beat-waves need not be equal to each other.

If the electron density could change discontinuously between neighboring regions, the accelerating efficiency of this mechanism would be unity. In fact, one expects that border domains will develop in which the electron density will change continuously between $\rho_A$ and $\rho_B$. The actual efficiency will therefore be less than unity. In the most general case, the phenomena which appear at boundaries can be quite involved. For purposes of the present discussion, however, it suffices to assume that the boundary between various regions is perpendicular to the direction of wave propagation, so that all refracted and reflected waves will continue to move along a common line. Furthermore, it may be postulated here that the change of density between neighboring regions approaches the adiabatic case, which allows one to neglect reflected waves to a good approximation. Then in the transition domains between regions the dominant effect due to density change will be the detuning of $\omega$, which will lead to a decrease and eventual disappearance of the electrostatic waves in these regions. Assuming the
worst case, namely that no accelerating wave exists anywhere in any transition region, one finds that the efficiency of the device will be multiplied as a result of transition domains of length $L_x$ between neighboring regions of length $L_\nu$, by a factor $>(L_\nu-L_x)/L_\nu$. To obtain a more precise value, one has to know the dependence of the electron density in each transition domain as a function of time and position, as well as the shape of the electron resonance as a function of $\omega$.

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References


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Paul L. Csonka received the Ph.D. degree in theoretical physics (without prior degrees) from Johns Hopkins University, Baltimore, MD, in 1963. From 1964 to 1966 he held a Postdoctoral Appointment at the Lawrence Livermore Laboratory in CA, and in 1966 and 1967 he was an NSF Postdoctoral Fellow at CERN Laboratories in Geneva, Switzerland. He was the NORDITA Visiting Professor to Scandinavia from 1972-1973, and he is currently a Professor in the Department of Physics at the University of Oregon, Eugene.

Dr. Csonka was the 1970-1972 recipient of the Alfred P. Sloan Fellowship. He is a member of the Institute of Theoretical Science at the University of Oregon, where from 1977-1979 he was Director of that Institute.