An NBS Calibration Procedure for Providing Time and Frequency at a Remote Site by Weighting and Smoothing of GPS Common View Data

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Abstract—The National Bureau of Standards (NBS) Time and Frequency Division now performs precision time and frequency transfer using common view measurements of Global Positioning System (GPS) satellites as a calibration service. Using this service, we have been able to transfer time with time stabilities of a few nanoseconds, time accuracies of the order of 10 ns, and frequency stabilities of one part in $10^{14}$, or better, for measurement times of about four days and longer. The full accuracy of the NBS primary frequency standard is now available at a remote site. This paper describes the technique used for weighting and smoothing the data to produce these levels of stability and accuracy. All of the primary frequency standards used in the generation of International Atomic Time (TAI) now use this technique.

I. INTRODUCTION

It is now possible to compare a clock with universal time coordinated (UTC(NBS)) anywhere in common view of a Global Positioning System (GPS) satellite with Boulder, CO at the full level of accuracy and stability of the National Bureau of Standards (NBS) atomic time scale for integration times of about four days and longer via the NBS Global Time Service. This availability includes East Asia, Europe, and North, Central, and South America. The service includes a dial-up line to one of the NBS computers for current estimates for the user’s clock performance, and a monthly report for improved estimates after the fact. We discuss here the method by which the common view time transfer values in the monthly reports are computed.

There are several important aspects to the method NBS uses in obtaining state-of-the-art time transfer: making the measurements and checking the data for outliers, characterizing the random and systematic variations in the data, determining nominal optimum weights for the data, and Kalman smoothing the weighted averages and interpreting the results. First, simultaneous measurements are taken using a satellite in common view of NBS Boulder and a second location. These measurements are repeated each sidereal day so that the geometry at measurement time remains fixed. The data are checked for statistical outliers, and these are removed by interpolation or extrapolation. Second, we characterize the random and systematic variations. A measurement geometry which repeats each sidereal day defines a time series for each of the common view GPS satellites employed. The type of measurement noise and the noise level of each time series is determined using a decomposition of variances from independent satellite comparisons. The noise type is determined initially in defining the model, and should stay the same unless something in the system changes. The level of the noise is determined independently for each monthly report. Also, a comparison of the biases between pairs of comparisons can be checked. Third, a nominally optimum set of weighting factors can be calculated knowing the noise levels. These are used to combine the measurements via different satellites into a single weighted average. Fourth, a Kalman smoothed estimate of time and fractional frequency offset of the remote clock is separately computed for each time series, using the measurement noise estimates along with the noise characteristics of the clocks involved; and finally, these are also combined using the weights previously determined to define optimal estimates of time and frequency offsets of the remote clock.

Using the above technique we have been able to transfer time with a standard deviation on the measurement noise residuals of the order of 1 ns and with time accuracies of the order of 10 ns.

II. DATA SELECTION AND REJECTION

Locations interested in comparing a clock with UTC(NBS) via GPS should measure their clock against GPS satellites according to an NBS tracking schedule. Recently NBS in support of the Bureau International de l’Heure (BIH), has transferred the technique for developing this tracking schedule to the BIH. A satellite is tracked for a 13-min interval and the data taken during that time are reduced to a value of GPS minus reference clock and a rate offset. It has been shown elsewhere [1] that tracking longer than about 10 min is of little value since the fluctuations appear to be flicker noise phase modulation (PM) limited for Fourier frequencies smaller than about one cycle per 10 min. One gains significantly by averaging the white phase noise at the higher Fourier frequencies. Tracks are extended to 13 min to ensure use of the most recent ionospheric correction, since that parameter is transmitted every 12.5 min. The tracking
schedule tells which satellites to track at what time on a
certain modified Julian day (MJD) for all locations in a
given large area of earth. Each track in the tracking sched­
ule is assigned to at least two areas and is chosen to max­
imize the elevation of a GPS satellite as seen from those
areas. The elevation of a satellite changes little over a
large area of earth since the satellite orbits are 4.2 earth
radii. The track times decrement by 4 min a day to pre­
serve the geometric relationship between the satellites and
the ground location at each measurement. This follows
since the satellites are in 12-h sidereal orbits. A sidereal
day is not exactly 4 min less than a solar day, but this is
close enough since the satellites deviate somewhat from
exact sidereal orbits. The tracking schedule is recomputed
from time to time (about once or twice a year) in order to
maintain good geometry.

GPS minus reference measurements are gathered to­
gether at NBS in Boulder from many locations in the
United States and around the world. UTC(NBS) can be
transferred to any location having common view data with
NBS. A track is in common view between two locations
if both locations have received the same signals from a
satellite, i.e., both locations have tracked the same sat­
eellite at the same time. In this way many sources of error
cancel or nearly cancel upon subtraction of the GPS minus
reference measurements [2]. If there is some discrepancy
between the times of tracking at the two locations the noise
of the difference measurement may degrade. A common
view track repeats every sidereal day and in this way de­
defines a time series comparing the clocks at the two lo­
cations. Each time series represents a different path from
one location to the satellite to the other location repeated
every sidereal day and used to compare the two reference
clocks. Each satellite in common view can be used for
such a time series. Indeed, a satellite can give rise to two
time series if there are two different common view paths
each day: one when the satellite is nominally between the
two locations, and one about 12 h later when observed
over the pole. Time transfer between two locations is ac­
complished by determining measurement noise and
weights for each path, using this to smooth each time se­
ries separately and combine them into a weighted aver­
age.

We often find that the entire time series of common view
measurements via one satellite is biased from the data via
another satellite. This is not entirely understood, but it
must be due to repeated errors in the transmitted
ehemeris or ionospheric model, or repeated errors in the
tropospheric model, coordinate errors at the local re­
ceiver, or a frequency offset between the reference clocks.
Because of these biases, we work with the time series se­
parately before they are combined into a weighted average.
Also, the presence of the biases makes choosing the
weights very important since the resultant average can
change with different weights. First, each time series is
studied for apparent bad points. These may be due to a
reference clock time or frequency step, in which case
points in the time series may seem bad but are actual mea­
urements of the clock. If this happens when there are
missing data from several satellites it can be difficult to
interpret. Bad points can also be caused by either troubles
in the data transmitted from the satellite or problems in
the receiver. A knowledge of clock characteristics and ex­
pperience with the GPS performance is necessary in order
to make proper judgements in dealing with any of the
above bad points. When these are found in a given time
series—and they are not attributable to the reference
clock—they are replaced by a value either linearly inter­
polated from neighboring good points, or, if it is an end
point, by a value extrapolated from the entire time series
by a quadratic curve fit. In this way a bad measurement
is replaced with a value which maintains the bias of the
time series when it is included in the weighted average.
Extrapolated data have to be used very cautiously.

III. MEASUREMENT NOISE AND WEIGHTS

The measurement noise and the weight of each time
series is estimated using a decomposition of variance or
what is called a “N-corner hat” technique with the mod­i­
ified Allan variance. The N-corner hat technique is a gen­
eralization of the three-corner hat [3], where the variance
of the stability of a particular clock is estimated using var­
iances of the stability of difference measurements among
three clocks. The generalization is that we have N clocks
instead of three. We apply this technique to differences of
the time differences of our GPS data, i.e., differencing the
common view measurements across satellites. The equa­
tions are as follows. A time difference is a difference of
measurements at two locations via satellite $i$:

$$ (\text{Ref}_2 - \text{Ref}_1 + \text{Noise})_i = (\text{GPS} - \text{Ref}_1 + \text{Noise})_i - (\text{GPS} - \text{Ref}_2 + \text{Noise})_i $$

where “Noise” on the left denotes the common view
measurement noise for that path.

The difference of the differences is

$$ \text{Noise}_i - \text{Noise}_j = (\text{Ref}_2 - \text{Ref}_1 + \text{Noise})_i - (\text{Ref}_2 - \text{Ref}_1 + \text{Noise})_j. $$

Thus we see that the set of variances of the difference of
differences is the set of variances of noise differences. We
may apply N-corner hat to this to find the variance of a
particular process just as we apply N-corner hat to the set
of variances of clock stability differences to find the sta­
bility of a particular clock. Let us consider the equations
for the composition of variances [4]. We want to find

$$ \sigma_i^2 = \text{estimate of the variance of process } i, $$

$$ i = 1, 2, \ldots, N $$
given

$$ s_{ij}^2 = \text{measurement of the variance of } i - j \text{ difference } $$
and under the assumption that the common view noise via
satellite $i$ is uncorrelated with that via $j$. 
We choose the $\sigma_i^2$ to minimize

$$A = \sum_{j=2}^{N} \sum_{i=1}^{j-1} (s_{ij}^2 - \sigma_i^2 - \sigma_j^2)^2.$$  

The result after solving $\partial A/\partial \sigma_i^2 = 0$ is

$$\sigma_i^2 = \frac{1}{N-2} \left( \sum_{k=1}^{N} s_{kk}^2 - B \right)$$  

where

$$B = \frac{1}{2(N-1)} \left( \sum_{k=1}^{N} \sum_{j=1}^{N} s_{kj}^2 \right), \quad \text{with} \quad s_{jj}^2 = 0.$$  

If we use the modified Allan variance [5], [6] in these equations we see that the common view noise has a spectrum consistent with the hypothesis of white noise PM. This means that the square root of the variance as a function of time interval $\tau$ should be proportional to $\tau^{-3/2}$, or where $\tau = m \tau_0$ ($m$ is an integer and $\tau_0$ is one sidereal day). Because of this we may multiply $\sigma_i^2 (m \cdot \tau_0)$ by $m^3$ and take the mean over the number $M$ of variance computations. Thus the common view noise squared $n_i^2$ for path $i$ is proportional to

$$n_i^2 \sim \frac{1}{M} \sum_{k=0}^{M-1} \left[ \sigma_i^2 (2^k \tau_0) \right] \left( 2^k \right)^3].$$  

The constant of proportionality is

$$\tau_0^3 \cdot 3p_i, \quad \text{where} \quad p_i \text{ is the percentage of good points.}$$  

The factor of $\tau_0^3/3$ comes from the relationship between time and frequency stability with the modified Allan variance in the case of white noise PM (see Appendix). The percentage of good points comes in because the confidence of the estimate gets worse with fewer points [7]. The weight of path $i$, $w_i$, is the reciprocal of the normalized noise estimate of path $i$, $w_i$, is the reciprocal of the normalized noise estimate

$$w_i = \left( 1/n_i^2 \right) \left( \sum_j 1/n_j^2 \right).$$  

These are the weights which are used to combine the time series for the different satellite paths into a single weighted average. The result is that more stable the series, the heavier it is weighted. This is optimum for unbiased data. If the bias of a path is proportional to its instability, then the weighted average will also be optimum. There are some mechanisms in the GPS that make this a good assumption. If a bias is due to local coordinate errors this assumption will fail. Biases due to coordinate errors are estimated to be only a few nanoseconds if the coordinates used are self-consistent with the GPS. Biases of several nanoseconds can occur if an independent coordinate determination is used. If the bias is due to error in transmitted data it is possible the bias would be unstable from attempts to correct the error and the above assumption would be valid. A study is in progress to better understand these biases. Despite the biases, this optimum weighting procedure leads to minimum measurement noise for frequency calibration and frequency transfer at a remote site.

The above procedure does not consider weighting due to the confidence of the estimate as that has not been developed yet for the modified Allan variance. For the Allan variance and the case of white noise PM it has been shown that the confidence of the estimate remains fairly constant [8]. It is believed that a similar result will be true for the modified Allan variance in which case the above equations are properly weighted.

IV. Kalman Estimates

Once the measurement noise level and type are characterized, a forward-backward Kalman smoother is designed and used to optimize the signal to noise for time and frequency transfer. Each path is smoothed separately using the estimates of the noise for each path, as well as estimates for the noise characteristics of the two clocks, as input to the Kalman smoother. The state vector consists of two elements: the time $x$ and fractional frequency $y$ of a clock offset from UTC(NBS). Interpolated or extrapolated values are not used by the Kalman, rather it replaces these with its own optimal estimates. The $x$ and $y$ values from the different paths are combined using the weights to generate a smoothed estimate of the time and frequency offset of a reference clock from UTC(NBS).

V. Results

Results of approximately 10-ns accuracy and 1 part in $10^{14}$ stability for integration times of four days have been reported elsewhere [9]. Here we simply note results on more recent data. We consider time comparisons between NBS in Boulder, CO, and Physikalisch-Technische Bundesanstalt (PTB) in Braunschweig, West Germany during a 94-day period from MJD 46490 to 46583, March 1 to June 2, 1986. We use measurements via paths for six satellites (SV’s) 3, 9, and 11. These are all biased

Table III tabulates the average biases for each track against a weighted mean and the computed measurement noise for each of the satellite paths. We look at Table III to consider our hypothesis that a larger bias is associated with a larger measurement noise. The worst measurement noise values are for SV’s 3, 9, and 11. These are all biased
do know that for time stability and frequency accuracy we are optimally weighting the data.

the same way, though the values are quite different. The smaller measurement noise values for SV’s 8, 12, and 13 seem to go with smaller biases, but the results are marginal. We simply do not know the right answer here for time accuracy. We believe that since each common view path should be an independent estimate, the standard deviation should be a good measure of the accuracy. It is not certain that the biases are independent. In fact, in some cases there are mechanisms for dependency between some of the biases. They can occur through the Kalman estimation process, for example. More study is needed. We

The rms standard deviation of the different SV paths averaged over the 94 days is 9.2 ns. This is due primarily to the biases, since the composite measurement noise is only 3.2 ns. The latter is an indication of the measurement noise remaining in the weighted average. The weighted average for the time transfer across the satellites is in Fig. 2. The $\sigma_f(\tau)$ for these data is plotted in Fig. 3. Knowing something about the clocks involved we see there can be little remaining measurement noise. We attempt to remove this with a Kalman Smoother. Fig. 4 shows the time residuals for UTC(PTB) – UTC(NBS) after the smoother, and Fig. 5 shows the associated $\sigma_f(\tau)$.

A good check of the consistency of the time transfer system via GPS in common view is to perform time transfer around the world using several different reference stations. In Fig. 6 we see the results of this using NBS, PTB, and the Tokyo Astronomical Observatory (TAO) in To-
Fig. 2. A plot of the weighted average of the time difference of UTC(PTB) - UTC(NBS). (The rms standard deviation about the weighted mean over the 94 days was 9.2 ns. The white noise PM was estimated to be 3.2 ns. The difference between these two numbers is primarily due to the biases in the system.)

Fig. 4. A plot of the Kalman smoothed GPS common view time difference between the PTB and NBS time scales. (Choosing the appropriate Kalman parameters reduces the GPS common view noise contribution so that all values or more nearly representative of the time scales involved.)

Fig. 3. The fractional frequency stability (square root of the two sample or Allan variance) as a function of the sample time $\tau$ is plotted using the data shown in Fig. 2. (The GPS common view noise contributes to the instabilities plotted an estimated amount of $\sigma(\tau) = 6.4 \times 10^{-14}/\tau$, where $\tau$ in this equation is in units of days. The sample times plotted are for $\tau = 1, 2, 4, 8, 16, \text{and} 32$ days. Hence, the common view noise only contaminates somewhat the time scale comparison stability for $\tau = 1$ and 2 days.)

Fig. 5. A plot of $\sigma(\tau)$ for the data shown in Fig. 4. (If the Kalman smoothing parameters are well chosen and the models appropriate to the clocks' stochastic processes, the fractional frequency stabilities plotted will be good estimates of the relative time scale stabilities even over the 8050-km baseline between the scales.)

kyo, Japan over a period of 569 days from MJD 46014 to 46582, November 10, 1984 to June 1, 1986. We see that the values stay biased away from zero for long periods of time. Here we see clearly the effect of the biases on time accuracy.

An example of a state-of-the-art time and frequency transfer experiment is illustrated in Figs. 7 and 8, which is a time and frequency stability comparison of the National Research Council (NRC), Ottawa, Canada, primary cesium clock and the NBS prototype passive hydrogen maser utilizing the above outlined technique via seven independent GPS satellite clocks. The theoretical noise estimated from the $N$-corner hat analysis is plotted along with the theoretical white frequency modulation noise from the NRC primary cesium and from the passive maser for comparison with the measured values. The excellent agreement is apparent.

In addition to using this service for the above-mentioned Global Time Service, it is used in computing the data sent to the BIH for the NBS input of the Système International d'Unités (SI) second and for the input of the times of the clocks in the NBS ensemble toward the generation of UTC and TAI. These data include comparisons of the time and frequency of UTC(NBS) with other principal timing centers, e.g., the NRC, PTB, the Radio Re-
Fig. 6. A plot of the consistency and the accuracy of the GPS common view approach over some nearly 600 days of data. (The errors plotted are the accumulation over three GPS common view paths: PTB-NBS, TAO-PTB and NBS-TAO, which would add to zero if the technique were perfect. There are clearly biases in the system of the order of several tens of nanoseconds, which walk in and out with time. Dividing the accumulated bias by three gives one common view comparison an accuracy of the order of 10 ns over the last several months. Whether that persists or not remains to be seen. Further understanding of these biases is needed.)

Fig. 7. A plot of the time difference residuals between the NRC primary cesium clock, Cs 5, and the NBS prototype passive hydrogen maser, PHM4 as measured via the weighted filtered average of seven GPS satellites utilized in the NBS-GPS common view time transfer mode. (M1D RMVD means the mean frequency was removed from the data by subtracting a mean first difference from the weighted average difference data. A mean time was removed (MNT: RMVD) for scaling convenience.)

search Laboratory (RLL) in Tokyo, and the U.S. Naval Observatory in Washington DC. The above technique was utilized in making a comparison across the Pacific of the NBS and RRL primary frequency standards. This is the first time (October–November 1984) they had been compared where their full accuracy was appreciated. The RRL primary standard Cs 1 was compared with the NBS primary standard NBS-6; the fractional frequency difference was measured to be

\[ \text{Cs 1} - \text{NBS-6} = (2 \pm 14) \times 10^{-14}. \]

The difference is, interestingly, about nine times smaller than the gravitational "red shift" that had to be applied in making the comparison.

VI. CONCLUSIONS

Time transfer via GPS satellites is possible at the level of accuracy of laboratory time standards for periods of four days or more, depending on the baseline and if it is done with care. Care is needed in making strictly simultaneous measurements at two locations repeated every sidereal day to maintain a common view measurement with a constant geometry. Care is needed in removing bad points from each of the time series. Weights for each path are very important since, due to biases in the system, a change in weights significantly changes the weighted mean. Finally, a Kalman smoother may be employed to remove measurement noise from the weighted average, but its results must be interpreted carefully. Understanding the biases in the system remains an important unsolved problem.

APPENDIX

CLASSICAL OR N-SAMPLE VARIANCE VERSUS \( \sigma^2(\tau) \), THE TWO-SAMPLE OR ALLAN VARIANCE

We will derive the relationship between the classical phase variance and the Allan variance. The ratio of expected value of the classical or N-point fractional frequency variance to the Allan variance has been derived for an arbitrary power law spectrum \( \mu \), and as a function of \( N \), the number of data samples, \( T \) the period of sam-
pling, and \( \tau \) the sample time [10], [11]. Let us denote the expected value of the \( N \)-sample fractional frequency variance with \( T = \tau \) (no dead time) as

\[
\langle \sigma_y^2(N, \tau) \rangle.
\]

Then if we denote the ratio

\[
B_1(N, \mu) = \frac{\langle \sigma_y^2(N, \tau) \rangle}{\sigma_y^2(\tau)}
\]

it can be shown from results in [10], [11] that

\[
B_1(N, \mu) = \frac{N(N^\mu - 1)}{2(N - 1)(2^\mu - 1)}.
\]

For white phase noise we have \( \mu = -2 \). Hence as \( N \) grows large without bound we have

\[
B_1(N, \mu) \to 2/3, \quad \text{as } N \to \infty, \mu = -2.
\]

To complete our derivation we will show the relationship between the classical fractional frequency variance and the classical phase variance. Since the phase variations are proportional to the time variations \( (x = \phi / 2\pi\nu_0) \), the relationship between fractional frequency \( y \) and time \( x \) is

\[
y(t) = \frac{x(t + \tau) - x(t)}{\tau}.
\]

If we compute a classical variance of both sides, and if the phase points \( x(t) \) are consistent with a white phase noise model so \( x(t + \tau) \) and \( x(t) \) are uncorrelated, we have as the number of samples \( N \) approaches \( \infty \)

\[
\sigma_y^2(N \to \infty, \tau) = \frac{\langle x^2(N \to \infty, \tau) \rangle}{\tau^2} = \frac{2}{\tau^2} \sigma_x^2(\tau).
\]

If we substitute this into our result above

\[
\frac{\langle \sigma_y^2(N \to \infty, \tau) \rangle}{\sigma_y^2(\tau)} = \frac{2}{3}
\]

we have

\[
\frac{2\sigma_y^2(\tau)}{\tau^2 \sigma_x^2(\tau)} = \frac{2}{3}.
\]

hence

\[
\sigma_x^2(\tau) = \frac{\sigma_y^2(\tau) \cdot \tau^2}{3}.
\]

REFERENCES


