Abstract—A solid-state reference waveform filter has been developed which uses the Maxwell-Wagner capacitor effect. This filter is realized in a stripline configuration with a lossy dielectric consisting of a thick (5-μm) layer of SiO₂ on Si. The equivalent circuit of this filter is equivalent to that for previously developed filters which used a lossy liquid dielectric. A preliminary design has been completed and a filter fabricated for which the design characteristic impedance, 38 Ω, and transition duration (rise time), 300 ps, agree with measured values to within 2 and 17 percent, respectively. The temperature dependence of the filter transition duration has been estimated from the temperature dependence of the filter conductance to be about 1 percent/°C.

Introduction

A TIME waveform history of various phenomena may now be acquired with much greater precision and convenience than ever before. This is due in large part to the inexpensive and convenient signal handling capability of microprocessors.

The question remains as to what degree this precision represents the truth. To help resolve such questions the National Bureau of Standards (NBS) has been charged with the responsibilities to 1) develop accurate measurement standards with which other measurements can be compared, and 2) encourage the use of standard measurement procedures.

Some of the basic standards developed by NBS include the second for time measurement, the meter for length measurement, and the volt for potential difference measurement. Standards which are derived from these basic quantities are now being developed for use in recording the time history of electrical waveforms. Two basic approaches have been used. The first method models the distortion caused by some measurement system and then removes it mathematically (deconvolves) to obtain an estimate of the true waveform [1]. The second method models the output waveform of a waveform generator whose output is applied to the measurement system; the measurement system’s response is then compared to the predicted available waveform of the generator [2]. The generator with the modeled output is called a reference waveform generator.

A reference waveform generator can be used to evaluate an oscilloscope or waveform recorder by determining how closely the display resembles the “available waveform” of the reference generator for a load impedance equal to the measurement system’s nominal input impedance.

The reference waveform is modified by the system transfer function $H(s)$ and the relation between $Z_{11}(s)$ and $Z_{2}(s)$ to produce the displayed waveform $e_d(t)$ as shown in Fig. 1. The specific nature of $H(s)$ may be derived by stable deconvolution techniques [1] using the measured $e_d(t)$, the reference waveform $z_d(t)$, together with a frequency-domain or time-domain measurement of $Z_{11}(s)$.

One of the more popular waveforms used in testing waveform recording devices is a steplike waveform (which makes a transition from an initial level to a final one) since it provides a more complete system characterization than some other commonly used waveforms. The time it takes for a steplike waveform to go from 10 percent of the transition to 90 percent is called the transition duration or rise time. (Note this parameter has meaning only when dealing with steplike waveforms.) Over the years NBS has developed a set of transition duration standards which consist of lossy filters excited by fast step generators (tunnel diodes).

In the nanosecond and picosecond regions of the time domain, reflectionless (constant impedance) filters for waveform shaping cannot be built using discrete circuit elements. Distributed circuit elements must be used to avoid reflections and their effects on impedance and the resultant transmitted waveforms [2]. The present waveshaping filters employed by NBS in the NBS reference waveform generators [2]-[4] use distributed elements comprised of polar liquid filled coaxial transmission lines approximately 24 cm in length. The resultant waveshaping is due to the Debye relaxation phenomena in the dilute polar solutions which serve as the dielectrics in the coaxial transmission lines. The present set of filters provide three steplike resultant waveforms having 50-, 100-, and 200-ps transition durations.

The simple geometry makes it possible to develop an accurate model of the filter transient response. In addition, the waveform generator output is independent of the input excitation if the input excitation has a much shorter transition duration than that of the filter step response. The case where the input excitation is not fast enough is discussed elsewhere [5]. With the simple geometry and a fast input step one can produce a modeled reference waveform generator.

These filters have the additional attribute that the transition duration can be varied simply by varying the filter length while maintaining a constant characteristic impedance. However they are limited in their use as a portable transfer standard and are intended for use only as laboratory standards. In

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addition they are not well suited for transition durations longer than about 1 ns.

To extend the transition-duration range into the 1-10-ns range using the present liquid dielectric design would require unwieldy (long) transmission lines and/or higher concentrations of polar solute molecules in the dielectric solution. Higher solute concentrations are limited by second-order polarization effects which are not mathematically represented by the Debye (dilute solution) equations; consequently, such concentrations would not produce an a priori model based upon the well known and characterizable Debye model parameters.

To cover this extended range a new waveshaping filter has been developed which contains solid dielectric layers in a microstrip transmission line structure [6]. The filter size is much smaller than the liquid dielectric filters and can be fabricated as an integrated semiconductor circuit making it a very suitable candidate for portable standards covering the extended ranges, and also for improving the present standards in the tens to hundreds of picoseconds region. This new filter uses the Maxwell-Wagner two-layer dielectric capacitor concept [7] and is shown schematically in Fig. 2.

**Theory**

The equivalent circuit for the Maxwell-Wagner Capacitor stripline filter, assuming planar skin effect, is given in Fig. 3 and is similar to that for the liquid dielectric lines. In the layered dielectric, subscript 1 refers to the silicon dioxide dielectric and subscript 2 refers to the silicon dielectric. The characteristic impedance $Z_0(j\omega)$ is given by

$$Z_0(j\omega) = \sqrt{Z(j\omega)/Y(j\omega)}$$  \hspace{1cm} (1)

where $Z(j\omega)$ is the distributed series impedance per unit length and $Y(j\omega)$ is the distributed shunt admittance per unit length. The propagation constant $\gamma(j\omega)$ is given by

$$\gamma(j\omega) = \alpha(\omega) + j\beta(\omega) = \sqrt{Z(j\omega)/Y(j\omega)}.$$  \hspace{1cm} (2)

Since $\alpha(\omega)$ is the real part of the propagation constant, it will determine the filter dispersion while the imaginary part $\beta(\omega)$ will determine the phase shift or time delay. Let us now consider the behavior of $Z(j\omega)$ versus frequency. At zero frequency, $Z_0(0) = R$. At high frequency the resistance $R$ is negligible and $Z(j\omega)$ is given by $j\omega L$ with the resulting functional form for $Z(j\omega)$ shown in Fig. 4.

Similarly, at very low frequencies, $Y(j\omega) = j\omega C_1$ and at very high frequencies it is equal to $j\omega C_\infty$, where $C_\infty$ is the capacitance at infinite frequency and is given by $C_\infty = C_1 + C_2$. The result is the functional form given in Fig. 5 for the case of $C_1 \gg C_2$. In this case $C_\infty \approx C_2$.

In Fig. 4, the resistance of the transmission line $R$ may be safely neglected in the frequency range of $\omega > 10(R/L)$. Similarly $C_2$ can be ignored for $\omega < 0.1 G_2/C_2$ in Fig. 5.

Now the effective values of the line parameters found by Hasegawa [8] for the stripline of Fig. 2 are

$$L = \mu_0(d_1 + d_2)^2/w$$
$$C_1 = \varepsilon_0 \varepsilon_{SiO_2} w/d_1^2$$
$$C_2 = \varepsilon_0 \varepsilon_{Si} w/d_2^2$$
$$G_2 = \sigma_2^2 w/d_2^2.$$  \hspace{1cm} (3)

The symbols $\mu_0$ and $\varepsilon_0$ in the above expressions represent the permeability and permittivity, respectively, of free space. In addition

$$d_1^2/w = (1/2\pi) \ln [8(d_1/w) + (4d_1/w)^{-1}] \hspace{1cm} \text{for } d_1/w > 1$$
$$= [(w/d_1 + 0.242 - 0.44 (d_1/w) + (1 - d_1/w)^{1/2})^{-1} \hspace{1cm} (4)$$

for $d_1/w < 1$ where $d_1^2$ can be either $d_1^2$, $d_2^2$ or $(d_1 + d_2)^2$

$$\varepsilon_1^2 = (\varepsilon_1 + 1)/2 + (\varepsilon_1 - 1)/[2(1 + 10d_1/w)^{1/2}].$$

where $\varepsilon_1^2$ can be either $\varepsilon_{SiO_2}$ or $\varepsilon_{Si}$.

Finally,

$$\sigma_2^2 = (\sigma_2/2)[1 + (1 + 10d_1/w)^{-1/2}]$$
FILTER DESIGN

A prototype microstrip filter has been realized by growing a thick (5.5-μm) layer of silicon dioxide on a 5-Ω-cm, 300-μm-thick silicon wafer. A 7-μm-thick film of aluminum was then evaporated on both sides of the resulting wafer and several 70-μm-wide lines were then etched on one side forming the strip of the microstripline. These strips were then separated with a diamond saw, mounted on brass plates and connected to coaxial lines as shown in Figs. 6 and 7. A time-domain reflectometer (TDR) picture of the microstrip and transitions is given in Fig. 8 and a photograph of the output reference waveform as measured on the NBS automatic pulse-measurement system (APMS) [9] is given in Fig. 9.

For our prototype filter

\[
T = 7 \mu m
\]
\[
W = 70 \mu m
\]
\[
l = 4.6 \text{ cm}
\]
\[
d_1 = 5.5 \mu m
\]
\[
d_2 = 300 \mu m
\]
\[
\varepsilon_{\text{SiO}_2} = 4
\]
\[
\varepsilon_{\text{Si}} = 12
\]
\[
\sigma_{\text{Si}} = 31 \text{ S/m}
\]
\[
\sigma_{\text{Al}} = 3.7 \times 10^7 \text{ S/m.}
\]

From these parameters we calculate

\[
L = 7.14 \times 10^{-7} \text{ H/m}
\]
\[
C_1 = 504 \text{ pF/m}
\]
\[
C_2 = 115 \text{ pF/m}
\]
\[
G_2 = 31.7 \text{ S/m}
\]
\[
R = 55.2 \Omega/m
\]

which in turn are used to determine that \( R \) can be neglected for frequencies (\( \omega/[2\pi] \)) above 200 MHz and \( C_2 \) can be neglected below 2 GHz. Therefore, in the frequency region from 200 MHz to 2 GHz, we have the equivalent circuits shown in Fig. 10 for which

\[
Z_0(j\omega) = \sqrt{j\omega L[1/G_2 + 1/j\omega C_1]}
\]
\[
= R_0\sqrt{1 + j\omega C_1/G_2} \quad \text{where } R_0 = \sqrt{L/C_1}. \]
Similarly,
\[
\gamma(j\omega) = \sqrt{j\omega L/[1/G_2 + 1/i\omega C_1]}
\]
\[
= j\omega \sqrt{C_1/L} / \sqrt{1 + j\omega C_1/G_2}
\]
\[
\approx j\omega \sqrt{L/C_1} \left[ 1 - j\omega C_1/G_2 \right]
\]
(9)
since \(\omega \ll G_2/C_1\). Therefore, with (2) in (9)
\[
\alpha(\omega) = C_1 \sqrt{L/C_1} (1/2G_2) \omega^2
\]
(10)
and the electromagnetic energy will be attenuated along the transmission line according to \(e^{-\alpha(\omega)H}\) or \(e^{-(\omega/\omega_0)^2 H}\) where \(\omega_0 = \sqrt{2G_2/(C_1\sqrt{L/C_1})}\). Note that this is a Gaussian function and as such has some special properties as an antialiasing waveform filter since an antialiasing filter that has a Gaussian rolloff characteristic can be designed with a cutoff frequency much closer to the Nyquist frequency than most other filters. Consider, for example, an antialiasing filter that will reduce the error due to the folded spectrum to less than one half of a least significant bit (\(\frac{1}{2}\) LSB) for a 1024-point sampled waveform in a 5-ns time window. For a simple RC filter, the cutoff frequency can be no greater than 3 MHz while for a Gaussian filter the cutoff frequency is 18 GHz.

To calculate the filter transition duration, consider the Gaussian filter voltage transfer function with arbitrary parameters \(D\) and \(a\)
\[
H(j\omega) = e^{j\omega D} e^{(j\omega/2a)^2}
\]
(11)
as a function of frequency. The transform of this function in the time domain gives the filter impulse response
\[
h(t) = (\pi/2a) e^{-\pi^2 (t-D)^2}
\]
(12)
from which the step response is found to be
\[
v_s(t) = \frac{1}{2} \text{erf} \left[ a(t-D) \right] + \frac{1}{2}
\]
(13)
for which the first transition duration (10-90 percent) is
\[
t_1 = 1.8a.
\]
(14)
For the filter being analyzed
\[
H(j\omega) = e^{j\omega L/C_1} e^{-C_1 \sqrt{L/C_1}/(2G_2) \omega^2}.
\]
(15)
Therefore,
\[
a = \sqrt{G_2/2IC_1 \sqrt{L/C_1}}.
\]
(16)
Substituting the calculated values for \(L\), \(C_1\), and \(G_2\) in (16), and (16) in (14) yields
\[
t_1 = 300 \text{ ps}
\]
(17)
which compares favorably with the measured transition duration of 362 ps.

Similarly, substituting the calculated values for \(L\) and \(C_1\) in the equation for \(R_0\) yields
\[
R_0 = 38 \Omega.
\]
(18)
From the measurements described earlier, \(t_1\) and \(R_0\) of the prototype filter are found to be 362 ps and 40 \(\Omega\), respectively. This compares favorably with the modeled values and provides a promising confirmation of the theoretical development of the filter model. Note that the filter corner frequency, the point where the response is 3-dB down, occurs at 780 MHz in good agreement with a measurement of \(S_{21}\) made with the APMS for which the 3-dB point is at 800 MHz. Due to its Gaussian nature, the filter response will decay an additional 13 dB before the 2-GHz limit of our approximation region is reached and an additional 135 dB at the frequency where \(G_2\) is comparable to \(\omega C_1\). Therefore, the approximation should yield good results.

Low-frequency measurements have also been made of \(L\) and \(C_1\) from 10 kHz to 10 MHz which yield
\[
\begin{align*}
L & = 21.7 \text{ pF} \quad \text{or} \quad C_1 = 472 \text{ pF/m} \\
L & = 30 \text{ nF} \quad \text{or} \quad L = 6.5 \times 10^{-7} \text{ H/m}
\end{align*}
\]
(19)
which compare favorably with the calculated values of 504 pF/m and 7.14 \times 10^{-7} H/m.

**Temperature Dependence**

In order for this filter to be useful in producing precision reference waveforms, an accurate account must be made of any temperature dependence. According to (16), for the transition duration of the filter, the temperature dependence of the transition duration is primarily determined by the effect of temperature on the capacitance and conductance of the silicon layer (assuming negligible change of line length and line inductance with temperature).

To measure the capacitance and conductance as a function of temperature, a capacitance bridge was used, as shown in Fig. 11. An oven controlled the temperature and a thermocouple attached to the base plate of the filter measured the temperature. As resistance measurements were also necessary, a voltage source and a known resistance were put in series with the filter, as shown in Fig. 12, and the voltage across the filter was measured with four-digit accuracy. In the figure \(V_S\) and \(V_L\) are the reference and measured voltages from which the unknown resistance \(R_L\) can be calculated in terms of a reference resistor \(R_S\) with the formula
\[
R_L = R_S V_L/(V_S - V_L).
\]
(20)

Two mounts were used for the measurements. The first was the mount of Figs. 6 and 7 described earlier. The center conductor of each coaxial line makes direct contact with the aluminum. The second was a mount which used a modified SMA-to-stripline adapter, and is shown schematically in Fig. 13.
The 1300- and 52-Ω resistors in parallel create the 50-Ω termination for the stripline.

In determining the dependence of conductance on temperature, it was necessary to measure the effect of temperature on the aluminum and contact resistance. To do this, the resistors on the filter of Fig. 13 were replaced with a short. The filter was placed in the oven and the resistance was measured using the circuit of Fig. 10 at temperatures near room temperature. The readings were taken at 20 V with a reference resistance of 577 Ω. With the short disconnected from the filter this same circuit was then used to measure the temperature dependence of the silicon conductance and capacitance. The bridge voltage was 10 V at 100 kHz.

To obtain small changes in temperature around room temperature, the oven blower only was turned on. After the oven reached the maximum desired temperature, the blower was turned off and, as the oven cavity cooled, a second set of measurements was taken. The later measurements took longer but allowed the temperature to stabilize before taking the thermocouple voltage reading. The results were all plotted against temperature over the measurement temperature range from 24 to 31°C. Since the results were basically linear, linear regression was used to get a best fit line for each measurement and the results were extrapolated to 20°C.

The coaxial line connector filter mount measurements gave a capacitance temperature coefficient of 0.01 percent/°C at 20°C and a conductance temperature coefficient of -1.1 percent/°C at 20°C. The corresponding results using the mount with the SMA adaptor were 0.0034 and -2.9 percent/°C, respectively. The measurement of the contact and aluminium resistance yielded a temperature coefficient of 0.34 percent/°C.

Even though the conductance temperature coefficient is different for the two mounts, the slopes of the lines are approximately equal. With the mount of Fig. 11, the conductances seen by the bridge are of the order of 0.04-µS range. For the second mount, the conductances are around 0.01 µS. Series resistance has the effect of increasing the observed shunt conductance. The coaxial lines can roll when connecting and disconnecting the connectors, weakening the contact and increasing the resistance with the result that the resistance is greater in the coaxial line filter mount. For this reason, the measurements of the filter on the mount with the coaxial lines were considered unrepresentative of the filter's characteristics and were disregarded, while the measurements made using the SMA connectors were used to determine the transition duration temperature dependence.

Because the temperature coefficient of the capacitance is less than 0.01 percent/°C and the temperature dependence of the aluminum and contact resistance is 0.34 percent/°C compared to 2.9 percent/°C for the conductance, the transition duration temperature dependence is primarily determined by the temperature dependence of the conductance.

Using the measured values of line length, inductance, capacitance, conductance and temperature dependence of conductance, the temperature dependence of the filter transition duration was estimated to be 1 percent/°C. This is approximately the same as the temperature coefficient of the liquid dielectric waveform filters [10].

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