A 50-ppm AC Reference Standard which Spans 1 Hz to 50 kHz

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Abstract—A digital sinewave generator which spans 1 Hz to 50 kHz is described. The rms amplitude is characterized by an internal thermal converter and corrected by a microcomputer to an uncertainty of 50 ppm. Amplitude is programmable from 0 to 7.07 V rms.

I. INTRODUCTION

HIGH-ACCURACY rms voltage measurements have traditionally been made by comparing the unknown voltage to a dc reference with a thermal voltage converter (TVC). RMS accuracies of better than 100 ppm can be achieved over a frequency range of 10 Hz to 50 kHz, although the time required to perform a single measurement may exceed several minutes even with semiautomated systems. Fast rms voltage measurements have been possible with electronic multipliers in the 0.2-percent uncertainty range. Recently, a new class of medium-speed rms voltmeters with a variety of operating principles extending from hybrid thermal converters to waveform samplers have become commercially available. They claim audio-frequency accuracies between 100 and 500 ppm and advertise response times of less than 10 s. The ac reference standard described in the following sections is an effort to provide fast (1-s response), accurate (50 ppm), programmable calibration support for these voltmeters, over a frequency range of 1 Hz to 50 kHz.

The standard consists of a microcomputer-controlled generator which uses digital synthesis techniques to produce a 7.07 V rms (10-V peak) sinewave. Digital waveform generation has been used in precision measurements for a number of years, and the technique employed here is similar to that described in [1]. As shown in Fig. 1, the waveform is synthesized from digital sine values stored in memory which are sequentially applied to a digital-to-analog converter (DAC). The result is

Fig. 1. Block diagram of the ac reference standard.
a staircase approximation of a sinewave whose frequency is
programmed by selecting the number of sample steps per pe-
period and the sampling frequency or rate at which the DAC
is updated. This technique is capable of fast response, easily
settling to a new frequency setting in less than 1 s. In addition
to excellent frequency and amplitude stability, this approach
provides the user with the ability to predict the rms value
of the waveform based on the dc reference of the DAC. Ideally,
the uncertainty of this prediction is due only to the quantization
error in the sampling process and is less than the value of one
least significant bit of the generating DAC. It can be shown
that the rms value of a perfectly sampled sinewave, constructed
of three or more steps of equal duration, is identical to the rms
value of a pure sinewave which passes through the sample
points [2]. If $V_{\text{peak}}$ is the peak value of the pure sinewave, the
rms value of either waveform is given by

$$V_{\text{rms}} = V_{\text{peak}}/\sqrt{2}.$$  (1)

The sampled waveform, however, contains sampling harmonics
which contribute to this rms value. The spectrum is calculated
by mixing the sampling frequency $F_s$ with the fundamental
time period of the generated waveform $F_1$, and the result is an
infinite series of harmonic pairs around integral multiples of
$F_s$. Harmonics obey

$$F_h = kF_s \pm F_1, \quad \text{for } k = 1 \text{ to } \infty$$  (2)

where $h$ denotes the harmonic number of the fundamental $F_1$, and

$$h = \lfloor kF_s/F_1 \rfloor \pm 1.$$  (3)

The harmonic amplitudes are defined by

$$A_h = \frac{\sin(\pi F_h T/\pi F_1)}{\pi F_h T} V_{\text{peak}}$$  (4)

where $T$ (equal to $1/F_s$) is the sampling period. The relation-
ship given in (4) is the frequency-response characteristic of
the DAC (zero-order hold) used to generate the waveform. From
these relationships, the total harmonic distortion (THD)
may be expressed as

$$\text{THD} = \left( \sum A_h^2 \right)^{1/2}/V_{\text{peak}} = \left( V_{\text{peak}}^2 - A_1^2 \right)^{1/2}/V_{\text{peak}}$$  (5)

where $A_1$ is the peak amplitude of the fundamental component.

A knowledge of the spectral density is particularly important
if the generator is used to calibrate voltmeters with limited
bandwidth. Some filtering may be required to eliminate har-
monics which contribute a measurable amount to the rms value
but are beyond the bandwidth capability of the voltmeter. A
sinewave constructed of 256 errorless steps, for example, from
(5) will have a THD of 0.7 percent. Since the first distortion
component occurs at the 255th harmonic, a low-pass filter can
effectively eliminate the distortion with negligible attenuation
of the fundamental. The removal of 0.7 percent of sampling
harmonics, while significantly improving the purity of the
waveform, reduces its rms value by only 25 ppm, correctable
with a proportional increase in gain. At 32 steps per period,
however, the THD is 5.7 percent, which, if perfectly filtered,
would reduce the rms value by 0.16 percent. Thus it can be seen
that the filter transfer function becomes increasingly critical
as the number of steps per period decreases.

![Fig. 2. RMS error versus frequency of the DAC generated sinewave; these practical limitations are based on an evaluation of several DAC's.](image)

The discussion so far has considered ideal waveform gen-
eration where quantization noise is the limiting factor. In
practice, static DAC parameters such as offset, gain, and
linearity will also degrade the ability to predict the rms value.
It is the dynamic errors of the DAC and associated amplifier,
however, which seriously limit the predictability of rms value
with frequency. Effects such as bandwidth limiting, glitches
(switching skews and digital feedthrough), overshoot, and
ringing distort the waveform and cause proportionally larger errors as the frequency is increased
[3].

Four configurations of 16-bit current output DAC's
followed by fast settling amplifiers were tested, and the difference
in the output voltage from the predicted rms value (based on a
dc calibration of the DAC-amplifier pair) was measured.
Differences of 50 ppm or less up to sinewave frequencies of 10
kHz were attained in several cases by carefully trimming the
voltage step response of each converter. However, consistent
performance at this accuracy was limited to about 2 kHz. By
utilizing a faster 12-bit voltage output DAC, differences in the
100-ppm range could be achieved through 20 kHz.

Fig. 2 gives the practical limits to which this approach can
be used to predict rms value. The curves are based upon the
evaluation of a number of DAC's and amplifiers, making use of
16-bit converters for the lower frequencies and 12-bit con-
verters above about 5 kHz. Accuracy limitations are a function
of DAC settling time which determines the maximum sam-
ping frequency. At higher fundamental frequencies, as the
number of steps per period decreases, harmonics provide a
larger portion of the signal power, and errors due to amplifier
bandwidth limiting predominate. It is possible to improve the
frequency response by measuring and correcting for this
bandwidth limitation. However, a more effective method,
which compensates for other sources of error as well, is to
characterize the waveform using a thermal voltage converter.
Digitally coded gain corrections are then applied to the
generated DAC as the frequency is changed.

### III. RMS-TO-DIGITAL CONVERTER

A modified TVC with a digital output has been incorporated
into the standard and is used to characterize the rms error as
a function of frequency. Shown in Fig. 3, the circuit uses a
TVC (which consists of a single-junction thermoelement and
a current-limiting film resistor) to sequentially compare the
generated sinewave at 7.07 V rms to a dc reference. The
thermoelement output is nulled with the output of a stable
DAC. The small-difference current is then amplified and converted to a frequency with a resolution of approximately 1 ppm/Hz. The output is integrated in a counter for a period of 1 s for parts-per-million precision; however, longer periods may be used to minimize the effects of low-frequency noise in the electronics.

Repeated measurements of the generated waveform, made over several hours, show standard deviations of less than 5 ppm. This performance can be attributed in part to the inherent stability of DAC-generated waveforms and in part to the particular measurement approach used in which the thermoelement heater is maintained at a nearly constant temperature, while the interval between measurements is determined with millisecond precision by the system controller.

The ac/dc difference of the TVC can be calibrated to within 10 ppm through 50 kHz, and the total uncertainty of a single measurement is estimated to be less than 25 ppm. Thus rms error of the generated waveform can be measured to high accuracy over a frequency range of approximately 10 Hz to 50 kHz. Measurements are made off-line while the instrument is idle, and a least squares fit to the data provides the digital gain settings necessary to correct the waveform at any frequency. This characterization correction approach is particularly effective for the following reasons:

1) It defers the time-consuming TVC measurements to a standby mode when the instrument is not being used, allowing fast settling in the normal operating mode.

2) The built-in TVC previously described allows the characterization to be continually updated resulting in higher confidence levels.

3) The rms value of the waveform is very repeatable, demonstrating residual standard deviations to the frequency response fit of less than 10 ppm. This performance implies correction capability to within 50 ppm of the dc reference while maintaining response times under 1 s. If higher accuracy is required, the thermal voltage converter can be used in real time to correct the waveform to within 25 ppm of the dc reference while slowing the response somewhat.

The curves in Fig. 4 show the improved performance typically achieved by characterizing the waveform with the internal TVC and correcting rms errors based on a series of linear fits to the frequency response data.

IV. SCALING

The generated sinewave is scaled between 0 and 7.07 rms, depending on the frequency, with resistive or inductive voltage dividers. While both types are capable of scaling unfiltered (sampled) waveforms, their response to sinusoids is more predictable, and for most applications it is the filtered sinewave which is attenuated.

Between 1 Hz and 1 kHz, scaling is done with the multiplying DAC shown in Fig. 5. A relay switched R-2R ladder provides the four most significant bits with an integrated circuit-multiplying DAC providing the lower 14 bits. This composite 18-bit multiplying DAC has static linearity errors measured to be less than 6 ppm [4]. Its linearity with an ac reference, measured with low-frequency inductive voltage dividers, appears to be within 20 ppm through 2 kHz.

A 20-bit, binary inductive divider has been constructed to scale the waveform above 1 kHz. This divider is based on a design by Hoer and Smith [5] with relay switching added to provide programmability. The first 10 bits are configured as two-stage ratio transformers to reduce the excitation current and thereby minimize the errors caused by loading of each binary stage by subsequent stages. The resulting inductive divider shown in Fig. 6 has measured ratio errors less than 2 ppm between 100 Hz and 10 kHz (the upper frequency of inductive divider calibrations at NBS, Gaithersburg). It is difficult to accurately extrapolate to higher frequencies, but with binary dividers it is possible to demonstrate the symmetry of each 1:1 winding with a simple reversing technique. The combined asymmetry of the four most significant windings appears to be less than 5 ppm through 50 kHz. Superposition
errors, which occur when the lower ordered bits do not add up to the next higher ordered bit (e.g., 0111 + 0001 ≠ 1000), are also less than 5 ppm through 50 kHz. These tests and comparisons with other binary inductive dividers of different design have led to the assumption that ratio errors over the frequency range of 1–50 kHz will probably not exceed 10 ppm. The divider’s output impedance is approximately 1 Ω, allowing it to drive relatively low impedances; however, at loads under 50 to 100 kΩ, it is generally advantageous to buffer the output. Errors imposed by the buffer-amplifier bandwidth can be minimized by characterizing its frequency response with the internal thermal converter, as previously described.

V. CONCLUSIONS

A microcomputer-controlled waveform generator, which synthesizes 7.07 V rms sinewaves from 1 Hz to 50 kHz, has been described. The rms value of the waveform can be predicted to within 100 ppm through the audio-frequency range, based on a dc calibration of the generating DAC. Uncertainties of 50 ppm through 50 kHz can be achieved by characterizing the waveform off-line with an rms to frequency converter (which utilizes a TVC), and applying gain corrections based on linear fits to the frequency response curve. The THD of the sampled waveform ranges from less than 0.01 percent at power frequencies to nearly 6 percent at 50 kHz. Low-pass active filters optimized for flat passband and maximum attenuation of the sampling frequency reduce the distortion to negligible levels at low frequencies and to less than 0.5 percent at 50 kHz. The passband response is also characterized by the internal thermal converter, and appropriate gain corrections bring the rms value of the filtered sinewave to within 50 ppm of its predicted value. Precision, programmable dividers scale the signal with only slight degradation in accuracy. With occasional monitoring by the thermal converter, an uncertainty of 50 ppm of full scale can usually be maintained between 0 and 7.07 V rms over the entire frequency range. Waveform parameters such as frequency, amplitude, and filtering are normally entered through the controller keyboard or through software routines. Remote operation over the IEEE-488 and RS232 buses is also possible. Settling time (to rated rms accuracy) after a change in any of the previously mentioned parameters is generally less than 1 s due mainly to calculation time and output delays in the controller.

Hardware to extend programmable voltages to 100 V rms is being constructed and designs for a high-speed generator capable of synthesizing sinewaves out to 1 MHz are being investigated. While precise generation and measurement of fixed amplitude sinewaves between 1 Hz and 1 MHz is of primary interest, it is accurate amplitude scaling which makes the standard particularly useful as a calibrator. Binary inductive dividers appear to offer the solution to this scaling problem by providing accurate programmable ratios over a wide frequency range in a relatively small package. With this in mind, measurement techniques to characterize these devices more thoroughly between 10 kHz and 1 MHz will be pursued.

REFERENCES