Coherent Image Fringe Contrast Measurement with Sensing Arrays

WILLIAM W. SEEMULLER, MEMBER, IEEE

Abstract—Two methods are described for measuring interferometric fringe contrast with sensing arrays. The absence of air turbulence allows contrast to be determined quickly by measuring the intensity of the fringe pattern at phases $90^\circ$ apart. The presence of turbulence requires that the peak-to-peak fringe intensity be found by sampling the fringes as they are scanned across the array. An analysis is presented showing the relation between maximum error in fringe contrast measurement and the number of samples taken. Experimental results are presented which show the effect of turbulence on fringe contrast measurement.

INTRODUCTION

The technology of solid-state light-sensitive arrays is sufficiently advanced at this time to make these devices useful in a variety of optical measurement applications. Initially, arrays were useful only for low-resolution imaging applications because of small element count and poor dynamic range. Presently, arrays having resolution equivalent to a videcon with a dynamic range of $10^3$ are available. This performance opens up applications for arrays in the areas of high-resolution image digitization, densitometry, and image mensuration. The chief advantage of the array as an optical transducer for these applications (as opposed to a videcon or image dissector) is the fixed and accurate element geometry which insures a fixed and known spatial relation between pixels within the array area. Because the array has many elements and because it is scanned electronically, it is also a very rapid device and eliminates the need for mechanical scanning.

This paper treats the problem of using arrays to measure the contrast of fringes produced by an interferometer, when the fringe pattern is varied in time across the array, by varying the path length of one arm of the interferometer. Although this requirement arose because of the Heterodyne Optical Correlator (HOC), the work described is applicable to any situation where the accurate measurement of fringe position and contrast is required, such as in optical component testing [1]. The HOC was developed to provide a rapid means for generating elevation contours from stereo, aerial-photograph pairs for map production. The HOC has been described elsewhere [2] and will not be discussed in detail here. Basically, the HOC is an interferometer in which one member of a stereograph pair is inserted in each arm. The two photographs are superimposed on the output image plane. Areas of good match between the photographs result in high fringe contrast in the output plane. Since high metric accuracy is required in the contour generating process, an array was chosen to measure the position and contrast of fringes in the HOC output plane.

The HOC correlation process is extremely rapid. The speed of the overall HOC system is determined chiefly by the speed at which image data can be extracted from the array and processed by the computer which is a part of the HOC system. This paper describes two methods for determining fringe contrast when a self-scanned array is used in the HOC system. An analysis is presented showing the relation between data rate and accuracy in fringe contrast measurement. Experimental data are presented showing the effect of turbulence on fringe contrast measurement.
The Measurement Problem

Fig. 1 is a block diagram of the HOC system emphasizing the data-acquisition and processing section. A Hewlett-Packard 21MX series computer is used for system control and data collection and reduction. From [2] the light-intensity distribution on the image plane due to the superposition of the two aerial transparencies is

\[ I(x, y) = t_1(x, y)^2 + t_2(x, y)^2 + 2t_1(x, y)t_2(x, y)\cos[k\delta(x, y)] \]  

(1)

where \( t_1 \) and \( t_2 \) are the amplitude transmissions of the transparencies, \( \delta(x, y) \) is the path-length difference between the two interferometer arms, and \( k = 2\pi/\lambda \), where \( \lambda \) is the wavelength of the laser light. If heterodyning is introduced so that the path length in one arm is varied linearly in time the intensity becomes

\[ I = t_1^2 + t_2^2 + 2t_1t_2\cos[k\delta + \omega t] \]  

(2)

where \( \omega \) is the angular frequency resulting from the linear path-length variation. Each array element integrates intensity over its active area \( A \), hence, the intensity on an element can be expressed as

\[ I = \int t_1^2 dA + \int t_2^2 dA + 2\int t_1t_2\cos(\theta + \omega t) dA \]  

(3)

The path-length variation over a single-array element can be made very small by adjusting the tilt of the interferometer. Thus \( k\delta \) in (3) can be set equal to a constant \( \theta \), and the intensity can now be expressed as

\[ I = \int t_1^2 dA + \int t_2^2 dA + 2\cos(\theta + \omega t)\int t_1t_2 dA \]  

(4)

In the HOC system, image correlation is determined by computing a normalized correlation coefficient which is expressed as

\[ C_n = \frac{\int t_1t_2 dA}{\sqrt{\int t_1^2 dA\int t_2^2 dA}} \]  

(5)

This quantity expresses the degree of correlation or match between the two transparencies and is a maximum of 1 when \( t_1 = t_2 \) over an element area. The first two terms in (4) are the intensities \( I_1 \) and \( I_2 \) on an array element due to the photographs in arm 1 and 2, respectively. The third term is the intensity due to the interference between the two photographs and is directly proportional to the correlation between them. The coefficient in (5) can be computed with the data from three measurements; \( I_1, I_2 \), and the amplitude of the cosine term in (4) which can be found by measuring the peak-to-peak (P–P) variation of (4).

The array used in the HOC and all other available large-area arrays are self-scanned devices with the video output multiplexed on a single output line. The output is usually a train of pulses or boxcar steps. The amplitude of a particular step is directly proportional to the time integrated light intensity on the corresponding array element. The array is scanned electronically, usually in a raster mode (row by row), and the integration time for each element is the time required to scan the entire array (the frame period). Because the sensitivity of an array element is dependent on the integration time, self-scanned arrays cannot be randomly accessed.

The measurement of \( I_1 \) and \( I_2 \) with the array is straightforward, requiring only a single frame to input the steady intensity values into the computer. This can be accomplished rapidly with a direct memory access channel to the computer. The measurement of the P–P value of (4) is not straightforward because the fringe pattern must be scanned continuously across the array because of air turbulence. This situation requires that the P–P variation be found by sampling continuously the time-varying pattern and looking for minimum and maximum intensity values. The spatial variation in the fringe pattern will be nonuniform in general so that each array element is at an unknown phase point on the time-varying sinusoid.

The peak-to-peak values can be determined by using the system in Fig. 1 in the following manner. Two array buffers are set aside in the computer memory: one for the minimum (min) values and one for the maximum (max) values. With the fringe pattern varying on the array, one frame is stored in both buffers. On the next frame, the computer inputs and compares, element-by-element, the value to the stored max and min. If an input element value is less than the min, it replaces the min. If it is greater than the max, it replaces the max; otherwise it is discarded. This process is done for each element and it is repeated for a predetermined number of frames. The only way to be sure of the accuracy of the max and min value is to take a sufficient number of samples, yet to be determined. The P–P sampling process is the bottleneck in the computation of the
correlation coefficients. Therefore, the speed at which map compilation can be done with the HOC is largely determined by the speed at which the P-P fringe values can be determined. The factors determining the speed of the P-P sampling process are the computer speed and the number of frames sampled. The computer speed can be maximized by efficient assembly level programming or microprogramming. More important is the number of samples taken to determine the P-P variations. Here, there is a tradeoff between speed and accuracy and this sets an upper bound on the compilation speed. The next section contains an analysis showing the relation between the number of samples taken and accuracy in P-P determination.

**P-P Measurement Error**

As the fringes are scanned sinusoidally across the array, the computer samples consecutive array frames to determine the P-P level for each element. Error arises in this process because of the array integration time and because the min and max values might not be sampled during the measurement time. If \( \tau_f \) is the array frame period, and \( \omega \) is the angular frequency of the fringes scanning across an element, then the element integrates the sinusoid over an angle \( \alpha = \omega \tau_f \) as shown in Fig. 2. If \( \theta \) is the angle to the center of the integration strip of width \( \alpha \), the element response can be expressed as

\[
\frac{1}{\alpha} \int_{\theta-(\alpha/2)}^{\theta+(\alpha/2)} \sin \xi \, d\xi \tag{6}
\]

where \( \xi \) is a variable of integration.

When evaluated, (6) becomes

\[
\frac{2}{\alpha} \sin \frac{\alpha}{2} \sin \theta \tag{7}
\]

or

\[
\sin \frac{\alpha}{2} \sin \theta \tag{8}
\]

The frame integration time has the effect of reducing the value measured on the sine way by the factor, \( \sin (\alpha/2) \). This in itself is a source of P-P measurement error. However, greater error will occur because of sampling at random points on the fringe cycle. The number of samples per cycle is \( N = 2\pi / \alpha = 2\pi / \omega \tau_f \). At least two samples must be taken to determine the P-P value. Since the \( N \) samples may be taken at any phase on the fringe cycle the error will vary widely. Fig. 2 shows the condition resulting in maximum error. It is assumed that an even integer number of samples is taken per cycle. The maximum error will occur when samples are taken at equal angles from 90° but not at 90°. If the element integration time is neglected, the maximum error in percent is

\[
E_m = 100 \left[ 1 - \frac{\pi}{N} \right] \frac{\sin \frac{\pi}{N}}{\sin \frac{\pi}{N}} \tag{9}
\]

Fig. 3 shows the maximum percent error in P-P sampling as a function of \( N \), the number of samples taken per fringe cycle (10). The maximum error decrease initially (very rapidly) with increasing \( N \) and a point is soon reached where very little is gained in taking more samples.

**P-P Determination by 90° Phase Shift**

If air turbulence is very small, an alternate and much quicker method exists for determining the P-P intensity variation on each element. If heterodyning is eliminated, the third term in (4) becomes

\[
M_1 = [2 \int t_1 t_2 \, dA] \cos \theta \tag{11}
\]

shifting the phase of one arm of the interferometer by 90° and taking another reading yields

\[
M_2 = [2 \int t_1 t_2 \, dA] \cos \left( \theta + \frac{\pi}{2} \right) = -[2 \int t_1 t_2 \, dA] \sin \theta. \tag{12}
\]
Thus the correlation term in (4) can be extracted as below

$$\sum t_1 t_2 = \frac{1}{2} \sqrt{M_1^2 + M_2^2}. \quad (13)$$

Only four measurements are required to compute the normalized correlation coefficient using this method; $I_1$ and $I_2$ as mentioned before, $I_3$ (both arms interferring at one phase angle), and $I_4$ (both arms interferring at a 90° phase shift from $I_3$). The quantities $M_1$ and $M_2$ are expressed as

$$M_1 = I_3 - I_1 - I_2,$$
$$M_2 = I_4 - I_3 - I_2. \quad (14)$$

The phase shift can be accomplished by rotating a piezoelectric (PZ) mirror a fixed distance between measurements.

Theoretically, the only error in this method arises because of the PZ displacement which can be made very small by calibration. However, turbulence present in an actual operating system may render this method impractical because of the unknown phase changes between measurements $I_3$ and $I_4$.

**THE EFFECT OF AIR TURBULENCE**

Air turbulence causes random fluctuations in the fringe pattern when both arms of the interferometer are open. This random-phase fluctuation can be modeled as low-frequency noise $N(t)$. If turbulence is considered, the third term in (4) becomes

$$2 \cos(\omega t + N(t)) \sum t_1 t_2 \, dA. \quad (15)$$

If the highest significant frequency component in the power spectrum of $N(t)$ is small compared to $\omega$, then $N(t)$ can be considered constant over one cycle of the heterodyne signal, and the P–P error analysis holds. Likewise, if $N(t)$ is essentially constant during the measurement of $I_3$ and $I_4$, the 90° phase shift method can be used. It may be possible to shield the HOC system so that turbulence can be controlled to allow the use of the quicker, 90° phase-shift, method. Moderate turbulence can be tolerated in the heterodyne method, but it prohibits the use of the 90° phase-shift method. Even if turbulence is severe, it is possible to make measurements with the heterodyne method if enough samples are taken. The P–P error analysis presented earlier would not apply in this case and the relation between samples and error would have to be determined experimentally.

**SAMPLING FRINGES WITH AIR TURBULENCE—EXPERIMENTAL RESULTS**

Fig. 3 shows the maximum P–P error as a function of the number of samples per fringe cycle for the ideal case of no air turbulence in the interferometer. This condition can be met only by shielding the light paths in order to eliminate air currents. In a more practical situation it is suspected that air turbulence would degrade the performance depicted in Fig. 3. Fig. 4. shows the experimental setup used to study the effect of air turbulence. The HOC is used with open light paths to produce broad fringes on the array. A beamsplitter is used just in front of the array to image fringes on a phototransistor. This enables the frequency of the fringes on the Reticon 32- × 32-element array to be determined by measuring the frequency of the phototransistor output with the electronic counter. The array-video output is monitored with an oscilloscope and the light intensity on the array is varied with the neutral-density step wedge. The size of the fringes on the array is monitored with the video display. The video is digitized with the A/D converter for input to the computer. The fringes are scanned across the array by varying the path length of one arm with a piece of glass rotated by a mirror scanner. A triggerable ramp generator drives the scanner. The computer is used for control, data reduction and storage; experimental results are output to the teletype.

This setup was used to sample fringes many times to obtain a statistical distribution of sampling error. A Fortran program was written to take a specified number of P–P measurements, store the results, and compute the probability of percent of error. During the experiment, the amplitude and duration of
the ramp was adjusted so that the frequency of the fringes, indicated by the counter, resulted in the desired number of samples per cycle for the experiment. For example, the array had an input frame period of 75 ms. If four samples per cycle were desired, the ramp would be adjusted to give a fringe period of $4 \times 75 = 300$ ms. Turbulence prevented the exact adjustment of fringe period. The ramp was set to give the approximate, average period over a short time span. The helium–neon laser light was attenuated with the neutral density wedge to place the fringe amplitude within the dynamic range of the array. The parameters required for an experimental run are 1) the coordinates of the array element where P–P error is to be tested, 2) the number of samples per fringe cycle, and 3) the number of P–P measurement trials. After the parameters are input with the TTY, data acquisition and result output are done under computer control. The measurement cycle is as follows: The computer sends a trigger signal to the ramp generator to initiate fringe scanning. P–P measurements are made on the selected array element for the specified number of samples per cycle. The duration of the ramp is not computer controllable, and a fixed number of measurements can be made before the ramp returns. The computer stops taking measurements before the end of the ramp and waits for a period of time sufficient to insure the return of the ramp before triggering the ramp again. This process is repeated until the specified number of measurements have been taken and stored.

The maximum P–P measurement is taken as the true value and the percent error is computed in terms of this true value. The set of measurements is divided into percent error bins, and the probability of the error being in the percent range of the bin is computed by dividing the population of the bin by the total number of measurements. Fig. 5 (a) and (b) shows the probability of P–P measurement error for 4 and 10 samples per cycle, respectively. Each point gives the probability of the error being between the indicated error and the next lower error. The maximum theoretical error as given in Fig. 3 is shown in the plots. The turbulence results in a possibility of errors greater than the theoretical maximum as the figures clearly indicate. As expected, turbulence results in a greater relative variation from the maximum error for the slower 10-sample rate than for the 4-sample rate. The effect of turbulence could be eliminated if the computer could sample at a sufficiently fast rate. Although air turbulence is the major factor, the figures reflect all the random fluctuations in the array image, such as laser-intensity variations and electronic noise. The turbulence present during the measurements is fairly typical of a laboratory environment containing electronic equipment with cooling fans.

**Conclusions**

The solid-state-sensing array is attractive as a device for the measurement of interferometer fringe contrast and position because of its fixed and accurate element geometry. Fringe contrast measurement requires the measurement of P–P intensity variation on the array. If air turbulence is eliminated, this can be accomplished quickly by making measurement of the fringe pattern differing by 90° phase shift. The presence of turbulence requires the random sampling of a time-varying fringe pattern to determine the P–P intensity. In this case, there is a tradeoff in speed (the number of samples taken) and the maximum allowable error in P–P measurement. If samples can be taken fast enough to make the effect of turbulence insignificant, the maximum error can be predicted theoretically. Significant turbulence increases the likelihood of errors larger than the theoretical maximum.

**References**
