Understanding the Subject-Specific Effects of Pupil Dilation on Iris Recognition in the NIR Spectrum

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Abstract—A recent report from the National Institute of Standards and Technology (NIST) showed that changes in pupil dilation affect the performance of iris recognition algorithms. Hence, there is a need to explore the effects of pupil motion from a biological standpoint. Our work looks at the pupil’s response to light, otherwise known as the pupil light reflex (PLR). By modeling the PLR using a nonlinear delay differential equation while considering images acquired in the near infrared (NIR) spectral band, we study both average and subject-specific pupil dilation effects. Experiments conducted on the WVU iris video dataset† convey the efficacy of our work in describing and evaluating pupillary response for both general and individual responses. The results of this work can be used to develop robust iris recognition algorithms that handle the effects of pupil dilation.

I. INTRODUCTION

The iris is the annular region of the eye that is located behind the cornea and in front of the lens. The rich textural details of the multi-layered iris can be used as a biometric cue for recognizing individuals [1], [2]. A number of iris recognition algorithms have been reported in the literature and most of them operate on iris images acquired in the near infrared (NIR) spectrum. This is because the texture of even dark-colored irides is better discernible in the NIR spectrum. Due to the richness of the iris texture and its variability across the subject population [3], iris-based biometric systems have been used in several applications ranging from border control systems to military applications.

Although iris recognition methods have demonstrated good matching accuracy on large datasets [17], a number of challenges are yet to be satisfactorily resolved. One such challenge is the impact of pupil dilation on the performance of iris recognition systems. The musculature of the iris constricts and dilates the pupil thereby controlling the amount of light entering the eye. Researchers have observed a degradation in matching accuracy when the two images to be matched are at different dilated levels [15]. Hence, there is a need to counteract this effect in iris recognition.

A. Literature Review of Pupil Dynamics

Pupil motion has been explored and studied extensively over several decades in several fields including ophthalmology, mathematical biology, and computer graphics. It is known that pupil motion is dictated by several factors such as focal length, heart rate, and emotional factors; however, physiological models were based off of variations of light intensity due to the availability of vast amounts of data available supporting this factor. Originally, these models were based on empirical studies. For example, the model proposed by Moon and Spencer [4] explored changes in dilation based off of background luminance. Subsequent models, however, attempted to incorporate the overall dynamics of the pupil rather than just changes in the pupil size. Ellis [5] performed observational studies to model the maximum constriction and dilation velocities as well as the latency2 at a given light intensity level. Later, Link and Stark [6] empirically described the pupil latency via the following equation:

$$\tau(R, L_{FL}) = 253 - 14 \ln(L_{FL}) + 70R - 29R \ln(L_{FL})$$

(1)

where $\tau$ is the latency in ms, $L_{FL}$ is the luminance in foot $-Lambert$, and $R$ is the frequency in Hz.

From the empirical models emerged models derived from physiological and anatomical observations without the reliance on experimental data. Usui and Stark [7] developed a parametric model of the iris to describe the pupil characteristics in response to light by using probability distribution functions to model the random fluctuations. Later, Stark [8] concluded that the pupillary response can be viewed as a negative feedback dynamical system. Although this characterization of the pupil dynamics is attractive, it was Longtin and Milton [9] who were the first to express the dynamics of the pupil light reflex (PLR) as a nonlinear delay differential equation given by

$$\frac{dg}{dA} + \alpha g(A) = \gamma \ln \frac{\phi(t - \tau)}{\phi}$$

(2)

where $\tau$ is the lag time between the light pulse reaching the retina and the pupillary reaction.

1All iris images displayed were taken from the WVU iris video dataset and used with permission.

2The lag time between the light pulse reaching the retina and the pupillary reaction.
where $A$ is the pupil area, $\phi$ is the retinal light flux, and $\bar{\phi}$ is the light level when there is no pupillary response. Furthermore, $\tau$ is the time delay due to retinal processing, which includes responses from the midbrain nuclei, and $\gamma > 0$ is a form fitting parameter. In the derivation of equation (2), $g(A)$ takes into account the inverse relationship between the pupil area and iris muscle activity. The direct relationship is given by

$$A = f(x) = \frac{\beta_1 \theta^n}{\theta^n + x^n} + \beta_2$$

(3)

where $x$ is the iris muscle activity\(^3\), $\beta_1$ is the minimum pupil area, $\beta_1 + \beta_2$ is the maximum pupil area, and $\theta$ and $n$ are positive constants. In equations (2) and (3) Longtin chose the Hill function because it best represents the fact that the pupil area is positive and has finite limits while accounting for the elastic properties of the iris muscle. With the purposes of increasing facial animation in characters, Pamplona [10] revisited Longtin’s equation (2) and noted that though this model is well cited, there are ambiguities in estimating the various parameters ($\alpha, \beta_1, \beta_2, \gamma, \theta, n, \bar{\phi}$). Using the steady-state formulation of (2), comparing it with the Moon and Spencer model [4], and making the appropriate conversions to the dynamic case Pamplona proposed the following formulation:

$$\frac{dM}{dt} = \frac{dD}{dt} + \bar{m}(D) = 5.2 - \gamma \ln \left( \frac{\phi(t - \tau)}{\bar{\phi}} \right) \quad \text{for } t \geq 0$$

(4)

$$D(t) = \psi_D(t) \quad \text{for } 0 \leq t \leq \tau$$

where $\bar{\alpha} = 2.3026$, $\bar{\gamma} = 0.45$, $D$ is the pupil diameter, and

$$M(D) = \tanh^{-1} \left( \frac{D - 4.9}{3} \right).$$

(5)

We note that $\psi_D(t)$ is the initial history function to specify the value of the solution prior to time $t = 0$. Pamplona further expanded on equation (4) to produce a general methodology to describe individualized pupil dynamics.

B. Related Work within the Iris Recognition Community

The impact of pupil motion on iris recognition was first noted by Ma et al. [11] who observed a number of false non-matches due to pupil dilation. Later, Thornton et al. [12] used Bayesian estimation to estimate and recover the level of iris deformation. Their results show that estimating the level of deformation leads to an improvement in iris recognition performance. Wei et al. [13] also explored this topic by investigating variations in deformation. They adopted Wyatt’s model [14] to experimentally model iris deformation as the sum of a linear stretch and a Gaussian deviation term. These realizations spawned the need to further explore the overall effects of pupil dilation and its impact on iris recognition. Hence, Hollingsworth et al. [15] conducted extensive experiments demonstrating the impact of varying degrees of dilation on iris recognition performance. Their results concluded that the differences in dilation affect overall matching accuracy. Recent theoretical and observational investigations have explored this topic from deformation and template standpoints. From a deformation viewpoint, Clark et al. [16] to abstractly model the effects of deformation due to the impact of the iris musculature. To show efficacy in their approach, existence and uniqueness conditions were determined followed by different numerical simulations to show the deformation response of the iris region. Additionally, template effects were explored by Grother et al. [17] who investigated the temporal stability of iris recognition systems. Their results confirm with the works of [18] and [15] that the difference in pupil dilation remains evident in iris recognition algorithms. Furthermore, Tomeo-Reyes and Chandran [19] performed bit error analysis to explore the consistence of texture information of the iris region from various degrees of dilation. Their results showed an increase in bit errors by by over 10% when comparisons were made between extreme and intermediary pupil sizes.

C. Our Motivation and Contribution

Our motivation for this work comes from the observations of [15], [17] and [19] that the dilation differences between the acquired and enrolled iris images affect iris matching accuracy. Furthermore, pupil dilation levels are different for each individual and these variations need to be considered. Previous investigations, such as from the works of [12], [13], [15] and [17], are mainly empirical characterizations, which are limiting because they are dependent on the particular dataset and technology at hand. Additionally, the aforementioned works neglect to consider the complete effects of pupil activity that are dependent on the iris muscle activity and the neurological delay.

The contributions of our work are two-fold. First, we revisit the work of [10] to consider both the average and subject-specific effects of dilation that are present in the NIR spectrum. Here, we consider the case of constant illumination to propose a mathematical formulation to model the general pupil response. From this, we propose an exact formulation and perform numerical comparisons to show the similarity in behavior. Next, we propose a methodology to determine the individually-based parameters where we test of our proposed process on a database of subject-specific near infrared (NIR) iris videos to show efficacy in our approach. Our work is an advancement of the prior art because we provide an analytical basis for describing the effects of pupil motion that are also present in the NIR spectrum, which can be expanded to explore other dilation defects such as aging [17]. Also, our model can be applied to understanding the impact of dilation on iris recognition from a state perspective [20]. The rest of this work is presented as follows: Section 2 provides the theoretical foundation; Section 3 provides the experimental results and comparison; and Section 4 describes our conclusions and future directions.

II. ANALYTICAL FORMULATION

Our analytical formulation begins by first assuming an iris capture scenario in an ideal operational environment such as an indoor border crossing station. Here, we make the assumption that the external illumination $I(t)$ is constant. Hence, we first examine the steady-state formulation of (4) given by

\(^3\)This activity is a consequence of the sphincter and dilator muscles.
\[ \bar{\alpha} M(D) = 5.2 - \bar{\gamma} \ln \left[ \frac{\phi^*}{\bar{\phi}} \right] \]  
\[ \text{where } D^* \text{ and } \phi^* = (\pi/4)I^*(D^*)^2 \]  
are the steady-state dilation and retinal light flux, respectively, and \( I^* \) denotes constant illumination. From (5) we obtain the inverse relationship

\[ D = 3 \tanh(M) + 4.9. \]  
(8)

and the retinal light flux can be approximated as

\[ \phi(t) \approx (\pi/4)I^*(D(t))^2. \]  
(9)

As a result, the dynamic model (4) can be expressed in the following form

\[ \frac{dM}{dt} = -\bar{\alpha}M(t) + f(M(t - \tau)) \text{ for } t \geq 0 \]

\[ M(t) = \psi_M(t) \text{ for } 0 \leq t \leq \tau \]  
(10)

where \( f(M(t - \tau)) \) is expressed as

\[ f(M(t - \tau)) = 5.2 - \bar{\gamma} \ln \left[ \frac{\pi I^*}{4 \bar{\phi}} \right] - 2 \bar{\gamma} \ln(D(t - \tau)) \]  
(11)

and \( D \) is defined by equation (8). In equation (10), \( \psi_M(t) \) is the initial history function to specify the value of the solution prior to time \( t = 0 \). We make the conjecture that this history function corresponds to \( \psi_D(t) \) in equation (4) via equation (5). From our model, we obtain an exact solution by first rewriting equation (10) as

\[ \frac{dy}{dt} = e^{\bar{\alpha}t} f(y(t - \tau)e^{-\bar{\alpha}(t - \tau)}) \]  
(12)

where \( y(t) = e^{\bar{\alpha}t}M(t) \). Next, employing the method of steps [21], we achieve our solution recursively via dividing \( t \) into several subintervals such that \( t \in [(k - 1)\tau, k\tau] \) for \( k \in \mathbb{N}^+ \). As a result, we obtain the following recursive expression:

\[ y_k(t) = y_{k-1}((k-1)\tau) + \int_0^{t-(k-1)\tau} \mathcal{K}(y, s, k, \tau)ds \]  
(13)

for \((k-1)\tau \leq t \leq k\tau\) where

\[ \mathcal{K}(y, s, k, \tau) = e^{\bar{\alpha}s} f \left( y(s + (k - 2)\tau)e^{\bar{\alpha}(s+(k-2)\tau)} \right) \].  
(14)

From the recursive expression, given by (13) and (14), we calculate the general dilation behavior from equation (8). We note that existence and uniqueness are preserved due to the continuity of the initial history function \( \psi_M(t) \) [22].

### A. Numerical Simulation of Average PLR Effects

To illustrate the effectiveness of our model, we focus our attention on simulating the average PLR response, given by equations (8) and (13), for both constriction and dilation cases. For convenience sake, we set the neurological delay \( \tau = 250 \) milliseconds as well as the illumination \( I^* = 10^{-8} \text{ lux} \) and \( I^* = 9 \times 10^{-5} \text{ lux} \) for dilation and constriction, respectively. In our simulations, we consider the initial history function \( \psi_M \) to be constant for \( t \in [0, \tau] \). This was chosen to correspond with the steady state dilation values \( D^* \) of 3.41 mm and 7.11 mm for the cases of dilation and constriction, respectively. To simulate the constriction response, we note that constriction speed is approximately three-times that of dilation speed [10].

![Fig. 1. Simulation of the average dilation response of the PLR for the neurological delay \( \tau = 250 \text{ ms} \).](image1)

![Fig. 2. Simulation of the average constriction response of the PLR for the neurological delay \( \tau = 250 \text{ ms} \).](image2)

Figures 1 and 2 show the simulation of the average pupillary response for both cases of constriction and dilation. Notice that for each case, the response begins at \( t = 0.25 \) seconds to account for the neurological delay. These responses for generalized pupil motion are similar to the work in [10]. Hence, we conclude that investigating the special case of Pamplona’s model for constant illumination can serve as an approximation to the general description of pupil activity.

### B. Accounting for Subject-Specific PLR Effects

To account for subject-specific dilation effects, we recall that there exists a one-to-one mapping \( B : D \mapsto D_i \) from the average dilation \( D \) to the subject-specific dilation \( D_i \) given by [10]:

\[ D_i = B(D) = C_{bD}(D) + r_1(C_{1D}(D) - C_{bD}(D)) \]  
(15)

where \( C_{bD}, C_{1D}, \) and \( r_1 \) are determined by
$$C_{BD}(D) = -5.44 \times 10^{-3} D^5 + 1.39 \times 10^{-1} D^4 - 1.34 D^3 + 6.22 D^2 - 13.2 D + 12.2, \quad (16)$$

$$C_{ID}(D) = -13.0 \times 10^{-3} D^5 + 3.22 \times 10^{-1} D^4 - 3.10 D^3 + 13.65 D^2 - 25.35 D + 18.2, \quad (17)$$

and \( r_I \in [0, 1] \). In the interpolation equation (15), \( r_I \) is known as the individualized parameter that determines the subject-specific response. We also note that because \( B(D) \) is a bijective function, existence and uniqueness are preserved when accounting for the subject-specific dilation effects \( D_i \).

### III. Empirical Evaluation

The goal of our empirical evaluation is to be able to quantify the subject-specific nuances in dilation in the NIR spectrum. We do this via our proposed parameter extraction methodology where we numerically estimate the individualized parameter \( r_I \) and the rate \( \dot{\alpha} \). Evaluations were conducted using the WVU iris video dataset\(^4\) to show the efficacy of the proposed formulation.

#### A. Data Acquisition and Experimentation

The interface used to collect the multispectral ocular videos consists of a multispectral camera, an annular ring light guide, a light source, two NIR light emitting diodes (LEDs), and an in-house voltage ramp generator. A Redlake MS3100 multispectral camera was attached to the mobile arm of the ophthalmologist’s slit lamp and could be easily maneuvered to position the camera for obtaining frontal iris images. This camera acquires color infrared images (CIR) with a resolution of \( 1040 \times 1392 \times 3 \) pixels that includes images from the visible light spectrum as well as the NIR spectrum. A StelarNet light source with a tungsten-krypton bulb is connected to an annular ring light guide to illuminate the eye. Additionally, two near-infrared LEDs\(^5\) were positioned on the left and right sides of the annular ring light guide to provide even illumination of the subject’s eye while focusing the multispectral camera prior to recording. Furthermore, a voltage ramp generator, developed at WVU, was connected to the tungsten-krypton bulb to produce variations in illumination. The ophthalmic videos were collected using Epix software\(^6\).

From this experimental set up, two studies were performed to understand pupillary excitation due to varying and sudden changes in illumination for time intervals of 20, 30, and 40 seconds. In the first experiment, shown in Figure 3(a), we investigate the case of gradual pupillary excitation via increasing the voltage linearly from a low value (close to zero) to a maximum voltage \( V_{\text{max}} \). The maximum voltage \( V_{\text{max}} \) is held constant beginning at the settling time \( T_S \) to the duration of the time interval [23]. This variation causes gradual constriction and dilation of the pupil for the specified intervals. In the second experiment, depicted in Figure 3(b),

\(^4\)The details on the physical set up are described in [23].

\(^5\)The central wavelength for these LEDs is 810 nm.

\(^6\)www.epixinc.com

![Fig. 3. Plots highlighting two voltage variations to illustrate gradual (a) and sudden (b) changes in illumination to excite pupil motion.](image)

Figure 4 provides an example of pupil variation from a single subject. To better observe pupil variation, we first segment the pupil region by fitting a circular contour \( C \{ (x_c, y_c), r \} \) where \( (x_c, y_c) \) and \( r \) are the center and radius of the circle, respectively. There were some subjects that had a tendency to blink during the acquisition process. In such cases, the pupil response signal is corrected as shown in Figure 5.

![Fig. 4. Examples of eye images with various pupil diameters. The contour of the segmented pupil (yellow line) is imposed on the original NIR component.](image)

#### B. Parameter Extraction Methodology and Results

Observing the bijective mapping \( B : D \mapsto D_i \), our process to account for the subject-specific effects can be described via the following stages:

1. **Perform Video-to-Data Conversion.** First, we convert the pupil diameters from pixels to millimeters.
2. **Determine the Average Initial History.** Next, we determine the average initial history. This is needed to properly account for the neurological response associated with the delay \( \tau \). We use equation (1) with \( R = 0.4 \) to estimate the neurological delay.
3. **Determine the Average PLR Response.** Third, we solve for the average model results via our solution given by (13).
We focus our attention on the gradual illumination scenario, shown in Figure 3 (a), where we investigate the case of steady illumination $I^*$ given by $V_{\text{max}}$. We do this by manually selecting a portion of the pupillary response signal associated with $V_{\text{max}}$. To ensure that we get adequate history information, we select the region before the settling time $T_S$ as depicted in Figure 3 (a). This careful selection was done for two reasons. One reason is that our model is built under the premise of a constant neurological delay $\tau$. Hence, care must be taken to ensure that this is considered in our experimentation. The other reason is that the case of acquisition of iris images under constant illumination presents a more realistic scenario for fixed indoor applications.

Table I provides a description of the WVU iris video dataset where we see the breakdown of subjects by eye color. Applying our methodology on the entire dataset yielded very good results with the typical standard error $\sigma < 0.05$. An example of this is shown in Table II where we see a sample of our results for eyes in Category I\footnote{Eyes in Category I include light brown, blue, green, and hazel.}. To further evaluate our proposed technique, we compared the theoretical response, with the calculated subject-specific parameters, to subject-specific data. Figure 6 provides an example of this where our model shows close agreement with a subject response (green line) and the actual observed subject data (blue line) where we note close agreement.

### IV. Discussion

We revisited the work of [10] to model and explore the general and subject-specific effects of pupil dilation in the NIR spectrum. By exploring the case of constant illumination, we see that the behavior of our solution, via the method of steps, is consistent with the work in [10] in illustrating the constriction and dilation behavior for the average subject. Having an exact solution is beneficial because we can directly apply it instead of dealing with the nuances of the delay differential equation that is dependent on the history function. We were also able to directly apply our solution to account for individual differences by transforming the average model to quantify subject-specific responses. As a result, we proposed a semi-automatic process to account for subject-specific effects and empirical testing on the WVU iris video dataset showed promising results. Hence, we conclude that our work provides a foundation for understanding the effects of pupil motion in the NIR spectrum.

The results of our work have several applications that can benefit the iris recognition community. Our model can serve as an analytical description for describing pupillary response,
which can be directly applied to explore the impact of dilation on iris recognition [20]. Furthermore, our process can be used to develop robust iris recognition algorithms that mitigate the effects of pupil dilation. Current iris recognition algorithms use Daugman’s rubber-sheet model, which uses a simple transformation to geometrically normalize the iris while accounting for dilation. Our model can be applied with the works of [12] and [16] to produce prior probabilities of the distortion vector produced by Bayesian estimation. Additionally, our work could serve to further understand the temporal stability effects of iris recognition systems. In their report, NIST researchers advocated that there is a need for modeling physiological changes of human subjects [17]. Our analysis addresses this need because our work provides an analytical foundation while testing that foundation via empirical analysis. Furthermore, our parameter extraction methodology, described in section III-B, can be applied to measure individual responses as well as ocular disorders.

Although our model is an advancement of the prior art and the empirical evaluations of our model yield promising results, there are several areas of future work that need to be explored. Firstly, our model is built under the assumption of constant illumination. Thus, one area of exploration would be modifying our model to consider the varying effects of illumination. Another potential area of exploration involves exploring sudden changes in illumination where a pictorial representation is provided in Figure 3(b). It is expected that, for sudden changes in illumination, there would be a different pupillary response and care must be taken to model this outcome. Lastly, there is a need for further experimentation on actual data where we consider the previous avenues of exploration. Exploring such avenues for future work enables us to attain more comprehensive knowledge of this physiological phenomenon.

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