SIGNAL-DEPENDENT KERNEL FOR REDUCTION OF TIME-DOMAIN CROSS TERMS IN TIME-FREQUENCY DISTRIBUTIONS

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ABSTRACT
We introduce a signal-dependent tapering function which is able to reduce time-domain cross terms for the class of impulsive signals. The reverse-correlation function magnitude is constrained to suppress increases away from the origin. In combination with the Cone-shaped Kernel approach, the resulting TFD demonstrates a reduction in both the time and frequency-domain cross terms.

INTRODUCTION
The Time-Frequency Distributions (TFD) of Cohen's bilinear class [1] are applied in nonstationary signal analysis. The properties of a specific TFD are defined by the explicit choice of a kernel. Interfering cross terms often appear in the TFD at times and/or frequencies when the signal is known to be zero. For the Wigner Distribution, these appear between every pair of signal components on the time-frequency plane. For the Cone Kernel (CK) distribution [2] the frequency domain cross terms are significantly attenuated and highly localized to a narrow bandwidth about the signal frequencies [3]. For both the Pseudo Wigner-Ville Distribution (PWVD) and Cone Kernel distribution, time domain cross terms are suppressed by the choice of a fixed tapering function, which is chosen to limit the time interval over which signal terms can interact to generate cross terms [3]. We introduce a signal-dependent tapering function which is able to adapt to the time interval of the signal terms. The approach is applicable to the class of signals where the instantaneous power amplitude is reduced in the time interval between signal terms, such as the analysis of multiple wide-bandwidth impulses. The following section contains the derivation of this approach. Results are presented to demonstrate the suppression of time-domain interference.

BACKGROUND AND DEFINITIONS
The Time-Frequency Distribution (TFD) of Cohen's bilinear class are obtained via a weighted, symmetric, nonstationary autocorrelation that is Fourier transformed in the lag variable \(r\),

\[
G(t,f) = \int_{-\infty}^{\infty} \phi(t-t',\tau) y(t',\tau) \exp(-2j\pi ft') dt' \quad (1)
\]

where \(y(t',\tau)\) is the reverse correlation in lag variable \(\tau\) about the time position \(t'\), defined as,

\[
y(t',\tau) = x(t'+\tau/2) x^{*}(t'-\tau/2)
\]

and \(x(t)\) is the signal of interest, \(\ast\) denotes complex conjugation.

The properties of a specific TFD are defined by the explicit choice of the kernel \(\phi(t,\tau)\).

For the Pseudo Wigner-Ville Distribution (PWVD),

\[
\phi(t-t') = \delta(t-t') g(\tau)
\]

Where \(\delta\) is the delta function and \(g(\tau)\) is the tapering function.

The PWVD is written as,

\[
W(t,f,g) = \int \tilde{g}(\tau) y(t,\tau) \exp(-j2\pi ft) d\tau
\]

For the Cone Kernel distribution, the reverse correlation is smoothed over a cone-shaped region prior to applying the tapering function

\[
\phi(t-t',\tau) = \left\{ \begin{array}{ll}
                g(\tau) & |t-t'| > |\tau| \\
                0 & \text{otherwise}
               \end{array} \right.
\]

Defining the smoothed reverse correlation as

\[
d(t,t',\tau) = \int y(t-t',\tau) d\tau \quad \text{from} \quad -|\tau| \text{to} \quad |\tau| \quad (5)
\]

The Cone Kernel TFD may be written as

\[
\text{CK}(t,f,g) = \int \tilde{g}(\tau) \ d(t,t',\tau) \exp(-j2\pi ft) d\tau
\]
ADAPTIVE TAPERING FUNCTION DESIGN

Cross terms usually position themselves between any pair of signal terms in the TFD. Fig. 1a consists of a pair of Gaussian impulses centered at time $t_1$ and $t_2$. In fig. 1b, the PWVD contains a strong cross term at time $t_c$, directly between the two impulses. The reverse correlation magnitude at time $t_c$ is shown in Fig. 1c. We observe that peaks, occurring at lags $\tau = \pm (t_2-t_1)$ are placed away from the $\tau=0$ origin. If a tapering function is designed to suppress amplitude increases away from the origin, the time domain cross term energy will be suppressed in the resulting TFD.

Defining the characteristic function as

$$C(t,\tau) = |g(t,\tau) y(t,\tau)|$$

we introduce a non-increasing in tau constraint as,

$$C(t,\tau_2) \leq C(t,\tau_1) \text{ for } \tau_2 > \tau_1$$

(9)

Applying this constraint, the cross terms in Fig. 1b are significantly attenuated, as shown in Fig. 1d.

The constraint of equation (9) can lead to sudden variations in the tapering function $g(\tau)$, especially when signals have multiple frequency components at the same time. Therefore, a limit on the rate of kernel change is introduced as a second
constraint:

\[ 0 < \frac{g(t2) - g(t1)}{g(t1)} - \alpha \quad (10) \]

A similar characteristic function is formed for the Cone Kernel distribution

\[ \mathcal{C}_{\text{ck}}(t, \tau) = \int g(t, \tau) \, d(t, \tau) \quad (11) \]

**RESULTS AND DISCUSSION**

Figure 2a contains the PWVD of three Gaussian Pulses, having both time and frequency-domain cross terms. Applying the constraints of equations (9) and (10), with a rate of change limit (\( \alpha \)) of 0.2, results in a 15 dB reduction of the time-domain cross terms as shown in figure 2b. Note that the frequency-domain cross terms are unchanged.

Figures 2c and 2d show the effects of applying the non-increasing in tau constraint to the Cone Kernel (CK) distribution. Here the frequency-domain cross terms have been suppressed by the CK method (in figure 2c). Applying the constraints of equations (9) and (10), with a rate of change limit (\( \alpha \)) of 0.3, results in a 15 dB reduction of the time-domain cross terms as shown in figure 2d.
CONCLUSION

We have introduced a signal-dependent tapering function which is able to reduce time-domain cross terms for the class of impulsive signals. In combination with the Cone-shaped Kernel approach, the resulting TFD demonstrates a reduction in both the time and frequency-domain cross terms.

REFERENCES