Control and Navigation of a Three Wheeled Unmanned Ground Vehicle by $\mathcal{L}_1$ Adaptive Control Architecture

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Abstract—This paper presents a modified navigation system and controller for an automatic UGV. It uses new variables to overcome the difficulty of dealing with cross coupling effects in dynamical equations due to the use of Cartesian coordinates. Then it proceeds with the introduction of a trajectory design algorithm in conjunction with $\mathcal{L}_1$ control architecture to accomplish the path following objective. This paper shows by illustrative simulations that using $\mathcal{L}_1$ adaptive controller can greatly facilitate controlling the UGV in uncertain environmental conditions.

I. INTRODUCTION

Unmanned ground vehicles have been the subject of intensive research and studies in the last decade. Their wide range of application include material handling [1], search and rescue [2], exploration [3], [4], transportation, crop harvesting [5] and combat vehicles [6].

UGVs (Unmanned Ground Vehicle) are fearless, tireless and often superior to humans in routine and tedious tasks. On the battlefield they can be utilized to complement and aid the soldiers and withstand enemy armaments. In hazardous and hard to reach environments, where humans cannot operate or may encounter real or possible danger to their health, UGVs can work with great speed and precision.

Power source, manipulator, sensory, control and navigation systems are parts of most of current UGVs. This paper mainly deals with control and navigation systems of a three wheeled UGV. Weiguo Wu et al. in [7] investigated time-optimal path planning of a three wheeled mobile robot in the presence of constraints. Masoud Ghaffari et al. in [8] described the design and implementation of an electric powered, unmanned three-wheeled vehicle, called the Bearcat III. In both of these cases the designers tried to use waypoint following to reach the destination. The main assumption in these and many other cases is that the whole nearby terrain is traversable except the few obstacles or potholes that should be avoided.

However, there are cases when the UGV needs to move along an already designated path or one that is determined by the pilot in charge. Alain Micaelli and Claude Samson in [9] proposed a method for generation of the reference trajectory in real time, so that in combination with the proposed algorithms for trajectory tracking, the mobile robot converges to the predefined path asymptotically and smoothly.

D. Soeanto et al. reformulated the method introduced in [9] with a feedback control law to overcome some of its constraints [10]. In the present paper a modified version of this algorithm is used with an adaptive augmentation loop based on $\mathcal{L}_1$ adaptive control theory [11], [12]. $\mathcal{L}_1$ control architecture is able to deal with the uncertainties in the plant and external disturbances and offers superb flexibility and performance.

The paper is organized as follows. Section II derives the kinematics of a three wheeled UGV. Section III derives the dynamical equations and tries to obtain variables that make the equations partially decoupled. Previous works dealt with Cartesian coordinates. Section IV tries to devise and update a trajectory in real-time to make the vehicle move along the predesigned path. Section V utilizes $\mathcal{L}_1$ adaptive control architecture to ensure that the automatic UGV track the commands issued by the path-following algorithm and compensate for uncertainties and disturbances introduced into the system. Section VII compares the results of $\mathcal{L}_1$ adaptive controller with a PID one in two different examples to show that even a well-tuned PID is no match for $\mathcal{L}_1$ adaptive controller when dealing with uncertainties in the system. Section VIII concludes the paper by summarizing the results and important points of the paper.

II. KINEMATICS

The UGV considered in this paper, which is based on the model considered in [8], has two rear driving wheels and one free caster. A schematic model of the structure is shown in Fig.1. The kinematic model with the center of mass C as the reference point is described as follows:

\[
\begin{bmatrix}
\dot{x} \\
\dot{y} \\
\dot{\theta}
\end{bmatrix}
= \begin{bmatrix}
\cos \theta & \sin \theta & 0 \\
\sin \theta & -\cos \theta & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
v_l \\
v_n \\
\omega
\end{bmatrix}
\] (1)

where $v_l$, $v_n$ and $\omega$ and $\theta$ are illustrated in Fig. 1. $v_l$, $v_n$ and $\omega$ can be related to angular velocity of the left and right driving
wheel by the following equation:
\[
\begin{bmatrix}
v_t \\
v_n \\
\omega
\end{bmatrix} = \begin{bmatrix}
\frac{r}{2r} & \frac{r}{2r} & \frac{r}{2r} \\
\frac{r}{2r} & \frac{r}{2r} & \frac{r}{2r} \\
0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
\dot{q}_t \\
\dot{q}_r
\end{bmatrix}
\]
(2)
where \( r \) denotes the wheel radius. This equation holds true because \( v_n = c\omega \).

Following the similar procedure for the left wheel yields:
\[
\begin{aligned}
f_l - F_l &= m_w \ddot{x}_l \\
\tau_l - f_l r &= J_w \ddot{q}_l \\
\dot{x}_l &= r \dot{q}_l 
\end{aligned}
\]
As shown in Fig. 2, the dynamical equations of the frame are:
\[
\begin{align*}
(F_l + F_r - F_T) \cos \theta + 2F_n \sin \theta &= M_c \ddot{x}_c \\
(F_l + F_r - F_T) \sin \theta + 2F_n \cos \theta &= M_c \ddot{y}_c \\
F_l d - F_r d - 2F_n e &= J_\theta \ddot{\theta}
\end{align*}
\]
(7)
where \( M_c \) denotes the frame mass, \((x_c, y_c, \theta)\) is the position of center of mass with respect to the fixed coordinates, \( J_c \) is the frame inertia relative to point \( C \) and \( F_T \) is the reaction force applied on the frame by the front wheel. For further details refer to [7], [8], [13]. Following equations can be obtained by using (3)-(7):
\[
\begin{align*}
J_c + 2J_0 v^2 & \ddot{\theta} = 2F_n e + \frac{d}{r} u_1 \\
(M_c + 2J_0 v^2) \dot{v}_1 &= \frac{1}{r} u_2 + 2F_n \sin 2\theta - M_c \dot{\theta}^2
\end{align*}
\]
(8)
(9)

III. DYNAMICS

Since the wheels’ inertia is negligible compared to frame’s, the Coriolis effect for wheels can be neglected. The Newton Euler equation for the right wheel is expressed as:
\[
\begin{align*}
f_r - F_r &= m_w \ddot{x}_r \\
\tau_r - f_r r &= J_w \ddot{q}_r
\end{align*}
\]
(4)
where \( f_r \) is the friction force arising from the contact between the right wheel and terrain, \( F_r \) denotes the reaction force exerted on the right wheel by the frame. \( M_w \) denotes the wheel’s mass. \( \tau_r \) is the torque acting on the right wheel. \( J_w \) is defined as the wheel inertia.

The following relationship holds if the wheel rolls without slipping:
\[
\dot{x}_r = r \dot{q}_r
\]
(5)

For the problem at hand, a reference trajectory for the orientation of the vehicle is designed, whereas a reasonable reference value for the tangential velocity is considered to be maintained at all times. Reference trajectory for \( \theta \) is defined as:
\[
\theta_{ref} = \theta_d + f(y) (\theta_c - \theta_d)
\]
(10)
where \( \theta_c \) and \( \theta_d \) are depicted in Fig. 3. \( f(y) \) can be any function that fulfills the following requirements:
\[
\begin{align*}
\lim_{y \to \infty} f(y) &= 0 \\
\lim_{y \to 0} f(y) &= 1 \\
\forall y \in [0, \infty) &\Rightarrow f(y) \text{ and } \partial f/\partial y \text{ exist}
\end{align*}
\]
(11)
V. $L_1$ CONTROL ARCHITECTURE

The main advantage of using $L_1$ adaptive controller in UGV is its ability for fast and robust adaptation which boosts system’s performance for both input and output compared to other controllers. This capability is achieved through the separation between adaptation and robustness. The controller estimates the uncertainties via a fast estimation algorithm and the resulting compensation for these uncertainties passes through a low-pass filter and constitutes part of the control signal. This low-pass filter not only ensures that the control signal stays in the reasonable frequency range, but also leads to separation between adaptation and robustness, and provides an efficient tool for adjusting the trade-off between performance and robustness [14]. The elements of $L_1$ adaptive controller are introduced next:

1) State Predictor: Consider the following state predictor:

\[
\dot{\theta}(t) = \dot{\omega}(t)
\]
\[
\dot{\omega}(t) = -a_\omega \omega(t) + b_1 u_1(t) + \hat{\sigma}_\omega(t)
\]
\[
\dot{v}(t) = -a_v \dot{v}(t) + b_2 u_2(t) + \hat{\sigma}_v(t)
\]
\[
\ddot{y}(t) = \begin{bmatrix}
    \dot{\theta}(t) \\
    \dot{\omega}(t) \\
    \dot{v}(t)
\end{bmatrix}
\]
\[
\begin{bmatrix}
    \dot{\theta}(0) \\
    \dot{\omega}(0) \\
    \dot{v}(0)
\end{bmatrix}^T = \begin{bmatrix}
    \theta_0 \\
    \omega_0 \\
    v_0
\end{bmatrix}^T
\]

where $a_\omega$ and $a_v$ have arbitrary positive real values.

2) Adaptive Laws: To estimate the system disturbances, a piecewise continuous adaptive law is designed. The accuracy of these estimates can be increased by reducing the time between states measurement that consequently leads to smaller time steps. These governing equations are:

\[
\hat{\sigma}_\omega = \frac{a_\omega e^{a_\omega T_n}}{(e^{a_\omega T_n} - 1)} (\dot{\omega}(t) - \omega(t))
\]
\[
\hat{\sigma}_v = \frac{a_v e^{a_v T_n}}{(e^{a_v T_n} - 1)} (\dot{v}(t) - v(t))
\]

where $T_n$ is time step.

3) Control Laws: The control signal generated as:

\[
u_1(s) = k_\omega \omega_{ref}(s) \frac{C(s)}{b_1} \hat{\sigma}_\omega(s)
\]
\[
u_2(s) = k_v v_{ref}(s) \frac{C(s)}{b_2} \hat{\sigma}_v(s)
\]
\[
\omega_{ref}(t) = -a_\omega (\theta(t) - \theta_{ref}(t))
\]

where $C(s)$ is a low pass filter with the following equation:

\[
C(s) = \frac{k}{s + k}, \quad k > 0
\]

$\nu_1$ and $\nu_2$ are defined as: $k_\omega = \frac{a_\omega}{b_1}$, $k_v = \frac{a_v}{b_2}$.

VI. ANALYSIS

In this section, the system is analyzed to show that it is stable and ensure that $\theta(t)$ follows $\theta_{ref}(t)$. The state predictor, adaptive law and control law for the orientation of the vehicle that was introduced in the previous section can be recast in the frequency domain. We must ensure that $\theta(t)$ follows the desired system:

\[
\theta(s) \approx M_0(s) \theta_{ref}(s)
\]
\[
M_0(s) = \frac{a_\omega^2}{s^2 + a_\omega s + a_\omega^2}
\]

which is the realization of the desired system embodied by (12) and (14). The dynamical equation of the system in (8) can be rewritten as:

\[
\theta(s) = G_0(s) (u_1(s) + z_0(s))
\]

In terms of the desired system behavior this equation takes the following form:

\[
\theta(s) = M_0(s) (u_1(s) + \sigma_0(s))
\]

where the uncertainties due to $G_0(s)$ and $z_0(s)$ are lumped in the signal $\sigma_0(s)$. $A_m$ is defined as follows:

\[
A_m = \begin{bmatrix}
    0 & 1 \\
    -a_\omega^2 & -a_\omega
\end{bmatrix}
\]

Let $P_\theta = P_\theta^T > 0$ be the solution to the algebraic Lyapunov equation:

\[
A_m^T P_\theta + P_\theta A_m = -Q_\theta, \quad Q_\theta = Q_\theta^T > 0
\]

Lemma: Given the $L_1$ adaptive controller defined via (12)-(14) if

\[
\| \theta_{ref}(t) \|_{L_\infty} \leq \gamma_{\theta_{ref}}, \quad \| \dot{\theta}_{ref}(t) \|_{L_\infty} \leq \gamma_{\dot{\theta}_{ref}}
\]

and also the initial condition verifies

\[
\left\| \begin{bmatrix}
    \theta(0) - \theta_{ref}(0) \\
    \dot{\theta}(0) - \dot{\theta}_{ref}(0)
\end{bmatrix} \right\|_{L_\infty} <
\frac{2 \| P_\theta b_1 \|}{\lambda_{\min} (Q_\theta)} (\omega_0 \gamma_{\dot{\theta}_{ref}} + \dot{\gamma}_{\theta_{ref}})
\]

then it follows that

\[
\| (\theta - \theta_{ref}) \|_{L_\infty} \leq \gamma_\theta + \gamma_{\theta} + \dot{\gamma}_{\theta}
\]

\[
+ \frac{2 \| P_\theta b_1 \|}{\lambda_{\min} (Q_\theta)} (a_\omega \gamma_{\dot{\theta}_{ref}} + \dot{\gamma}_{\theta_{ref}})
\]

with

\[
\lim_{T_n \to \infty} \gamma_\theta = 0, \quad \lim_{C(s) \to 1} \dot{\gamma}_{\theta} = 0
\]

Proof: The proof of the result can be found in [15].

VII. SIMULATIONS

In this section two different examples are presented to compare the performance of $L_1$ adaptive controller to that of a simple PID.
4) Example 1: Consider the following values for the UGV, reference path and $L_1$ controller parameters:

\[
\begin{bmatrix}
  x_0 & y_0 & \theta_0 \\
\end{bmatrix} = \begin{bmatrix}
  0 & 3 & -\pi/4 \\
\end{bmatrix},
\]

\[
J_c = 43,
J_0 = 0.274
\]

\[
J_w = 0.055,
J_o = 0.2095
\]

\[
e = 0.338,
d = 0.432,
F_n = 629.45
\]

\[
a_w = a_v = 15,
T = 10^{-3}
\]

\[
k_w = 329.7,
k_v = 1001,
f_{\text{path}}(x) = \sin(0.1x)
\]

After tuning, two PI controllers with the following transfer functions are used:

\[
G_\omega(s) = 300 + \frac{50}{s}
\]

\[
G_v(s) = 250
\]

The vehicle’s position controlled by $L_1$ and PID are shown in Fig. 4 and Fig. 5. Both of the controllers show satisfactory performance in dealing with this nonlinear system.

5) Example 2: Assume that all of the conditions are the same as example 1 except:

\[
\begin{bmatrix}
  x_0 & y_0 & \theta_0 \\
\end{bmatrix} = \begin{bmatrix}
  0 & 8 & -\pi/4 \\
\end{bmatrix},
\]

\[
F_n = 3776.7 \sin (0.2(x + y))
\]

responses of both systems are plotted in Fig. 10, Fig. 11, Fig. 12 and Fig. 13. $L_1$ is still able to control the vehicle despite the nonlinear uncertainties that were introduced in the system, whereas PID could not make the vehicle follow the path in a reasonable amount of time. Fig. 12 and Fig. 13 clearly show that the vehicle equipped with $L_1$ efficiently follows the reference velocity, whereas PID fails to converge. The results reveal the fact that $L_1$ adaptive controller has greater capability to compensate for disturbances and deal with nonlinear uncertainties. They also show that although $L_1$ has
faster and better performance, it does not necessarily lead to higher input values.
Fig. 13. Tangential velocity of the vehicle compared to reference velocity controlled by PID adaptive controller

Fig. 14. Right wheel torque using $\mathcal{L}_1$ adaptive controller

VIII. CONCLUSION

A strategy for generating and updating the desired trajectory according to the reference path was developed. The adopted architecture proves to be superior to conventional way-point navigation method, enabling an automatic UGV to follow a predetermined path. An $\mathcal{L}_1$ adaptive controller was designed and shown that it can outperform a common PID controller when there are uncertainties in the system and unknown disturbances introduced by the environment. The effect of changes in the inputs coefficients caused by uneven loading and terrain conditions on the wheels may be the subject of future studies.

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REFERENCES


