Noise-Robust Synchronized Chaotic Communications

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Abstract—Until now, research on the applications of self-synchronized chaotic circuits to communications has been hindered by the high sensitivity of self-synchronized chaotic systems to additive noise. In this paper, I demonstrate a self-synchronized chaotic system that synchronizes even in the presence of noise much larger than the signal. This system works because it generates signals with two different time scales, allowing noise added to the shorter time scale system to be averaged out by the longer time scale system. I demonstrate a simple communications scheme with this system, and I show that the curve of bit error rate as a function of (energy per bit)/(noise spectral density) is invariant with respect to bit length, allowing this system to operate in arbitrarily low signal-to-noise environments.

Index Terms—Chaos, communication, spread spectrum, synchronization.

I. INTRODUCTION

Much of the initial research into self-synchronized chaotic systems was motivated by possible applications to communications [1]–[11], for chaotic circuits are a naturally simple way to produce broadband signals. Self-synchronized chaotic systems have suffered from the same problem as transmitted reference spread spectrum systems [12], however; the reference signal is contaminated by noise in the channel. It is probable that self synchronizing methods can never perform as well as stored reference methods (such as CDMA [13]) in the presence of noise, but, as I show in this work, it is possible for self synchronizing chaotic systems to be noise robust according to the definition of Abel et al. [14], which states that a communications system is noise robust if the curve of bit error rate as a function of $E_b/N_0$ is invariant with respect to bit length. If a communications system is noise robust, operation in arbitrarily low signal-to-noise ratios is possible simply by increasing bit length.

The communications scheme described in this paper is not a practical system, but rather, was chosen because it was simple to describe. The purpose of this paper is to demonstrate that a self-synchronizing chaotic system can be noise robust, so I did not want to complicate the description by also having to explain a complicated communications scheme. The power efficiency of the system described below is not very high, but it is hoped that other modulation schemes might improve efficiency.

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The signal that was transmitted from drive to response, \( x_t \), was a linear combination of signals from the drive system. The response system was described by

\[
x_t = \sum_{i=1}^{6} k_i x_i + \eta \quad x_r = \sum_{i=1}^{6} k_i y_i
\]

\[
\frac{dy_1}{dt} = -\alpha_1 (0.02 y_1 + 0.5 y_2 + y_3 + 0.5|y_1|)
\]
\[
\frac{dy_2}{dt} = -\alpha_1 (-y_1 + 0.02 y_2 + y_6)
\]
\[
\frac{dy_3}{dt} = -\alpha_1 (-g(y_1) + y_3)
\]
\[
\frac{dy_4}{dt} = -\alpha_2 (y_1 + 0.05 y_4 + 0.5 y_5 + y_6 + b_4 (x_t - y_r))
\]
\[
\frac{dy_5}{dt} = -\alpha_2 (-y_1 + 0.11 y_5 + b_5 (x_t - y_r))
\]
\[
\frac{dy_6}{dt} = -\alpha_2 (-g(y_1) + y_6 + b_6 (x_t - y_r))
\]

where the \( x \) variables represent the drive system, the \( y \) variables represent the response system, and \( \eta \) is a Gaussian-noise signal filtered to match the bandwidth of the response system (the determination of the bandwidth is described below). Note that the error signal \( x_t - y_r \) is fed back only into the high frequency part of the system, \( y_t \) to \( y_6 \).

The parameters \( k_i \) and \( b_i \) are set to minimize the largest Lyapunov exponent for the response system corresponding to (2) [21], [22]. The \( k_i \)'s and \( b_i \)'s are varied by a linear optimization routine in order to minimize the largest Lyapunov exponent for the response system. The optimization routine yields many local minima, so there are many sets of \( k_i \)'s and \( b_i \)'s which will give a stable response system, and many of these sets yield approximately equal Lyapunov exponents. For the parameters listed in Table I, the largest Lyapunov exponent for the response system was \(-0.22\) when \( \alpha_1 = 0.2 \) and \( \alpha_2 = 10.0 \). Varying \( \alpha_1 \) and \( \alpha_2 \) changed the stability of the response system, but the largest Lyapunov exponent remained negative for a large range of \( \alpha \)'s, so the same \( k_i \)'s and \( b_i \)'s were used for all simulations.

### III. COMMUNICATION

The systems of equations (1)–(2) were simulated in order to test the communications performance of this type of system. The information signal \( s_1 \) in (1) was set to \( \pm 1 \) to simulate a binary signal. The value of \( s_1 \) determined the phase of the low frequency drive variable \( x_t \), while the dotted line (difficult to separate from the solid line in this figure) is the corresponding response variable \( y_1 \), showing rapid synchronization as \( s \) is switched. The noise level is 0. (c) The difference between \( x_1 \) and \( y_1 \).

![Fig. 2. (a) Information signal \( s \) from (1). (b) The solid line is the low frequency drive variable \( x_t \), while the dotted line (difficult to separate from the solid line in this figure) is the corresponding response variable \( y_1 \), showing rapid synchronization as \( s \) is switched. The noise level is 0. (c) The difference between \( x_1 \) and \( y_1 \).](image_url)
The receiver of (2) was used to measure the phase of the low frequency part of (1) in order to determine the value of $s_1$. The phase detector part of the receiver was

$$\frac{d\theta}{dt} = \omega, \quad \frac{du}{dt} = \frac{dy_1}{dt} - \frac{\alpha_1}{100.0} u$$

$$v = \text{sgn}(\sin(\theta)) u$$

$$\frac{dw}{dt} = \frac{\alpha_1}{10.0} (v - 0.1 w)$$

(3)

The frequency $\omega$, the frequency of the local reference oscillator in the response system, was the same frequency as in (1). As long as the phase of the local reference oscillator does not drift by much over one cycle, exact phase synchronization between the local oscillator and the chaotic response system is not necessary. In a more realistic implementation, differential phase-shift keying [23] could be used to eliminate problems with uncertainty in the phase of the local oscillator, but the system described here has been kept as simple as possible in order that the new concepts are clear. The signal $v$ was a high-pass filtered version of $y_1$, high-pass filtered because the absolute value function in (2) produced a dc offset in $y_1$ when the noise signal was large. The “sgn” function is the signum function (+1 for the argument $>0$, -1 for the argument $<0$). The variable $w$ in (3) is set to zero at the start of each bit interval, and $w$ is measured at the end of each bit interval to determine the value of the received bit. The measured value of $w$ will be $>0$ or $<0$, depending on the bit value. For a practical implementation, phase drift in the local oscillator and phase shift in the transmitted signal make it uncertain which sign of $w$ corresponds to a $+1$ bit and which corresponds to a $-1$ bit, so differential phase shift keying [23] would be necessary to eliminate this uncertainty. Differential phase-shift keying will yield the same results as the simple phase shift keying demonstrated here, except that to get an equal bit error rate, $E_b/N_0$ must be 3 dB larger. It should be noted that the BPSK curve shown in Fig. 4 also suffers from these same limitations, so that in a real communication system a differential version of BPSK would be necessary, also resulting in a 3-dB loss in performance.

The necessary bandwidth for the chaotic signal is found by measuring the bit error rate. A low-pass filtered noise signal is added to a low pass filtered version of the signal $x_1$ and the resulting bit error rate is measured at the receiver as the filter breakpoint is lowered. At some given breakpoint, the bit error rate is seen to increase, so the filter breakpoint is set larger than this value. For the system in this paper, the minimum filter breakpoint was 7.5.

Fig. 4 shows the probability of bit error $P_b$ for the system of equations (1)–(3). The dark circles show $P_b$ for a bit length of $L = 653.9$, in which case $\alpha_1$ was set to 0.2 and $\alpha_2$ was set to 10.0. The open circles show $P_b$ for a bit length of $L = 1307.84$ (twice as long), with $\alpha_1 = 0.1$ and $\alpha_2 = 10.0$. The open squares show $P_b$ for $L = 20 924.8$, with $\alpha_1 = 6.25 \times 10^{-3}$ and $\alpha_2 = 10.0$. The actual bit rates depend on the overall scale of the $\alpha$ values. The energy per bit/noise power spectral density ($E_b/N_0$) is calculated for a 2-sided noise power spectrum. The three sets of data lie along the same curve, demonstrating that the curve of $P_b$ vs. $E_b/N_0$ does not depend on bit length, a property not yet seen in other transmitted reference chaotic communication systems. The solid line in Fig. 4 shows the probability of bit error for binary phase shift keying (BPSK) [23] for comparison.

The noise robustness of this system may be further explored by changing the bit length $L$ and finding the probability of bit error at a constant value of $E_b/N_0$. In Fig. 5, $E_b/N_0$ is held constant at 14.3 dB while the bit length $L$ is varied by a factor of 32, from 653.9 to 20 924.8. These bit rates are slow, yet higher bit rates may easily be achieved by rescaling both $\alpha$ values. The slow time constant $\alpha_1$ is varied at the same time the bit length is varied, so $\alpha_1$ varies from 0.2 to 0.2/32 = 0.00625, while $\alpha_2$ is held constant at 10.0. The upper scale in Fig. 5 shows the signal to noise ratio in decibels. The probability of bit error is seen to be roughly constant when the bit length $L$ varies by a factor of 32, demonstrating that this self-synchronizing chaotic system is noise robust for added Gaussian noise. In addition, the performance of this communications system does not degrade when the signal to noise ratio is below 0 dB.

In order to demonstrate that the transmitted signal is truly being buried in the noise, Fig. 6(a) shows the power spectrum $S$ of the transmitted signal $x_t$ when $\alpha_1 = 0.00625$ and $\alpha_2 = 10.0$. Fig. 6(b) shows the same signal with added Gaussian noise so that the $S/N$ is approximately $-43$ dB (corresponding to the longest bit length used in Fig.
For different bit lengths when the $E_b/N_0$ of 14.3 dB.

Fig. 5. Probability of bit error $P_b$ for different bit lengths $L$ at a constant $E_b/N_0$ of 14.3 dB. $L$ varies by a factor of 32. The top axis is the corresponding signal to noise ratio.

5). The added noise covers even the peaks in the power spectrum of $x_t$. The high frequency peak is about 20 dB below the noise level, while the low frequency peak is at approximately the same level as the noise.

IV. CONCLUSION

Transmitted reference-communications systems should not be noise robust, since noise is added to the reference signal, and studies for chaotic transmitted reference signals confirm this supposition [14]. Self-synchronizing chaotic systems are not transmitted reference systems such as DCSK [24], [25], but are nonlinear filters. In this way they are more general versions of periodic methods such as BPSK, which uses linear filters to isolate the signal from noise. Self-synchronizing chaotic systems are not as efficient as purely periodic systems because the nonlinear filter (the chaotic response system) has a larger bandwidth, so less noise is excluded.

Phase modulation was used in this paper as a simple example to demonstrate that communication is possible with two-frequency synchronized chaotic systems, but it is probably not the most efficient way to use these systems. A two-frequency system might be more useful as a filter for communication involving symbolic dynamics, in which chaotic trajectories form the communications symbols [26].

REFERENCES


