A Note on the Distribution of Atmospherically Ducted Signal Power Near the Earth's Surface

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Abstract—Interference fields caused by low-level atmospheric ducts are of concern at UHF frequencies and above. This note develops a long-term median distribution of received signal power during ducting conditions from observations taken on a 175 km, 5 GHz path in New Jersey in 1966. A simple worst case estimate for the received signal power during ducting is also derived.

I. INTRODUCTION

During an average year, ducting layers that occur at or near the earth's surface can be expected between about 1 and 15 percent of the time, depending upon one's location in the United States [1]–[3]. If these layers are of sufficient horizontal homogeneity, and any UHF and higher frequency radio links are in the vicinity, the potential is high for enhanced signal power reception at either a desired or undesired cochannel receiver for part or most of the time the ducting layers are present. There is, however, to the knowledge of this author, only extremely sparse information and data on the conditional distribution of received signal level (RSL) in the presence of ducting.

II. LONG-TERM DISTRIBUTION ESTIMATION

Fig. 1, which was presented by Crane [4], shows the distribution of the RSL's for the ducting cases on the HW path. The line drawn through these data is an "eyeball" fit. These data are, according to Crane [4], 5-min median RSL's observed once per hour. Hence, Fig. 1 represents the "long-term" distribution of the median power \( x_m \) for the 5-min observation period. Thus, the straight line in Fig. 1 should be a lognormal distribution of the form, conditional on the presence of ducting \( D \):

\[
P(X_m \leq x_m | D) = \frac{4.3429}{\sigma_D x_m \sqrt{2\pi}} \int_{-\infty}^{x_m} \exp \left[ -\frac{(10 \log_{10} x_m - \mu_D)^2}{2 \sigma_D^2} \right] dx_m.
\]

In the case of Fig. 1, we can obtain \( \mu_D = -58 \text{ dBW} \) and \( \sigma_D = 13.26 \text{ dB} \). Equation (1) is not advocated as the only long-term distribution applicable to median observations, although the lognormal form tends to be commonly used for many applications [5], [6]. Crane [4] also considers RSL data taken on the HW path during a different time period (August 19–September 1, 1966) in which further ducting, as well as troposcatter results, were obtained. Without going into detail (because the mathematical results would not be particularly clear without an accompanying long and distracting derivation), the troposcatter observations, which are also available for the August 2–August 9, 1966 period, can be combined with the ducting observations of Fig. 1, as described by (1). Also, a combined ducting-troposcatter conditional distribution can be obtained to represent August 19–September 1, 1966 HW path RSL data. Finally, the two distributions can be compared, as shown in Fig. 2, and it is noted that the maximum difference between the two independently derived distributions is about 3.5 dB. This adds a little credence to the use of the lognormal distribution (1). Note that neither

\(^{1}\) There is an admitted lack of rigor in an "eyeball" fit as opposed to use of certain statistical procedures to test goodness of fit. This was done because the values for the data points were unavailable and because this correspondence is intended primarily to illustrate methodology.

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distribution in Fig. 2 is a straight line. This is because the combination of two essentially lognormal conditional distributions, for ducting and troposcatter conditions, is not normal.

It is clear that \( \mu_D \) and \( \sigma_D \) will be different for different paths and carrier frequencies. It is especially difficult to evaluate the distribution standard deviation \( \sigma_D \) since, as stated earlier, there is apparently so little information on RSL distributions during ducting conditions. Thus, we have little choice other than to assume the value of \( \sigma_D = 13.26 \) dB stated earlier. We can, however, make some assumptions concerning \( \mu_D \) to give it a path and frequency dependency. Dougherty and Hart [7] note that RSL in a duct tends to vary inversely with the square of frequency (which is standard), but inversely with distance. If this variation is taken as typical of average ducting conditions, then we have

\[
\mu_D = P_T + G_I + G_r - 92.45 - 20 \log_{10} f_{\text{GHz}}
- 10 \log_{10} d_{\text{km}} - 0.03 d_{\text{km}} - A
\]  
(2)

where \( f_{\text{GHz}} \) is the carrier frequency in gigahertz (=5 on the HW path), \( d_{\text{km}} \) is the path length in kilometers (=175 on the HW path), and \( \mu_D \) is the mean RSL. The value of \( A \) can be evaluated for the August 2-August 9, 1966 HW path by comparing the left- and right-hand ordinates in Fig. 1. A "worst case" value of \( \mu_D \) will result, however, when \( A = 0 \). It is this worst case estimation that we desire to obtain. In (2), \( P_T \) is the transmitted power (in decibel units) and \( G_I \) and \( G_r \) are the transmitter and receiver gains in dB. Thus, for worst case or extreme condition situations, (2) with \( A = 0 \) can be used to estimate \( \mu_D \). It is further assumed that the frequency \( f_{\text{GHz}} \) has been checked to see what percent of the time it is expected to be trapped in a duct [8]. Although it does not affect the conditional distribution (1) per se, it is important in assessing the percent of time that ducting problems are expected to be consequential on a given link.

### III. SHORT-TERM AND TOTAL DISTRIBUTION DETERMINATION

We can hypothesize the RSL inside a surface or low-elevated duct as being composed of several modes. As the order of the modes increases, the attenuation rate of the modes also increases, as shown by Wait and Spies [9]. Assuming that these modes can be treated as phasors, combination of even the two lowest order modes with the long-term distribution (1) results in an essentially intractable integral, obtained by using the so-called "two component" distribution [10] for the short-term distribution. A "worst case" (or extreme condition) distribution can, however, be obtained by use of the Rayleigh distribution and it is this worst case situation that we wish to consider next. This approach circumvents the necessity of having to use intractable convolution techniques which can usually only be handled by numerical integration procedures via computer.

### IV. A WORST CASE DISTRIBUTION FOR UPFADING

Upfading occurs when the RSL exceeds its expected median value and is usually the property of concern in ducting analysis since upfading is tantamount to a higher interference field arriving at another cochannel receiver. The Rayleigh distribution hypothesizes that a large number of components, all with nearly\(^2\) equal amplitudes, are contributing to the RSL [11]. Since the ducting modes are more strongly attenuated the higher the mode, the use of a Rayleigh distribution represents an extreme-condition short-term distribution in a ducting situation because none of the modes, treated as phasor amplitudes, suffers any attenuation with respect to any other mode.

If we use the Rayleigh distribution to represent the short-term distribution of total received power \( x_p \) at some percent of time \( P = P(x_p > x_p | D) \), we obtain

\[
x_p = \frac{x_m \ln P}{\ln 0.5} = 1.4427 x_m \ln P.
\]  
(3)

In decibels, (3) becomes

\[
(RSL)_p = 10 \log_{10} x_p = 1.592 + 10 \log_{10} |\ln P| + 10 \log_{10} x_m.
\]  
(4)

Suppose, for example, we are interested in determining a "worst case" RSL, to be exceeded 1 percent of the time, on the HW path. Thus, \( P = 0.01 \), and (4) yields

\[
(RSL)_{0.01} = 8.2 + 10 \log_{10} x_m.
\]  
(5)

From Fig. 1, the value of \( 10 \log_{10} x_m \) exceeded 1 percent of the time is about -28 dBW. Thus,

\[
(RSL)_{0.01} = 8.2 - 28 = -19.8 \text{ dBW}.
\]  
(6)

Since we are dealing with 5-min medians, the RSL evaluated for about 0.001 percent (5.3 min) of a year would yield an effective "upper bound" value. Note that the largest value \( x_p \) should be determined from the minimum basic transmission loss, such as that given by Dougherty and Hart [7]. If results from (4) exceed the RSL obtained from the minimum basic transmission loss, the user should choose the minimum-basic-transmission-loss value.

\(^2\) The components can be of arbitrary amplitude as long as none is large enough to be conspicuous vis-a-vis the others.

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**Fig. 2.** Comparison of fits to the distributions of two different sets of data observed on a Highlands-to-Wildwood, NJ, 5 GHz, 175 km long path.
V. SUMMARY

A distribution of 5-min median RSL's was obtained using data taken on a Highlands to Wildwood, NJ, 175 km, 5 GHz path from August 2 to August 9, 1966 as the key ingredient. The distribution was made more general by suggesting (2) as a way to incorporate any given path length and frequency into the distribution determination. Then, a simple engineering-design procedure to evaluate an extreme-condition or "worst case" interference RSL during ducting conditions is evaluated, given the availability criteria for a particular link. This implies that convolution of the long- and short-term distributions to obtain the total RSL distribution, which is usually difficult to evaluate, may be unnecessary for many system designs where low-level ducting interference is of concern.

REFERENCES