Effects of a Transform Domain Filter Upon Error Rates and Detectability of a Spread Spectrum Signal

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Abstract

Interference and jamming can severely disrupt the performance of an intended communication receiver, as well as any intercept receivers. This paper examines the effects of a transform domain digital excision filter upon the performance levels of both types of receivers as determined through simulations using the Signal Processing Worksystem. The communication system considered in this research employed direct sequence spread spectrum modulation. Intercept receivers considered were the radiometer and chip rate detector. The performance of the communication and intercept receivers in the presence of narrowband jammers was measured with and without the excision filter in order to characterize the performance gains achieved with the filter. These gains are presented in the form of bit error probability and receiver operating characteristic curves.

LPI Scenario

Figure 1 illustrates a typical low-probability-of-intercept (LPI) scenario in which covert communication is attempted in the presence of jamming. LPI quality factors were developed in [3] to quantify the covertness of an LPI system, using the ratio of the communication and intercept ranges. Using link analysis techniques and assuming free space propagation, the LPI system quality factor is defined as follows:

\[
\frac{R_C}{R_I} = \frac{G_{CT}G_{TC}}{G_{IT}G_{TI}} \frac{L_I N_{0I}}{L_C N_{0C}} \frac{S_I/N_{0I}}{S_C/N_{0C}}
\]  

(1)

where

- \( R_C \) and \( R_I \) are the communication and intercept ranges, respectively
- \( G_{CT} \) and \( G_{CT} \) are the antenna gains at the communication transmitter and receiver, respectively
- \( G_{IT} \) and \( G_{IT} \) are the antenna gains in the intercept link
- \( L_C \) and \( L_I \) are other losses in the communication and intercept links, respectively
- \( N_{0C} \) and \( N_{0I} \) are the power spectral densities (PSDs) due to noise+interference in the communication and intercept receivers
- \( S_C/N_{0C} \) and \( S_I/N_{0I} \) are the signal power to noise/interference PSDs required at the communication and intercept receivers to meet their performance requirements (i.e., bit error probability, probability of false alarm, etc.)

The objective of the LPI system designer is to maximize this equation, providing for the largest possible \( R_C \) while forcing the interceptor to move closer to the transmitter. This is accomplished by using high gain antennas with low sidelobes, low noise receivers, and waveforms which are inherently less susceptible to interception.

To simplify the design and analysis of LPI communication systems, Equation 1 can be expressed as the combination of several quality factors, as shown:

\[
Q_{LPI} = Q_{ANT} Q_{ATM} Q_{MOD}
\]  

(2)

where \( Q_{ANT} \) is the antenna quality factor, which accounts for any advantages provided by using high quality antennas, and \( Q_{ATM} \) is the atmospheric quality factor, which accounts for the relative losses due to atmospheric effects in the communication and intercept links. These are scenario specific factors, which were not considered in this effort.

\( Q_{MOD} \) is the modulation quality factor, which allows a direct performance comparison between the intercept and communication receivers. \( Q_{MOD} \) depends only upon the type of modulation and detection employed and thus eliminates any scenario de-
dependent variables. It is defined as follows:

\[ Q_{MOD} = \frac{S_I/N_{0I}}{S_C/N_{0C}} \]  

(3)

where \( S_I/N_{0I} \) is the signal power to noise-plus-interference PSD ratio required by the interceptor to maintain a given probability of false alarm, \( P_F \), and probability of detection, \( P_D \). Likewise, \( S_C/N_{0C} \) is the signal power to noise-plus-interference PSD ratio required by the communication receiver to maintain a given probability of bit error, \( P_B \), data rate \( R_b \), and modulation scheme.

\( Q_{IS} \) is the interference suppression quality factor, which compares the ability of the intercept and communication receivers to suppress or minimize interference, thereby whitening their noise/interference backgrounds. The interference suppression quality factor is defined as follows:

\[ Q_{IS} = \frac{N_{0I}}{N_{0C}} \]  

(4)

Factors considered in determining \( Q_{IS} \) include receiver noise figures, null steering techniques, and spectral filtering [3]. The digital excision filter (DEF) used for this research is based on the fast Fourier transform (FFT). In the frequency domain, any frequency bins which exceed some given threshold are set to zero, and then an inverse FFT is performed. This process is shown in Figure 2. In part (a), the spectrum of a low-pass equivalent spread spectrum signal in the presence of a tone jammer and additive white Gaussian noise (AWGN) is shown. The output of the filter is shown in part (b), and as shown the tone jammer has been removed. An acceptable excision threshold for a variety of jamming scenarios has been determined to be 6 dB above the average signal level for all frequency bins [2].

**Simulations**

Due to the adaptive nature of the DEF, deriving an expression for \( Q_{IS} \) using the DEF would be difficult. Thus, the effects of using the DEF in both the communication and intercept links were determined via simulations using the Signal Processing Worksystem (SPW), by Alta Group, of Cadence Design Systems. The purpose of these simulations was to measure the combination of the interference suppression and modulation quality factors as a function of the DEF.

A typical simulation block diagram is shown in Figure 3, based on the scenario shown in Figure 1. All functions except the excision filter were implemented using blocks from built-in SPW libraries, and lowpass equivalent signals were used to make the simulations more efficient.

**DS-SS System**

A direct sequence spread spectrum (DS-SS) using binary phase shift keying was used for the communication link. The transmitter consisted of a random data source and a pseudorandom code generator, using a processing gain of 100. Code synchronization was not studied in this work, so the spreading code was simply passed on to the receiver. The channel consisted of an additive white Gaussian noise source and up to three complex tone jamming signals.

At the receiver, the code sequence was delayed by an amount equal to the processing time of the DEF in order to achieve code synchronization. The de-spread data was then passed onto a PSK matched filter demodulator. A PSK error counter then compared the demodulated data to the original data, and an estimate of the bit error probability was found by dividing the number of errors by the total number of bits used in the simulation. This process is collectively known as the "brute force" Monte-Carlo method [4]. To reduce the simulation times, the SPW Semi-Analytic Error Estimator Macro was also used, when appropriate. The communication system was validated by comparing simulated results with theoretical results for coherent BPSK modulation.

**Wideband Radiometer**

The implementation of the wideband radiometer is shown in Figure 4. As shown, the radiometer estimates the energy content of the signal of interest by integrating the square of the signal's magnitude and then scales the test statistic by \( 2/N_b \), where \( N_b \) is the power spectral density of the background AWGN. This estimate is then compared to a threshold, and a decision is made about whether the signal is present or not. The detection process is implemented using a sample and hold circuit and threshold crossing counter.

An estimate of the probability of false alarm was determined by dividing the number of declared detections by the total number of trials when there was only noise and interference at the input. An estimate of the probability of detection was found in a similar manner. For a given scenario with a fixed signal-to-noise ratio, receiver operating characteristics (ROC) curves were generated by storing the test statistics for the signal absent and present hypotheses to files for off-line analysis. Example histograms for the radiometer test statistics are shown in Figure 5(a). ROC curves were generated by varying the
decision threshold and estimating $P_F$ and $P_D$ for each
threshold. The resultant probabilities are then plotted as shown in Figure 5(b).

The radiometer model was validated by comparing its simulated performance to detectability equations such as those given in [1]. For example, with large time bandwidth products ($TW > 1000$), the distribution of the radiometer test statistic is approximately Gaussian, and the energy to noise PSD ratio required to obtain some desired $P_F$ and $P_D$ is given as

$$\frac{E}{N_0} = [Q^{-1}(P_F) - Q^{-1}(P_D)] \sqrt{TW} \quad (5)$$

where $T$ is the integration time, $W$ is the bandwidth of the detector, $E/N_0 = TS_I/N_0$, and $Q(x)$ is the tail integral of the Gaussian probability density function:

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-z^2/2} \, dz \quad (6)$$

Figure 5(b) illustrates ROC curves obtained using Equation 5 and the simulation model for $P_F = 0.1$ and $TW = 8192$.

**Chip Rate Detector**

The second intercept receiver considered was the chip rate detector, or delay and multiply detector. This detector creates a periodic signal at the chip frequency, which is isolated by a bandpass filter and detected using an energy detector. The component at the chip rate is formed by multiplying the input signal with a delayed version of itself, as follows:

$$x(t) = s(t)s^*(t - \tau) \quad (7)$$

where $\tau$ is typically half of the chip period. Its implementation was similar to the radiometer described earlier, except for the addition of a delay and multiply function and narrowband filter to isolate the chip rate line.

**Digital Excision Filter**

The implementation of the digital excision filter is summarized as follows: First, the time-domain data is converted to a vector of length $N$, which is then passed to a C-language custom coded block [5]. The custom coded block computes an $N$-point Fast Fourier Transform and estimates the average spectral content over all frequency bins and adds a threshold value to this average level. Any portions of the FFT which exceed this mean plus threshold value are then excised (set to zero). As stated earlier, the threshold value used for all simulations was 6 dB above the mean value of the FFT.

**Simulation Results**

Simulations were performed for several configurations. First, the effect of using the DEF on the communication link was measured for a single continuous wave (CW) jammer. The power level of the jammer was set such that the jammer-to-signal power ratio exceeded 40 dB, which easily overcame the 20 dB processing gain, and without the DEF, the probability of bit error was approximately 0.5, yielding a useless channel. However, when the DEF was employed at the communication receiver, the results shown in Figure 6(a) were obtained. As shown, for $P_B = 10^{-6}$ the single tone jammer with the excision filter is only about 1 dB worse than with no jamming.

The DEF was also shown to improve performance of the intercept receivers as well. Figure 6(b)—(c) show estimated receiver operating characteristic (ROC) curves for the radiometer and chip rate detector for constant signal and jammer power levels. It is desirable to have the ROC curves fall in the upper left corner of the ROC plot, since this would indicate the ability to achieve a large $P_D$ with a small $P_F$. The ROC for the chip rate detector without the DEF represents a worst case condition for the interceptor ($P_D \approx P_F$ for all detection thresholds), while the radiometer performed only slightly better without the DEF. Without the DEF, the only way to improve the performance of the two detectors would be to increase the transmitted signal power, which would in turn extend the communication range. In other words, the system LPI quality factor would increase, favoring the communicator.

However, when the DEF was used, both detectors experienced a dramatic improvement in performance, with no additional transmitter power. Alternatively, the signal power required at the interceptor to achieve some desired $P_D$ and $P_F$ decreases, which corresponds to a decrease in the LPI system quality factor. In other words, the interceptor could either move further away from the transmitter and maintain its current level of performance, or remain in the same position and obtain better performance.

Similar results were obtained with three CW jammers, as shown in Figure 7. Again, the communication link was only useful when the DEF was used. Likewise, the intercept receivers also greatly benefited from the excision process. However, note that more $E_0/N_0$ is required to obtain a given $P_B$ as compared to the single jammer case. This is expected since more signal energy has been excised along with the jamming signals.
Conclusions

For the various jamming scenarios, the insertion of the excision filter in the communication and intercept links translates to an improvement in performance for both communication and intercept receivers. If an excision filter is not used, the presence of very strong jamming may preclude acceptable operation of either receiver. Furthermore, if only one of the receivers uses an excision device, it will have a significant advantage over the other, which would translate to increased operational ranges.

For example, if the communication system were to operate in tandem with a cooperative jammer, the DEF would allow the communication receiver to operate with an acceptable bit error probability. If the intercept receiver does not have such an excision capability, it will have to move closer to the targeted transmitter, increasing its physical risk. However, if the intercept receiver has an excision capability, it can retain its ability to detect the LPI signal over a longer distance.

References


Figure 3: Overall System Block Diagram
Figure 4: Wideband Radiometer Block Diagram

Figure 5: (a) Histogram of Radiometer Test Statistics, and (b) Radiometer ROC Curves
Figure 6: Results for a Single CW Jammer: (a) Probability of Bit Error (b) Radiometer ROC (c) Chip Detector ROC

Figure 7: Results for Three CW Jammers: (a) Probability of Bit Error (b) Radiometer ROC (c) Chip Detector ROC