Analysis and Simulation of a Pseudonoise Synchronization System

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Abstract

This paper summarizes an investigation [1] of a pseudo-noise (PN) synchronization system that uses an auxiliary matched filter channel to speed acquisition time. The system of interest presents a combined matched filter/serial search rapid acquisition strategy in a direct sequence, multiple-access environment as described by Smirnov [2]. Attention is focused on initial acquisition. The measure of performance is the mean acquisition time and is determined analytically. A simulation model of the synchronization system using the Block Oriented Systems Simulator (BOSS) is developed. The matched filter channel is tested against the analytic expectations for false alarm performance via the simulation model.

Introduction

A commonly used method for PN code acquisition in direct sequence spread spectrum systems is the serial search of the relative positions of the received signal and the locally generated reference waveform in discrete steps of a fraction of a code chip. If no a priori timing information is available to the receiver, uniform search over the long pseudo-noise code periods lead to acquisition times that are prohibitive. Prior information about the approximate position of the received code leads to a decrease in the mean acquisition time by several orders of magnitude over a wide range of the receiver input signal-to-noise ratios [3]. In this system, the initial uncertainty is reduced with matched filter detection of shorter code segments of the received PN sequence.

The serial search of an active correlation device is the standard for comparison of a system that augments this technique with an auxiliary matched filter channel to speed code acquisition. A block diagram of the receiver synchronization section is shown in Figure 1.

Detection in the matched filter channel is based on the principle of passive correlation. The matched filter channel shown in Figure 2 consists of the matched filter, envelope detector, accumulator device, and a threshold detector. The matched filter, implemented via a tapped-delay line, continuously correlates a sub-sequence of the incoming signal with that of the reference sub-sequence of the filter. A main correlation peak appears at the output of the matched filter when the epoch of the incoming signal matches with that of the filter. The timing signal is obtained by rectifying the matched filter output through a square-law device and accumulating a number of successive correlation peaks until a threshold level is crossed. The timing signal, initiated at the instant of the threshold crossing, sets the epoch of the local PN generator and receiver synchronization is achieved.

This paper is organized as follows. First, the quantitative analysis is presented. The acquisition time performance of the active correlation device as a stand-alone synchronization circuit serves as the standard to judge improvement in acquisition time provided by the aux-

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iliary matched filter channel. Next, the simulation model is presented and the results of the false alarm performance tests are given.

**Acquisition Time Performance**

The active correlation device is a coherent, single dwell quadrature correlator. The correlator output signal is applied to a simple threshold device which in turn drives a serial search strategy. The mean acquisition time of a serial direct sequence search system is known. The expression given below assumes the search is performed in half-chip increments (cells) and in the absence of code Doppler and carrier instabilities.

\[
T_{acq} = \frac{(2 - P_d)(1 + K P_d)}{2 P_d} q \tau_d
\]  

(1)

With the dwell time set for full period correlation, the number of cells to search is \( q = 2n \) where \( n \) is the total number of code chips in a full PN code sequence. It follows that the integration time for a full period is \( T_c = n T \) where \( T \) is the chip duration [4].

To analyze the matched filter channel, the standards for acquisition time performance must be expressed in terms of receiver operating characteristics. When the matched filter channel initiates the local PN generator, for given probabilities of false alarm and detection, the quadrature correlator will indicate synchronization in the first dwell period. The ability of the matched filter to successfully initiate the synchronization process, as specified by the above probabilities, is a function of its output SNR. The matched filter may not achieve the required output SNR required for single-pulse detection. At lower output signal-to-noise ratios, the matched filter channel must process several correlation spikes prior to threshold detection and initiation of the local PN generator. The matched filter output SNR and therefore the acquisition time of the matched filter channel is influenced by the length of the filter, the number of interfering users, and level of thermal noise.

In addition to the main correlation peak and the partial cross-correlation peaks due to the PN synchronization waveform, additional signals appear at the output of the matched filter. The IF bandwidth includes thermal noise, the data waveform of the desired signal, and in a multiple-access environment, the data and PN code waveforms of other users. These additional signals degrade the matched filter output signal-to-noise ratio. To determine the peak output SNR \( \delta_{MF} \), the average signal power for each of the above interfering signals and the peak power of the desired PN code sequence must be determined.

The performance of the matched filter channel can be characterized in terms of the first two moments of the probability density function for the amplitude of the matched filter output. The partial correlation output of the matched filter is treated as a Gaussian random variable at the input to the square-law detector.

The input chip waveform, \( \sqrt{2} P \tau(t + \tau) \), correlated by the matched filter reference, \( c(t + \tau') \) over the period \( M T_c \) is given by:

\[
C = \sqrt{2} P \int_0^{MT_c} \tau(t + \tau) c(t + \tau') dt
\]  

(2)

where \( M \) is the number of stages in the matched filter [4]. The expected value of the correlation conditioned upon whether the incoming chip waveform is correlated with the reference waveform \( (H_1) \) or uncorrelated \( (H_0) \) is given by:

\[
E(C|H_i) = \left\{ \begin{array}{ll}
\sqrt{2} P M T_c (1 - |\Omega|) & i = 1 \\
0 & i = 0
\end{array} \right.
\]  

(3)

where \( \Omega \) is the fractional timing offset with respect to a chip between the two code waveforms [4].

The expected value reaches a peak value of \( \sqrt{2} P M T_c \) when the codes are correlated \((\Omega = 0)\). Since partial correlation is a Gaussian random variable with zero mean, the expected value of uncorrelated code segments is zero. The power of the uncorrelated waveforms is expressed by the variance:

\[
\text{var}(C|H_i) = 2 P M T_c^2 G_i(\Omega)
\]  

(4)

where

\[
G_i(\Omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^\infty e^{-x^2/2} dx
\]
The power of the interfering waveforms can now be determined. The data waveform of the desired signal will be synchronized with its associated PN code waveform and will therefore have no offset \((\Omega = 0)\) with the reference waveform. The average power is therefore equal to \(P_{D}MT_c^2\) where \(P_{D}\) represents the power in the data waveform. The data and PN code waveforms associated with other users will occur with an offset outside the correlation interval. Assuming that the timing offset for these waveforms is uniformly distributed over a chip period, the average value of \(G_i(\Omega)\) will be:

\[
G_i(\Omega) = \left\{ \begin{array}{cl} \Omega^2 & i = 1 \\ 1 - 2|\Omega| + 2\Omega^2 & i = 0 \end{array} \right. \quad (5)
\]

The average power for the data waveforms associated with other users is therefore \(\frac{3}{2}P_{D}MT_c^2\). The average power for the PN code waveforms associated with other users is \(\frac{1}{2}P_{C}MT_c^2\).

With thermal noise at the input to the matched filter expressed as \(N = N_oBW_{IF}\) and no correlation with the reference signal \((\Omega = 1\) or \(\Omega = -1)\), the noise power at the output of the matched filter channel is \(NMT_c^2\). The total average power of interfering sources can now be expressed:

\[
\sigma_{\text{total}}^2 = MT_c^2 \left[ N + P_{D} + \frac{3}{2}K(P_{D'} + P_{C'}) \right] \quad (7)
\]

where \(K \geq 0\) is the number of interfering users.

The peak signal power of the desired PN code waveform is the square of the peak \((\Omega = 0)\) expected value for a correlated waveform and is equal to \(2P_{C}(MT_c)^2\). The peak SNR at the output of the matched filter channel can now be expressed:

\[
\delta_{\text{MF}} = \frac{2P_{C}M}{N + P_{D} + \frac{3}{2}K(P_{D'} + P_{C'})} \quad (8)
\]

To simplify further analysis, the power of the data and PN code waveforms will all be set equal such that \(P_{C} = P_{D} = P_{C'} = P_{D'}\). Allowing the combined power of the data and PN code waveforms of a particular transmitted signal to be represented as \(S = P_{C} + P_{D}\), the matched filter peak output SNR reduces to:

\[
\delta_{\text{MF}} = \frac{M\delta_i}{1 + \delta_i\left(\frac{3}{2} + \frac{3}{2}K\right)} \quad (9)
\]

where \(\delta_i = S/N\) is the SNR of the target transmitter, \(K\) is the number of other users, and \(M\) is the length of the matched filter.

Acquisition time in terms of code periods is given by the number of pulses required for detection at specified probabilities of false alarm and detection. A pulse, or main correlation peak, occurs once every code period. The length of a code period is equal to the number of chips in the code sequence multiplied by the code chip duration or \(nT_c\). Letting \(N_p\) represent the number of pulses processed according to tabulated single and multiple-pulse detection requirements [5], the mean acquisition time is given by:

\[
T_{\text{acq}}(M) = N_pnT_c \quad (10)
\]

Acquisition time performance for a range of input signal-to-noise ratios, matched filter length, and number of other users has been determined [1]. Figure 3 shows performance characteristics for a matched filter length of 256 and a false alarm rate \(P_{fa} = 10^{-6}\) and detection probability \(P_d = 0.9\).

Improvement in acquisition time offered by the matched filter channel is evident in the ratio of the mean acquisition times for the correlator and that of the matched filter given in Eq (10). Recalling Eq (1) and recognizing that \(KP_{fa} << 1\) for the case of interest \((P_{fa} = 10^{-6}\) and \(P_d = 0.9\)), the mean acquisition time for the correlator acting alone is:

\[
T_{\text{acq}}(C) = 1.22n^2T_c \quad (11)
\]

where \(n\) is the number of code chips in the PN code period. The acquisition time is improved by a factor \((I)\) expressed in the following ratio:

\[
I = 1.22\frac{n}{N_p} \quad (12)
\]

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As shown in Figure 3, for signal-to-noise ratios in the range above 0 dB, a variation of the input SNR has no effect on the acquisition times. In this range, the primary influence on acquisition time performance for a given matched filter length is the number of users in the system. Improvement in the acquisition times over this range of input signal-to-noise ratios is shown in Figure 4 for a code length of $n = 2047$ chips.

As the input signal-to-noise ratio decreases 0 dB, the number of users in the system becomes less of an influence on the acquisition times. For further decreases in the input signal-to-noise ratio, the curves asymptotically approach a doubling in the acquisition time for each 1.5 dB decrease in input SNR [5]. For the input SNR range below $-18$ dB, noise is the dominant influence on acquisition time. In this range, increasing the number of users does not produce the degree of variation in the improvement factor that is seen at the higher input signal-to-noise ratios. In addition, the improvement levels are significantly reduced due to the high additive white Gaussian noise (AWGN) levels.

To summarize the quantitative analysis, it is the peak output SNR of the matched filter that determines acquisition time performance. For a selected $P_{fa}$ and $P_{da}$, the peak output SNR determines whether a signal can be detected within the tabulated single-pulse or multiple pulse detection requirements. If the output SNR does not meet single-pulse detection requirements, the number of pulses that must be processed to meet the multiple-pulse detection requirements determines the acquisition time. Therefore, the acquisition time performance is directly related to the matched filter output SNR as a function of the matched filter length, the number of other users in the system, and the AWGN level.

Simulation

The Block Oriented System Simulator (BOSS) provides for simulation-based analysis and design of any system which can be represented in block diagram form. Once a correct description of the system is expressed in block diagram form, BOSS supports a time-domain (waveform level) simulation [6].

The simulation model for the single-user system is shown in Figure 5. A single transmitter with an AWGN channel is implemented. At the end of the initial load sequence, the matched filter reference generator sends a ready signal to select the transmitter output. The output of the quadrature correlator is terminated since the focus of the simulation effort is to examine the initial acquisition characteristics.

Development of a model that will allow a realistic appreciation of actual system behavior under a variety of scenarios is the goal of the simulation effort. However, a simulation model must be proven against theoretical expectations before it can be freely exercised. The operating characteristics of the matched filter is the key to system performance. For a given signal-to-noise ratio and probability of detection, the matched filter output will be examined to determine if the correlation peaks occur at the expected value for the specified probability of false alarm.

Determination of false alarm performance is a multi-step process. First, the average noise power at the output of the matched filter is determined via simulation. The associated peak output signal-to-noise ratio is then determined analytically. The peak output SNR distinguishes whether the matched filter at a particular length and SNR can perform at single or multiple-pulse detection requirements for a specified $P_{fa}$. Threshold values are then set to confirm matched filter performance at the specified $P_{fa}$.

The $P_{fa}$ chosen for simulation is $10^{-3}$ and requires a minimum matched filter peak output SNR of 13.7 dB to meet single-pulse detection requirements. This false alarm rate can be simulated with reasonable time frames and memory requirements.

In order to confirm $P_{fa}$ performance, the threshold level must be accurately determined. The method for single-pulse detection tests sets the threshold level for the signal directly at the output of the matched filter. The governing equation for this method is [5]:

$$P_{fa} = \exp \left[ \frac{\beta^2}{2} \right] \quad (13)$$
where $\beta = \alpha / \sigma_T$. The threshold value ($\alpha$) necessary to test for a particular $P_{fa}$ at the output of the matched filter is dependent on the average noise power ($\sigma_T^2$) at the sampling instant. This method is simple and direct and offers a certain clarity in terms of the threshold value against the expected peak signal value.

The method for the multiple-pulse detection tests establishes the threshold level at the output of the accumulator. A table of bias levels $\delta = \delta(m, p)$ corresponding to the specified probability of false alarm, $P_{fa} = 10^{-p}$, as a function of the number of pulses $(m)$ required for detection exist for receivers using square-law detectors [7]. Given the bias level ($\delta$) from the table and the average noise power ($\sigma_T^2$), the threshold level is determined by:

$$\alpha = 2\sigma_T^2 \delta \quad (14)$$

The next step is to determine how long the simulations must run in order to confirm performance at a particular probability of false alarm. The rule of thumb is to run the simulation long enough to count at least ten errors [8]. For a specific $P_{fa}$, the period of simulation is equal to $10/P_{fa}$ times the interval over which a false alarm can occur. The interval over which a false alarm can occur is different for single-pulse and multiple-pulse detection sequences.

For the single-pulse detection sequences, a false alarm can occur over any two-chip period. The matched filter continuously correlates its reference sequence with the incoming signal and the main correlation peak occurs over a two-chip period. During all other two-chip intervals, a correlation peak exceeding the specified threshold can occur due to noise or the cross-correlation characteristics of the input signal. Therefore, the length of all simulations testing $P_{fa}$ for single-pulse detections is set according to:

$$\text{stop time} = \frac{10}{P_{fa}} DT \left[ \frac{\text{Samples}}{\text{chip}} \right] \cdot \left[ \frac{\text{chips}}{PNP\text{Period}} \right] (\#\text{pulses}) \quad (15)$$

Obviously, the length of simulations to characterize multiple-pulse detection requirements are excessive. A validation simulation will be run for the multiple-pulse detection sequences. In other words, the simulation will verify that the specified threshold is crossed in the expected time frame over one false alarm interval.

The results of the $P_{fa}$ simulations for the single-user system are given in Table 1. In the single-pulse detection tests, the result is a value for false alarm performance. The multiple-pulse detection tests are validation simulations which yield pass or fail results.

For a matched filter of length equal to 64 stages, the multiple-pulse detection tests failed with the single-user input. These failures are attributed to the high level of correlation noise produced by the selected code. This noise coupled with the AWGN consistently produce failure indications even with threshold level and time period adjustments. This filter length was therefore not simulated with the multiple-user input. At shorter matched filter lengths, performance becomes increasingly sensitive to the correlation noise produced by the selected PN code. It is reasonable to assume that a PN code with lower partial cross-correlation characteristics would produce favorable results at these matched filter lengths.

In the multi-user system, code selection is a significant consideration. Ideally, a set of orthogonal codes is used. The simulation effort did not entail a code selection process for the
multi-user system—detection of the single target transmitter was the objective. Therefore, random sequence generators substituted controlled code generation by the interfering users. The impact was increased cross-correlation noise and higher threshold levels. The effect was significant at a matched filter of length equal to 128 stages. Once again, it is reasonable to assume that careful code selection would produce favorable results at these matched filter lengths.

The remaining results confirm the expected performance characteristics. All single-pulse detection tests were successful. The validation tests for multiple-pulse detections were also successful at the longer matched filter lengths. Sensitivity to the correlation noise at these matched filter lengths was not a factor. Recognizing the impact of code selection and the correlation noise produced by the selected PN code, the simulation model is suitable to provide a realistic appreciation of actual system behavior.

Conclusions

The quantitative results indicate that the matched filter channel improves acquisition time. For signal-to-noise ratios above 0 dB and a particular matched filter length, the mean acquisition time was directly a function of the number of users in the system. Also, mean acquisition time was significantly improved for signal-to-noise ratios above 0 dB. As signal-to-noise ratios decreased below 0 dB, the variation in mean acquisition time caused by the number of users rapidly declines. For signal-to-noise ratios below -18 dB, mean acquisition time is almost directly related to the noise level regardless of the number of users. Mean acquisition time asymptotically approaches a doubling in time for each 1.5 dB decrease in the signal-to-noise ratio.

The simulation results confirmed the analytic expectations for false alarm performance of the matched filter channel. All single-pulse detection requirements at the specified $P_{fa}$ were favorably confirmed. Multiple-pulse detection requirements were favorably validated with shorter simulations. Exceptions to the multiple-pulse detection results were noted due to the high correlation noise produced by the selected PN code. The shorter matched filter lengths proved sensitive to the linearly-additive qualities of the correlation noise under the multiple-pulse detection tests.

The Block-Oriented Systems Simulator provided a highly flexible simulation environment. The system model includes flexibilities to adjust matched filter lengths, noise levels, and threshold levels. In this effort, the simulation model was used to statically confirm the analytic expectations. The successful static tests prove suitability of the model to accurately simulate dynamic applications of system.

References

[8]. Jeruchim, Michel C. “Techniques for Estimation the Bit Error Rate in the Simulation of Dig-
Figure 1. Synchronization System

Figure 2. Matched Filter Channel
Figure 3. MF Length = 256 ($P_{fa} = 10^{-6} / P_d = 0.9$)

Figure 4. Acquisition Time Improvement (Input SNR ≥ 0dB)
Figure 5. Single-User System

Table 1. Single-User $P_f$ Results

<table>
<thead>
<tr>
<th>MF Length</th>
<th>Target Transmitter SNR Levels</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No AWGN</td>
</tr>
<tr>
<td>64</td>
<td>Pass</td>
</tr>
<tr>
<td>128</td>
<td>Pass</td>
</tr>
<tr>
<td>256</td>
<td>0.69e-3</td>
</tr>
<tr>
<td>512</td>
<td>1.19e-3</td>
</tr>
<tr>
<td>1024</td>
<td>1.39e-3</td>
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</tbody>
</table>