the inverse of the noise variance at each point

$$
\sum_{l=1}^{n} \sigma_l^2 \Psi_j, \Psi_k, i = \delta_{jk}. \tag{24}
$$

Such functions can easily be constructed by a Gram–Schmidt or modified Gram–Schmidt technique [5] provided the noise variance at each point is known. Such a method of forcing orthonormality is not necessarily a hindrance. The periodic sampling of functions of a continuous variable which are orthonormal on an interval does not generally yield sequences which are orthonormal on the same discretized interval, and hence one is often compelled in such instances to use a Gram–Schmidt technique to generate a set of orthonormal sequences. In such instances, the sequences should be constructed to satisfy (24) rather than (2) if the noise variances are not constant on the interval. One obvious disadvantage of variances that are not constant is that algorithms such as the fast Fourier transform cannot be used to rapidly determine coefficient estimates, since the discrete sine and cosine functions would not be orthonormal on the interval with respect to the weights.

If the noise associated with the data is not Gaussian, as, for example, the time sequence of Poisson distributed counts from a radioactive source, then minimization of the weighted square error is not equivalent to maximization of the likelihood function. In such cases, the approximation is sometimes still made after carrying out a suitable transformation of the noise variance at each point [4]. Alternatively, one may minimize a different quantity, such as a non-Euclidean norm. The danger in this approach is that maximization of the likelihood function may not correspond to the minimization of a single quantity, as in the case of the least squares approximation.

In any event, one should be aware of the limitations in bias and accuracy when estimating coefficients in an orthonormal expansion, in order to assure that the results from such techniques are meaningful.

### REFERENCES


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**Performance Model for Square Law Detectors Followed by Accumulators and an ORing Device**

**W. A. STRUZINSKI**

**Abstract**—A mathematical model is developed to characterize the performance of square law detectors followed by accumulators and an ORing device. The performance of a Gaussian signal in Gaussian noise is determined for a specified number of channels ORed. The ORing loss is computed as a function of the number of samples in the accumulator preceding the ORing device for various fixed input signal-to-noise ratios (SNR) to the square law detector.

It is concluded that the ORing loss decreases with an increase in the number of samples in the accumulator preceding the ORing device.

### INTRODUCTION

Many signal processing systems produce a large quantity of data that are not feasible to display. It is desired to reduce the amount of data displayed, and thereby reduce the amount of hardware needed. One technique that provides a reduction of data is ORing, a process wherein one picks that single channel with the most energy.

The subsequent analysis characterizes the performance of square law detectors followed by accumulators and an ORing device. Some relevant past work on this topic is given in [1]–[3].

### MATHEMATICAL ANALYSIS

The system of interest is shown in Fig. 1. The input to the ORing device $X_1, X_2, \ldots, X_N$ contains either $N$ channels of noise or $N - 1$ channels of noise, and one channel of signal plus noise. The random variables $X_1, X_2, \ldots, X_N$ are statistically independent, and they are identically independently distributed (iid) for the noise-only channels. The accumulators following the square law detectors are electrical integrators that sum $M$ statistically independent samples.

It is assumed that the input to the square law detector is an envelope of a Gaussian signal in Gaussian noise, both having zero mean. The input signal-to-noise ratio (SNR) to the square law detector (SNR$_{ID}$) is given by

$$
SNR_{ID} = 10 \log (\sigma_s^2 / \sigma_n^2) \text{ (dB)}
$$

where $\sigma_s^2$ = signal power at the input to the square law detector and $\sigma_n^2$ = noise power at the input to the square law detector.

The input SNR to the ORing device (SNR$_{IOR}$) for Gaussian input statistics is

$$
SNR_{IOR} = 20 \log \left( \sqrt{M} (\sigma_s^2 / \sigma_n^2) \right) = 20 \log d_{IOR} \text{ (dB)}
$$

where $d_{IOR} = \sqrt{M} (\sigma_s^2 / \sigma_n^2)$ = input SNR to the ORing device in nondecibel form and $M$ = number of samples in the accumulator.

Finally, the SNR at the output of the ORing device (SNR$_{OOR}$) is given by

$$
SNR_{OOR} = 20 \log \left( \frac{\mu_{Y1} - \mu_{YO}}{\sigma_{YO}} \right) \text{ (dB)}
$$

where $\mu_{Y1}$ = mean of the signal plus noise after ORing, $\mu_{YO}$ = mean of the noise after ORing, and $\sigma_{YO}$ = standard deviation of the noise after ORing.

In order to evaluate (3), it is necessary to determine $\mu_{YO}$, $\mu_{Y1}$, and $\sigma_{YO}$. The output of the ORing device is mathematically defined as

$$
Y = \max (X_1, X_2, \ldots, X_N). \quad (4)
$$

For the noise-only case, the cumulative distribution of $Y$, $F_0(y)$ is

$$
F_0(y) = \text{prob} (Y \leq y | \text{all noise})
$$

$$
= \text{prob} (X_1, X_2, \ldots, X_N \leq y | \text{all noise})
$$

$$
= F_0^N (y). \quad (5)
$$

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Manuscript received April 23, 1982; revised November 11, 1982 and January 3, 1983.

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The probability density function $f_0(y)$ for the noise-only case is
\[ f_0(y) = \frac{dF_0(y)}{dy} = NP_0^{-1}(y)p_0(y). \]  

For the case in which one of the $X_i$'s is signal plus noise, the cumulative distribution of $Y$, $F_1(y)$, is defined as
\[ F_1(y) = \text{prob}(X_1, X_2, \cdots, X_N \leq y|\text{signal}) = \sum_{i=1}^{N} p_0^{-1}(y) P_i(y). \]  

The probability density function $f_1(y)$ for the signal plus noise case is defined as
\[ f_1(y) = \frac{dF_1(y)}{dy} = (N-1) P_0^{-2}(y) p_0(y) P_1(y) + \sum_{i=2}^{N} P_i(y). \]  

The mean values $\mu_{Y0}$ and $\mu_{Y1}$ in (3) for the signal-absent and signal-present cases are
\[ \mu_{Y0} = \int_{-\infty}^{\infty} y f_0(y) \, dy = N \int_{-\infty}^{\infty} y P_0^{-1}(y) p_0(y) \, dy \]  
\[ \mu_{Y1} = \int_{-\infty}^{\infty} y f_1(y) \, dy = \int_{-\infty}^{\infty} y \left[ (N-1) P_0^{-2}(y) p_0(y) P_1(y) + \sum_{i=2}^{N} P_i(y) \right] \, dy. \]

The standard deviation for the noise-only case $\sigma_{Y0}$ in (3) is
\[ \sigma_{Y0} = \left( \bar{y}_0^2 - \mu_{Y0}^2 \right)^{1/2} \]  

where
\[ \bar{y}_0 = \int_{-\infty}^{\infty} y^2 f_0(y) \, dy = N \int_{-\infty}^{\infty} y^2 P_0^{-1}(y) p_0(y) \, dy. \]

The only thing remaining is to determine the cumulative distribution functions $P_0(y)$ and $P_1(y)$ and the probability density functions $p_0(y)$ and $p_1(y)$ of the input to the ORing device.

It is assumed that sufficient post-detection integration has been performed in Fig. 1 to invoke the central limit theorem. Employing Gaussian input statistics, we obtain the mean values $\mu_{Y0}$, $\mu_{Y1}$, and the standard deviation $\sigma_{Y0}$
\[ \mu_{Y0} = m_0 + \sigma A_N \]  
\[ \mu_{Y1} = m_0 + \sigma C_N(d_{IOR}) \]  

The signal-to-noise ratio at the output of the ORing device ($SNR_{ROOR}$) is now obtained by substituting (12)–(14) into (3)
\[ SNR_{ROOR} = 20 \log \left[ \frac{C_N(d_{IOR}) - A_N}{(B_N - A_N^2)^{1/2}} \right] \text{ (dB)}. \]  

It is now desired to evaluate the performance of the system in Fig. 1. The integration over $M$ statistically independent samples produces a gain. However, this gain will be reduced by an amount equal to the ORing loss. The ORing loss relative to the input to the ORing device ($SNR_{lossof OR}$) is
\[ SNR_{lossof OR} = 20 \log \left( \frac{\sqrt{M} \sigma_0^2}{\sigma_{IOR}^2} \right) - 20 \log \left[ \frac{C_N(d_{IOR}) - A_N}{(B_N - A_N^2)^{1/2}} \right] \text{ (dB)}. \]  

The ORing loss relative to the detector input ($SNR_{lossD}$) is governed by (17)
\[ SNR_{lossof D} = 10 \log \left( \frac{\sqrt{M} \sigma_0^2}{\sigma_{IOR}^2} \right) - 10 \log \left[ \frac{C_N(d_{IOR}) - A_N}{(B_N - A_N^2)^{1/2}} \right] \text{ (dB)}. \]
A Distributed Gray–Markel Filter

FRED J. TAYLOR

Abstract—The distributed arithmetic philosophy has been shown to support very high data rate linear shift-invariant filtering. These filters traditionally exhibit a Direct I structure. A distinctly different architecture, namely the Gray–Markel filter, is studied in this note. Efficient and high-speed distributed arithmetic Gray–Markel filters are shown to be feasible.

I. LADDER ARCHITECTURES

Historically, one of the principal advantages of a ladder architecture is its low coefficient roundoff error sensitivity [1], [2]. This is a moot point in the context of a distributed filter. Here, the distributed filter tables are programmed using real (not rounded) coefficient values. Therefore, the coefficient roundoff sensitivity for a distributed ladder filter should be very similar to that of all admissible distributed architectures. However, there is a ladder filter attribute which cannot be emulated by the traditional Direct I distributed filter structure. An Mth order Gray–Markel filter, for example, will present to the user a set of M + 1 mutually orthogonal outputs [6]. This makes this class of filter especially useful in certain speech and signature analysis applications. A one and two multiplier Gray–Markel filter consists of 3M + 1 multipliers and 2M + log₂(M + 1) two input adders for the two-multiplier form, or 2M + 1 multipliers and 3M + log₂(M + 1) adders for the one-multiplier structure. A traditional Direct I distributed filter realization simply moves a copy of its shift-register image to its series successor [i.e., x(k) → x(k–1) for z system input or output variable]. In a Gray–Markel filter, each state (shift-register value) is independently defined. As a result, a Direct I realization is impossible.

Distributed Gray–Markel Filter

A Gray–Markel distributed filter is suggested in Fig. 1. In this figure it is shown that a distributed Gray–Markel filter can be architected by interconnecting L distributed nth order state subfilters and S distributed output filters of order T. Here

\[ L = \lfloor M/n \rfloor, \quad S = \lfloor M/T \rfloor, \quad \log_2 S = \text{output adder levels}. \] (1)

A detail of an nth order state subfilter can be found in Fig. 2. The ith subfilter is a function of states \( \{x_1(n), \ldots, x_L(n)\} \), and inputs \( u_i(k) \) and \( x_1(k) \). Based on a two-multiplier model the state and system variable dependence is given by

\[ q_n^i(k) = q_n^i(u_i(k), x_1^i(k) - 1, x_1^i(k - 1), \ldots, x_L^i(k)), \] \[ x_1^i(k) = f_1^i(u_i(k), x_1^i(k - 1), x_1^i(k - 1), \ldots, x_L^i(k - 1)), \] \[ x_L^i(k) = f_L^i(u_i(k), x_1^i(k - 1), x_1^i(k - 1), \ldots, x_L^i(k - 1)). \] (2)

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Manuscript received February 15, 1982; revised November 16, 1982. This work was supported in part by the National Science Foundation under Grant ECS-800A71.

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