Performance of the Generalized Cross Correlator in the Presence of a Strong Spectral Peak in the Signal

JOSEPH C. HASSAB AND RONALD E. BOUCHER, MEMBER, IEEE

Abstract—Under consideration is the effectiveness of various windowing functions in the generalized correlator when a strong spectral peak, i.e., a sinusoid, is present in the signal. The windows $W_{\text{IBF}}(\omega)$ and $W_{\text{SCOT}}(\omega)$ or $W_{R}(\omega)$ avoid the ambiguity problem that is encountered by the other windows when sinusoids are present in the signal.

I. INTRODUCTION

Much of the comparative analysis and simulation of windowing functions in the generalized cross correlator have been performed for signals that do not contain strong spectral components in their elements. Sinusoids in signals are encountered in many applications and originate usually from the rotary motion in the mechanism of the emitting source that is under passive acoustic observation.

This paper considers the inclusion of such a strong spectral peak in the signal elements and the resulting effect on the performance of various spectral weighting windows found in the literature. A given window is added to the basic cross correlator in order to improve its performance when measuring time delays between weak signals received respectively at two sensors in the presence of a noise field. It seems worthwhile to point out the variable effectiveness of the windows when a dominant spectral peak is present in the received signals. The basic correlator output, for instance, becomes highly oscillatory, thus causing ambiguity in the selection of the dominant peak that corresponds to the time delay. In this instance, Carter, Nuttall, and Cable [1] used a smoothed coherence transform window $W_{\text{SCOT}}(\omega)$ to reduce the influence of a strong tonal. However, for smooth signal and noise spectra, Hassab and Boucher [2], [3] have noted that the addition of $W_{\text{SCOT}}(\omega)$ has weakened the performance of the basic cross correlator while other windows have improved it. Now, it seems worthwhile to contrast the performance of all those windows for the type of signals where the usefulness of $W_{\text{SCOT}}$ is emphasized [1].

II. ANALYSIS AND RESULTS

Assume that the outputs of two spatially separated sensors are

$$z_1(t) = y(t) + n_1(t)$$
$$z_2(t) = ay(t - \tau) + n_2(t)$$

(1)

where $y(t)$ is the emitted signal which contains a wide-band element $y_1(t)$ and a narrow-band element $y_2(t)$. The noise $n_1(t)$ and $n_2(t)$ are real jointly stationary and uncorrelated random processes. The functions $z_1(t)$ and $z_2(t)$ are stationary over the observation time $T$ with much greater than both the time delay $\tau$ and the decorrelation time of the signal element $y_2(t)$ only.

A frequency domain implementation of the generalized cross correlator involves [2] the inverse Fourier transform ($F^{-1}$) of

$$Z_1^*(\omega)Z_2(\omega)W(\omega) = a\phi_\omega(\omega)e^{-i\omega\tau}W(\omega) + \phi_1 n_2(\omega)W(\omega) + \phi_n(\omega)W(\omega)$$

(2)

where

$$\phi_\omega(\omega) = |Y(\omega)|^2$$
$$\phi_1 n_2(\omega) = N_1^*(\omega)N_2(\omega)$$
$$\phi_n(\omega) = Y^*(\omega)N_2(\omega) + aY(\omega)e^{-i\omega\tau}N_2^*(\omega).$$

The lower case $\phi$ refers to a single realization of the function and uppercase $\Phi$ will refer to its expected value over the ensemble, i.e., $<\phi_\omega(\omega)> = <|Y(\omega)|^2> = \phi_\omega(\omega)$. The term $W(\omega)$ represents the windowing function and is equal to unity for the basic cross correlator.

General and particular forms of the various windows are listed in Table I. An ideal processor has an impulse response given by $F^{-1}e^{-j\omega\tau}$. As given by (2), the correlator output, however, is made up of disturbances from the amplitude modulation of the information term $e^{-i\omega\tau}$ by the noise spectrum $\phi_\omega(\omega)$, from the noise term $\phi_1 n_2(\omega)$, and from the cross term $\phi_n(\omega)$. In

TABLE I

<table>
<thead>
<tr>
<th>WINDOW FORM</th>
<th>W(\omega)</th>
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<tbody>
<tr>
<td>$W_{\text{IBF}}$</td>
<td>$1$</td>
</tr>
<tr>
<td>$W_{\text{SCOT}}$</td>
<td>$1$</td>
</tr>
<tr>
<td>$W_{\text{IBF}}$ or $W_{\text{SCOT}}$</td>
<td>$\tau$</td>
</tr>
<tr>
<td>$W_{R}$</td>
<td>$1$</td>
</tr>
<tr>
<td>$W_{\text{LWS}}$</td>
<td>$1$</td>
</tr>
<tr>
<td>$W_{\text{AC}}$</td>
<td>$1$</td>
</tr>
<tr>
<td>$W_{\text{LC}}$</td>
<td>$1$</td>
</tr>
</tbody>
</table>

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III. Applications

Several cases have been simulated. An example is given here to illustrate the behavior of the various windows when measuring the time delay for a signal that contains a strong spectral peak. The signal has also a random element with a spectral density

$$\Phi_y(\omega_k) = \frac{\alpha^2}{\left(\alpha^2 + \omega_k^2\right)}, \quad \omega_k = 2\pi k/N, \quad k = 0, 1, \ldots, N, \quad N = 255, \quad \alpha = 0.333,$$

and a sinusoidal element whose interfering frequency is at 30/(N + 1). The ratio of interference power in y_w to signal power in y_s is set equal to 25 and 1, respectively. The additive noise sequence is uncorrelated, white and Gaussian. The signal-to-noise ratio at each sensor equals 5 dB where

$$\text{SNR} = \left[\frac{\sum_k \Phi_y(\omega_k)}{\sum_k \Phi_n(\omega_k)}\right].$$

The echo strength $\alpha$ is set equal to 1.

The respective windowing functions are depicted in Fig. 1. It is observed that $W_{\text{HBII}}(\omega_k)$ and $W_{\text{SCOT}}(\omega_k)$ or $W_R(\omega_k)$ notch out the interfering sinusoid at the tonal frequency. On the other hand, $W_{\text{HBII}}(\omega_k)$ or $W_{\text{ML}}(\omega_k)$, $W_E(\omega_k)$ or $W_{\text{SAC}}(\omega_k)$, $W_{\text{HBS}}(\omega_k)$ enhance the amplitude of the interfering frequency. The notching process in $W_{\text{HBII}}(\omega_k)$ and $W_{\text{SCOT}}(\omega_k)$ yields a distinct peak at the generalized correlator output whose position corresponds to the correct time delay. The enhancement process in $W_{\text{HBII}}(\omega_k)$ or $W_{\text{ML}}(\omega_k)$, $W_E(\omega_k)$ or $W_{\text{SAC}}(\omega_k)$, and $W_{\text{HBS}}(\omega_k)$ yields a series of ambiguous peaks in the correlator output (Fig. 2). Fig. 3 gives the generalized cross correlator outputs when the power ratio in $y_w$ to $y_s$ is reduced to 1. All windows yield a correct time delay estimate, but with a better peak to background enhancement given respectively by $W_{\text{HBII}}$, $W_{\text{SCOT}}$ or $W_R$, $W_{\text{HBII}}$ or $W_{\text{ML}}$, $W_{\text{HBS}}$, $W_E$ or $W_{\text{SAC}}$. It may be observed that even though the signal $y$ is low in frequency, the windows $W_{\text{HBII}}(\omega_k)$ and especially $W_{\text{SCOT}}(\omega_k)$ give most weight to a higher frequency region. In this regard, we note that the window $W_{\text{HBII}}$ is designed to optimize detection for a nonlinear and not a linear system [2]. In an experimental simulation, the window $W_{\text{HBII}}$ has outperformed $W_E$ and $W_{\text{HBS}}$, even though the latter windows have emphasized the low frequency region [3]. It is also noted that the noise characteristic (i.e., the signal) is reduced to twice that of the low frequency signal $y_w$. As $W_R$ or $W_{\text{HBII}}$ or $W_{\text{ML}}$, $W_{\text{HBS}}$ gives the most weight to the high frequency region. $W_{\text{HBS}}$ continues to stress the low frequency region, since it has been designed to estimate the time delay as well as the signal. Finally, the windows $W_R$ and $W_{\text{SAC}}$ have respectively a similar effect to $W_{\text{SCOT}}$ and $W_E$ on the cross correlator output for the signal and noise characteristics used in the simulated cases of this paper.

IV. Discussions

The windows $W_{\text{HBII}}(\omega)$ and $W_{\text{SCOT}}(\omega)$ or $W_R(\omega)$ are effective for time delay measurements by a generalized correlator in the presence of a strong spectral peak in the signal. The win-
Fig. 1. The form of various windows as a function of $\omega_k$ for signal and noise at hand. Only $W_{\text{HBII}}$ and $W_{\text{SCOT}}$ or $W_R$ have a notch at the sinusoidal frequency in the signal.
Fig. 2. Generalized cross correlator outputs using various windows. Only $W_{HBI1}$ and $W_{SCOT}$ or $W_R$ avoid the ambiguity problem and yield correct time delay. Ratio of power in sinusoidal signal to wideband signal equals 25.

dows $W_{HBI1}(\omega)$ or $W_{ML}(\omega)$, $W_E(\omega)$ or $W_{SAC}(\omega)$, $W_{HBL}(\omega)$ are not. In order to exploit the main benefit of $W_{SCOT}(\omega)$, the presence of a strong tonal in the signal is necessary when using it. Otherwise, application of $W_{SCOT}(\omega)$ would display a higher $S/N$ threshold than $W_{HBI1}(\omega)$ [2], [3]. In addition, $W_{SCOT}(\omega)$ is observed to deteriorate the performance of the basic correlator instead of aiding it [3]. An independent confirmation of the last result is given in [10]. $W_{HBI1}(\omega)$ does not display this disadvantage and yields a definite improvement on the basic cross correlator in the absence of sinusoids. This characteristic of $W_{HBI1}(\omega)$ to operate effectively in either the presence or absence of sinusoids is a very desirable characteristic when contrasted to $W_{SCOT}(\omega)$ and, for that matter, $W_{ML}(\omega)$. In practice, a sinusoid may not be constantly present or constantly absent but fades in and out during a series of processing intervals, thus enhancing the desirability of a $W_{HBI1}(\omega)$ type window.

Naturally, if it is determined a priori that $\phi(\omega)$ is "well-be-
Fig. 2. Generalized cross correlator outputs using various windows. Only $W_{HBI}$ and $W_{SCOT}$ or $W_R$ avoid the ambiguity problem and yield correct time delay. Ratio of power in sinusoidal signal to wide-band signal equals 25.

Fig. 3. Generalized cross correlator outputs using various windows for equal power in sinusoidal signal to that in wide-band signal. Better time delay peak to background enhancement is given respectively by $W_{HBI}$, $W_{SCOT}$ or $W_R$, $W_{HBI}$ or $W_{ML}$, $W_{HBL}$, $W_E$ or $W_{SC}$. The variance of the peak, however, this information should be included in the design of the window and this reduces $W_{HBI}(\omega)$ to $W_{HBI}(\omega)$. $W_{HBI}(\omega)$ is then the optimum time delay detector according to the criterion in [2], and is also the optimum time delay estimator as defined in [4]-[7] where for a high output signal-to-noise ratio, the time delay variance would approach the Cramér-Rao bound, given a zero bias in the time delay estimates. This means that the detection process has occurred successfully in the immediate neighborhood of the correct peak denoting the time delay.
Finally, the theoretical predictions [1]–[10] on the performance of any window do not reflect the inaccuracies incurred in a certain implementation. If unattended to, such inaccuracies deteriorate differently the performance of each window and could have a more pronounced effect on a window optimized to a given set of conditions versus a robust one. Such inaccuracies can be minimized, for instance, through different partitioning of the estimated elements in a window and through optimization by parameter adjustment of the resulting window estimate.

Fig. 3. Generalized cross correlator outputs using various windows for equal power in sinusoidal signal to that in wide-band signal. Better time delay peak to background enhancement is given respectively by $W_{HBI}$ or $W_{ML}$, $W_{HLS}$, $W_{E}$ or $W_{SAC}$.

REFERENCES


Local Steam Transit Time Estimation in a Boiling Water Reactor

LILIANA KOSTIĆ

Abstract—A new correlation method, the so-called “rehocence” [1] or “smoothed coherence transform” (SCOT) [4], for transit time estimation is applied in a boiling water reactor (BWR). The transit time of a propagating perturbation of the coolant density between two points along the propagation path could be measured by means of stochastic signals. This is usually performed by directly observing the peak in the cross-correlation function of the stochastic signals measured at two points in the propagation path, or indirectly by determining the slope of the phase. The used rehocence method presents the transit time directly in the same way as in the ordinary cross correlation technique, but with a better resolution, even when the measured signals are contaminated by a narrow-band limited internal noise coming from the global noise of the neutron flux fluctuation.

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I. INTRODUCTION

BOILING water reactors are more noisy than other power reactors. The major noise source in these reactors, defined as “boiling noise,” originates from discrete steam bubbles delivered to the steam space. The formation and movement of the steam bubbles in these reactors give rise to a spatial correlation field of the neutron flux fluctuations, which may be detected by in-core detectors. The power spectral density function of the neutron flux fluctuations (measured by means of an in-core detector) exhibits two parts—a global part in the lower frequency range, and a local part in the higher frequency range. The global part results from reactivity effects on the whole core, and is limited by a break frequency of 1 Hz. The local part of the spectrum results