Fig. 2. (c) As in (b) for the phase error in the passband of interest.

In many applications of CCD transversal filtering, the CTI $e$ and the filter length $N$ are such that $Ne << 1$. In this case, the general expressions (5) for the transfer function deviation caused by the device CTI have been obtained. From these relations, the amplitude and phase errors of the filter frequency response can be derived in a straightforward way. The accordance between the computed errors and those obtained from computer simulations and/or those already known in the literature was verified for many filters.

Furthermore, the simple modification algorithm (9) of the filter coefficients derived from the preceding analysis has proved to be quite effective in reducing the amplitude and phase deviations by more than an order of magnitude. The residual errors are to be attributed to the terms neglected in the approximation (4) and to the fact that the charge-transfer loss actually transforms an FIR filter into an IIR one, as can be seen from the model of Fig. 1 and from relation (3).

IV. CONCLUSIONS

In many applications of CCD transversal filtering, the CTI $e$ and the filter length $N$ are such that $Ne << 1$. In this case, the general expressions (5) for the transfer function deviation caused by the device CTI have been obtained. From these relations, the amplitude and phase errors of the filter frequency response can be derived in a straightforward way. The accordance between the computed errors and those obtained from computer simulations and/or those already known in the literature was verified for many filters.

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Confidence Bounds for Signal-to-Noise Ratios from Magnitude-Squared Coherence Estimates

JOHN W. FAY

Abstract—Coherence is used frequently to determine the degree to which one observed voltage is related to another observed voltage. Typically, in practice, these observables are degraded by system noise that is often independent, white, and Gaussian. Often, in measuring coherence, the interest is to determine the fraction of the observed power that is due to coherent signals and the fraction that is due to the uncorrelated noise floor. The term "signal" as used here describes a component of voltage of interest to an observer. With accurate coherence estimates, uncorrelated noise power can be separated from coherent signal power. Therefore, the concern in this article is with the accuracy of signal-to-noise ratio (SNR) calculations made from magnitude-squared coherence (MSC) estimates. Use is made of work by Carter and Scannel [1] in which they determine confidence bounds of MSC estimates for stationary Gaussian processes. Their results are used in this article to derive corresponding confidence bounds for SNR calculations without recourse to the complicated details of the underlying SNR statistics.

DISCUSSION

The magnitude-squared coherence (MSC), or the coherence function as it is sometimes called, between two stationary pro-

Manuscript received October 29, 1979; revised March 6, 1980. The author is with New London Laboratory, Naval Underwater Systems Center, New London, CT 06320.

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the fraction in (1) is the magnitude squared cross spectrum \( \gamma_{xy} \), which, for the most part, shows spectral components that are correlated between voltages \( x(t) \) and \( y(t) \). Typically, the cross spectrum has deterministic errors due to such things as gain differences. Quantities in the denominator of the fraction are autospectra of observed processes \( x(t) \) and \( y(t) \). The coherence in (1) measures the fraction of power in \( y(t) \) that is related in a linear way to \( x(t) \). Note that the MSC is independent of gain or level difference between \( x(t) \) and \( y(t) \).

As pointed out by Carter and Scannel [1], the MSC is usually estimated in practice by means of a dual-channel fast Fourier transform (FFT) algorithm, as follows:

\[
\hat{\gamma}_{xy}^2 = \frac{\sum_{n=1}^{N} x_n y_n^*}{\sum_{n=1}^{N} |x_n|^2 \sum_{n=1}^{N} |y_n|^2} \tag{2}
\]

The asterisk denotes complex conjugation and \( N \) is the number of data points averaged. The factors \( x_n \) and \( y_n \) are FFT outputs of the \( n \)th data segments of periodically sampled \( x(t) \) and \( y(t) \), respectively. Typically, we have \( N \) independent data segments, each of length \( MT \), where \( \Delta T \) is the sampling interval. In general, this partitioning is done to gain statistical stability at the expense of increased bias and decreased frequency resolution. Bias and frequency resolution are not a concern here.

If we have uncorrelated noise and coherent signals between \( x(t) \) and \( y(t) \), it is possible to correct the spectrum of \( y(t) \) to determine the amount of power due only to uncorrelated noise and the amount due only to coherent signal. If the uncorrelated noise power in \( x(t) \) is assumed nonexistent, the power in \( y(t) \) as a result of a coherent signal between \( x(t) \) and \( y(t) \) at frequency \( f \) is

\[
\hat{\gamma}_{yy} \hat{\gamma}_{xy} \tag{3}
\]

Similarly, the power in \( y(t) \) due to uncorrelated noise is

\[
\hat{\gamma}_{yy} (1 - \hat{\gamma}_{xy}^2) \tag{4}
\]

The ratio of (3) and (4) yields the coherent signal-to-uncorrelated-noise ratio (SNR) [2] for \( y(t) \) at a single frequency in terms of measured coherence,

\[
\text{SNR} = \frac{\hat{\gamma}_{yy} \hat{\gamma}_{xy}}{\hat{\gamma}_{yy} (1 - \hat{\gamma}_{xy}^2)} = \frac{\hat{\gamma}_{xy}^2}{1 - \hat{\gamma}_{xy}^2} \tag{5}
\]

Note from (5) that the SNR is a function of frequency and is a random variable because of the randomness in the estimate \( \hat{\gamma}_{xy} \). Thus, we are limited in the use of (5) until we can put some confidence bounds on its estimate.

Using the results in Carter and Scannel [1], we can find corresponding confidence intervals for SNR. A computer program in [1] permits calculating arbitrary confidence limits for MSC estimates of stationary Gaussian processes. Table I is a sample output from this program.

Magnitude squared coherence values from Table I have been used to compute confidence bounds for SNR in (5) (see the Appendix). These bounds are given in Figs. 1 and 2 for 80 and 95 percent confidence, respectively. The outermost curve in each figure corresponds to \( N = 8 \), whereas the innermost curves are for \( N = 128 \). In order to use these curves, it is necessary to obtain an estimate of MSC from (2) for some value of \( N \), e.g., \( N = 8 \) and 95 percent confidence of MSC. This specifies a point on the abscissa in Fig. 2 at a value of 0.5, for example. The line normal to the abscissa at 0.5 intersects the \( N = 8 \) curves at ~6.73 and 6.44 dB. These SNR bounds correspond to the lower and upper MSC bounds, respectively. We can, therefore, make the statement that the true SNR is within this interval 95 percent of the time. Expressing this another way, true SNR is no less than the estimated SNR minus 6.73 dB, or not greater than the estimated SNR plus 6.44 dB, 95 percent of the time.

Overall, there is a tightening of the confidence interval as \( N \) increases, and a leveling off of the confidence curves with increases in estimated MSC. Confidence is poor for small MSC estimates. An obvious rule of thumb is to choose as large a value of \( N \) as is practically possible.

**Summary**

We have presented confidence bounds for SNR calculations from MSC estimates for Gaussian processes. Confidence
where $\hat{\gamma}_{xy}^2$ is estimated from (A1).

Our interest, here, is in determining the confidence bounds for SNR, given confidence bounds for $\hat{\gamma}_{xy}^2$. We can do this by noting from (A2) that SNR is a monotone increasing function of $\gamma_{xy}^2$. Let $u = \hat{\gamma}_{xy}^2$ and $V = 1 - \gamma_{xy}^2$. Then $\Delta u = \hat{\gamma}_{xy}^2(C_{\text{upper}}) - \gamma_{xy}^2$, and $\Delta V = \hat{\gamma}_{xy}^2 - \hat{\gamma}_{xy}^2(C_{\text{lower}})$, where $\hat{\gamma}_{xy}^2(C_{\text{upper}})$ is an upper or lower confidence bound for the coherence estimate in Table I.

From (A2),

$$\Delta \text{SNR} = \frac{u + \Delta u - u}{V + \Delta V} \cdot V \Delta u - u \Delta V$$

and

$$\Delta \text{SNR} = \frac{V \Delta u - u \Delta V}{u(V + \Delta V)}.$$  

Substituting for $u$, $V$, $\Delta u$, and $\Delta V$, we obtain

$$\frac{\Delta \text{SNR}}{\text{SNR}} = \frac{\hat{\gamma}_{xy}^2(C_{\text{upper}}) - \gamma_{xy}^2}{\hat{\gamma}_{xy}^2(1 - \hat{\gamma}_{xy}^2(C_{\text{lower}}))}.$$  

Taking $10 \log_{10} [1 + (\Delta \text{SNR}/\text{SNR})]$ provides us with the SNR confidence bounds, relative to 0 dB, presented in the main text, i.e., the interval within which the true SNR will exist a given percent of the time, which is defined by $\gamma_{xy}^2(C_{\text{upper}})$, $\gamma_{xy}^2$, and $\gamma_{xy}^2(C_{\text{lower}})$ from [1].

We have presented SNR confidence intervals for 80 and 95 percent confidence bounds in the main text. Note that estimated MSC values of zero and unity have been deliberately avoided because, for MSC confidence limits in [1], $\log_{10} [1 + (\Delta \text{SNR}/\text{SNR})]$ is undefined as MSC goes to zero and $\Delta \text{SNR}/\text{SNR}$ becomes indeterminate at unity MSC. However, the MSC values we have considered should cover most practical cases. If not, the reader must invoke a metric different from the one used here.

REFERENCES


A Continuous Recursive DFT Analyzer—The Discrete Coherent Memory Filter

B. G. GOLDBERG

Abstract—The discrete version of the coherent memory filter (DCMF) is introduced and it is shown that the device performs a (N point) DFT of the N input data samples. It is also shown that with a simple modification the device can be operated recursively, so that as the data samples flow in, the DFT of the last N data samples are performed continuously.

Manuscript received November 12, 1979; revised March 28, 1980. The author is with the Ministry of Defense, Haifa, Israel.

$$\text{SNR} = \frac{\hat{\gamma}_{xy}^2} {1 - \hat{\gamma}_{xy}^2}$$  

(2)

where $\hat{\gamma}_{xy}^2$ is estimated from (A1).