The Variation of Bistatic Rough Surface Scattering Cross Section for a Physical Optics Model

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Abstract—The general behavior of a rough surface scattering cross section is examined as a function of incident and scattering angles, surface roughness, dielectric constant, and polarization for physical optics (PO) conditions. Quite distinct and complicated variations are observed. For some conditions, deep nulls occur in the normalized bistatic cross section \( \sigma^* \) as a function of scattering angle, while other parameter sets yield no such pattern. These results are analyzed and interpreted. The differences in the angular variation in \( \sigma^* \) for different polarizations suggested that, for a given set of conditions, it would be possible to minimize the scattering from a rough surface. This topic is addressed in the second part of the paper. For the case of a given incident polarization, a technique is presented to optimize the bistatic observation of terrain scattering by using combinations of receiver polarization which vary as the scattering angles are changed.

I. INTRODUCTION

POLARIZATION DEPENDENT scattering of electromagnetic (EM) waves at arbitrary scattering angle has been discussed for many applications. The question of polarimetric techniques in signal processing has been summarized by Cloude [1]. He discussed the scattering matrix for a general target and examined the cross and co-polar nulls for observing a target in a backscatter mode. McCormick and Hendry [2] extended this approach to backscatter from an ensemble of randomly oriented targets. They derived expressions for nulls in terms of ensemble averaged scattering matrix elements. In contrast to the single target case which can have sharp zeros, they found that the partially polarized backscatter from the ensemble of scatterers exhibits minima whose depth and breadth are related to the degree of polarization and randomness of the ensemble of scatterers. McCormick and Hendry applied this to the particular cases of rain and hydrometeors. Agrawal et al. [3] have outlined a similar approach to backscatter from an ensemble average of scatterers. They provided generalized expressions for the depth and spread of the polarization nulls in terms of ensemble averaged quantities. They proposed that this approach applies to a wide range of random scattering conditions such as vegetation, snow, or terrain but no specifics as to how the appropriate ensemble averaging would be determined were given. Specific analytical expressions for the normalized cross section, \( \sigma^* \) of rough terrain at arbitrary angles have been developed by several authors [4]–[6]. In these analyses, general integral expressions for the average scattered power from a rough surface were derived under assumptions of physical optics (PO) (radius of curvature of surface irregularities large compared to a wavelength). Subsequently, closed form solutions neglecting multiple scattering are obtained for the high frequency limiting case of geometric optics (GO). Other authors also use this limiting condition in their results [7]–[9].

In the Appendix it is shown how the equivalence of these results gives some insight into the physics of the formalisms. Although these expressions are unrestricted in angle, most interest has been in the two cases of backscatter or forward scatter. One analysis [10] by Fung and Eom dealt with the case where the incident elevation angle, \( \theta_i \), and the scattered elevation angle, \( \theta_s \), are both equal to \( \theta \). The model includes multiple scattering. In a later paper [11], they used those results to identify co-polarization null locations on the Poincaré sphere for the monostatic geometry.

In this paper we will address two related topics. In the first instance we will present results that show how there are complicated angular dependencies for \( \sigma^* \) that depend on signal polarization, surface roughness, and dielectric constant. We introduce some simplified analytical relations that provide some understanding of these effects. It was observed that the behavior of the scattering pattern as a function of \( \theta \) and \( \phi \) (Fig. 1) differed for different signal polarizations; this suggested that there could be a received signal polarization that would minimize scattering for each particular orientation of the bistatic system. In the second part of the paper, we will discuss how, in a bistatic system with polarization diversity in the receiver and fixed incident polarization, optimal performance can be obtained by varying the co-polar and cross-polar relations as a function of the geometry. Alternatively, it could be useful in remote sensing applications to maximize the cross section of the surface using this approach.

Before presenting the details of the analyses, some discussion of the context of the paper may clarify the different forms used to obtain our results. The first question is that of motivation. For a variety of bistatic system configurations, the polarization of the incident wave may be determined by circumstances beyond the control of the receive system user. In a practical bistatic radar system, the angles of incidence and scattering are determined by the constraints of siting the system and the function of performing target search. Thus, we are addressing an optimization formalism that assumes fixed incident polarization and is concerned solely with the receive system polarization variability.

Next, we will discuss some distinctions between the physical optics and geometric optics aspects of the earlier
analyses [4]–[6] and how our own development reflects these considerations.

Barrick and Peake [5] obtained the following expression for the scattered field \( \vec{E}_s \) under the Kirchhoff tangent plane approximation:

\[
\vec{E}_s = \left[ -\frac{ik}{4\pi R_0} e^{ikR_0} E_i \right] \int_{-L/2}^{L/2} \int_{-L/2}^{L/2} \hat{F}(\vec{x}_s, \vec{y}_s) \cdot \exp \left( ik(\vec{k}_i - \vec{k}_s) \cdot \vec{r} \right) \, dx \, dy
\]

(1)

where

\[
k = 2\pi/\lambda
\]

\[
R_0 \text{ distance from origin to observation point}
\]

\[
\vec{r} = x\hat{x} + y\hat{y} + z\hat{z} = \text{distance from origin to point on the rough surface}
\]

\[
L^2 \text{ area of the rough surface contributing to the scattering}
\]

\[
\vec{k}_i, \vec{k}_s \text{ unit vectors in the directions of incident wave and scattered wave}
\]

\[
\xi_x, \xi_y \text{ local surface slopes in x and y directions at surface point } z = \xi(x, y)
\]

\[
E_i \text{ incident field intensity}
\]

The vector \( \vec{F}(\vec{x}_s, \vec{y}_s) \) is a complicated function of the local normal to the surface and the local Fresnel reflection coefficients. One way of calculating the average scattered power is to square (1) and then perform the ensemble average over surface heights and surface slopes \( \langle \vec{E}_s^* \cdot \vec{E}_s \rangle \). The heights and slopes are correlated through a two-point probability distribution function (pdf). The procedure also has to take into account the surface area integrations. Unless additional simplifying assumptions are introduced at this point, all these integrations would, in general, have to be solved numerically. The result, however, would be a "pure" physical optics solution.

We next consider the implications of the use of approximate solutions to avoid the numerical approach. The basic requirement in these solutions is to be able to separate the integrations over the surface heights and slopes at the two points. The rigorous justification for this is that it can be shown for a Gaussian surface that under certain conditions, the heights and slopes can be decorrelated. They then represent independent variates. One sufficient condition for this is the high frequency limit \( (k \to \infty) \). Another condition occurs when the correlation length, \( T \) is much greater than the wavelength \( (T \gg \lambda) \) and the surface slopes are small [12]. This does not require going to the high frequency limit and hence is an equivalent physical optics case.

Barrick and Peake [5] use the high frequency condition (GO) and carry out stationary phase evaluation of the integrals. Their solution takes \( \vec{F}(\vec{x}_s, \vec{y}_s) \) to be \( F(\vec{x}_{sp}, \vec{y}_{sp}) \) where \( (x_{sp}, y_{sp}) \) define the specular point for the scattering. They justify this by the argument that, if the remaining integrals are evaluated by stationary phase, then most of the scattering should be attributable to tilted facets having the slopes evaluated at the specular points.

In another case, a perfect conductor, Beckmann and Spizzichino [13] suggest that (1) can be approximated by assuming that \( F(\xi_x, \xi_y) \) has some constant over the surface area of integration. Mitmacher [14] generalized this to a dielectric rough surface. He calculated scattering matrix elements from tilted dielectric planes. If these procedures are used to evaluate the average scattered power, it is again necessary to have the heights and slopes independent. However, as indicated, it is not necessary to use stationary phase, provided \( T \gg \lambda \) and the surface slopes are small. The integrals in \( \vec{F}(\xi_{xsp}, \xi_{ysp}) \) can be approximated by \( \vec{F}(\xi_{xsp}, \xi_{ysp}) \) and the remaining integrals involving the phase factor

\[
\exp \left( -ik(\vec{k}_i - \vec{k}_s) \cdot \vec{r} \right) \cdot \exp \left( ik(\vec{k}_i - \vec{k}_s) \cdot \vec{r}' \right)
\]

can be independently ensemble averaged. This leaves integration over the surface area to be evaluated but, at this point, the result is still consistent with physical optics. This will be discussed further in the next section. A final comment can be made about the reasonableness of this approximate solution.

The approach assumes that the angles of incidence \( \theta_i \) and scattering \( \theta_s \) and \( \phi_s \), as well as the Fresnel reflection coefficients, do not vary significantly over the area of integration. As long as the area of integration is limited to several correlation lengths \( (L = 5T) \) and the source and observation points are sufficiently far from the surface region of integration, these are realistic requirements. Such conditions are typical for radar system configurations.

The next discussion concerns the transition from the general formalisms for average scattered power to specific expressions for scattering cross section of the rough surface.

II. Scattering Cross Section

The most commonly used form for the scattering cross section is that of Barrick [5]. In this form it includes the shadowing function \( S \) given by Sancer [9]. Both results depend on the GO high frequency limit:

\[
\sigma^o = |\rho_p|^2 JS
\]

(2)

where \( p \) refers to the polarization of the incident wave, \( q \) refers to the polarization of the reflected wave and

\[
\begin{align*}
\beta_{VV} & = \frac{a_2 a_4 R_4(i) + \sin \theta_i \sin \theta_s \sin^2 \phi_s R_{\perp}(i)}{a_1 a_4} \\
\beta_{HV} & = \sin \phi_s - \sin \theta_i a_3 R_1(i) + \sin \theta_s a_2 R_1(i) \\
\beta_{VH} & = \sin \phi_s - \sin \theta_i a_3 R_1(i) - \sin \theta_s a_2 R_1(i) \\
\beta_{HH} & = -\sin \theta_i \sin \theta_s \sin^2 \phi_s R_{\perp}(i) - a_2 a_3 R_{\perp}(i). \\
\end{align*}
\]

Here, the angles \( \theta_i, \theta_s, \phi_s \) are defined in Fig. 1, and \( R_4(i) \) and \( R_{\perp}(i) \) are the Fresnel reflection coefficients for vertical (V) and horizontal (H) polarizations, respectively. The angle \( i \) is defined as

\[
\cos i = (1/\sqrt{2})(1 - \sin \theta_i \sin \theta_s \cos \phi_s + \cos \theta_i \cos \theta_s)^{1/2}
\]
with

\[ a_1 = 1 + \sin \theta_i \sin \theta_s \cos \phi_s - \cos \theta_i \cos \theta_s \]
\[ a_2 = \cos \theta_i \sin \theta_s + \sin \theta_i \cos \theta_s \cos \phi_s \]
\[ a_3 = \sin \theta_i \cos \theta_s + \cos \theta_i \sin \theta_s \cos \phi_s \]
\[ a_4 = \cos \theta_i + \cos \theta_s . \]

The quantity \( J \) in (2) is proportional to \( p(\xi_x, \xi_y) \), the probability density function for the slopes. The quantities \( \xi_{xp} \) and \( \xi_{yp} \) are the surface slopes necessary to give rise to specular reflection of the incident field into the scattering direction:

\[ \xi_{xp} = \frac{\sin \theta_i - \sin \theta_s \cos \phi_s}{\cos \theta_i + \cos \theta_s} \]

and

\[ \xi_{yp} = \frac{\sin \theta_s \sin \phi_s}{\cos \theta_i + \cos \theta_s} . \]

As was pointed out in the preceding section, we can write an alternative form for the cross section which is not dependent on the GO limit, as long as the surface satisfies the restrictions that \( T \gg \lambda \) and \( \xi_x < 1 \) and \( \xi_y < 1 \). In this physical optics (PO) version, the cross section \( \sigma^o \) has the form given by (2). The same scattering matrix elements \( \beta_{pq} \), which were originally derived by going to the high frequency (GO) limit, are also valid for these particular physical optics conditions, while the slope related term \( J \) is given by the expression

\[ J = (8\pi^2 / \lambda^2) \int_{-\infty}^{\infty} J_0(u_{xy}\tau) [\chi_2 - \chi_1^* \chi_1] \tau \ d\tau . \] (3)

Here

\[ v_{xy} = \sqrt{v_x^2 + v_y^2} \]
\[ v_x = (2\pi / \lambda) [\sin \theta_i - \sin \theta_s \cos \phi_s] \]
\[ v_y = (2\pi / \lambda) [\sin \theta_i \sin \phi_s] \]
\[ \tau \] distance between two surface points
\[ \chi_2 \] bivariate two-point characteristic function
\[ \chi_1 \] univariate surface height characteristic function.

For our surface conditions (Gaussian),

\[ \chi_2 = \exp \{- \Sigma^2 \{1 - c(r)\}\} , \]
\[ \chi_1 = \exp \{- (1/2)\Sigma^2\}, \]

and

\[ \Sigma = \alpha k \ [\cos \theta_i + \cos \theta_s] . \]

Two points can be made here. First, neither the GO form of \( J \) nor the PO form depends on either the incident or scattered signal polarization. Second, in the limit \( \lambda \to 0 \), a steepest descent evaluation of the integral in (3) yields the well-known asymptotic expression (often referred to as the geometric optics solution):

\[ J = \left( 4\pi^2 T^2 / \lambda^2 \Sigma^2 \right) \exp \left\{ - \left( \frac{v_{xy}^2 T^2}{4 \Sigma^2} \right) \right\} . \] (4)

For computational purposes, this closed-form solution is convenient. These aspects are significant in terms of choosing the form in which the results of this paper are obtained. This is discussed in the next section.

III. RESULTS

In this section we will show some selected results for the variation of \( \sigma^o \) as a function of geometry, roughness, and dielectric constant for different polarizations. Before examining specific cases, some general comments may be useful. First, our purpose is to demonstrate that the physical optics \( \sigma^o \) variation with scattering angle has nulls at various angle conditions and that the behavior is sensitive to polarization and dielectric constant. If we consider (3), we see that the conditions for which \( J \to 0 \) in this physical optics form are not angle relations (except for \( \theta_i = \theta_s = 90^\circ \)); the conditions are surface dependent ones that would only apply to a few limited cases: \( \theta_i + \theta_s = 0 \). Consideration of the asymptotic geometric optics result for \( J \in (4) \) shows similar behavior. Therefore, in determining the angle dependent location of a null, the use of the geometric optics form for \( J \) does not change the null location from that for physical optics; it only affects the overall magnitude of the \( \sigma^o \) results. Hence, use of (4) to obtain the results in the \( \sigma^o \) figures does not affect their generality. This observed behavior also offers an explanation of why a single scattering physical optics \( \sigma^o \) does not have roughness dependence in its null locations. Finally, it should be noted that the results presented are for \( \theta_i = 75^\circ \) although additional cases were generated. The results do depend on the value of the incident angle; distinctly different behavior can be seen, and \( \theta_i = 75^\circ \) was used only as a typical instance.

In Fig. 2, the behavior of \( \sigma^o \) for vertical–vertical polarization as a function of scattering elevation angle \( \theta_s \) is shown for four azimuthal scattering angles \( \phi_s \). We used a representative dielectric constant \( \epsilon = 80 + j9.0 \). Four levels of surface roughness are depicted in terms of mean surface slope \( \alpha / T \). It can be seen that as the azimuthal angle increases from forward scatter, the minima occur at smaller values of \( \theta_s \) except for \( \phi_s = 135^\circ \) which shows no clear null. In fact, extensive studies showed that for \( \phi_s > 90^\circ \) the nulls always had disappeared.
In Fig. 3 the same variation is shown for horizontal–horizontal polarization. Again, there are nulls in the elevation dependence, and these show a similar trend to those of the vertical–vertical case. However, for a given azimuthal angle, the nulls occur at larger \( \theta_i \) values here as compared with Fig. 2. Again no nulls are apparent for large \( \phi_s \).

In Fig. 4 we introduce the cross polarized terms and show a consolidated history. For this case we selected \( \sigma/T = 0.2 \) and \( \epsilon = 20 + j0.1 \). This latter aspect will be examined further. In this illustration we also introduce the forward scatter case which shows (as expected from (2)) that the cross polarized components of \( \sigma^* \) are zero. Only the VV component has a null (at \( \theta_s = 80^\circ \)). At \( \phi_s = 45^\circ \), all four components (VV, HH, VH, and HV) of \( \sigma^* \) are about the same order of magnitude, with a sharp falloff as \( \theta_s \) increases beyond 60°. Only the VV component exhibits a null, which occurs at \( \theta_s = 45^\circ \). At \( \phi_s = 75^\circ \), both the HH and VV polarization components of \( \sigma^* \) exhibit nulls and, as before, the horizontal–horizontal null occurs at a larger \( \theta_s \) value. At \( \phi_s = 135^\circ \), no polarization components exhibit nulls, and all four components have approximately the same order of magnitude and show a monotonic decrease in \( \sigma^* \) value as \( \theta_s \) increases. If we compare these results with those in Figs. 2 and 3, we see that by changing the dielectric constant the nulls in the \( \sigma^* \) curves have shifted location.

In Fig. 5 we examine forward and backscatter results for vertical–vertical polarization with \( \sigma/T = 0.1 \). These curves represent a range of permittivity conditions for surfaces with...
small conductivity levels. The values are

\[ \varepsilon = 40 + j0.1 \]
\[ \varepsilon = 20 + j0.1 \]
\[ \varepsilon = 5 + j0.1 \]
\[ \varepsilon = 2 + j0.1. \]

For these parameter regimes, the VV component of \( \sigma^\circ \) exhibits a null for all four values of \( \varepsilon \) for \( \phi_s = 0^\circ \). As the real part of \( \varepsilon \) is decreased, the null occurs at smaller and smaller values of \( \theta_s \). Also, the depth of the null decreases as the real part of \( \varepsilon \) decreases.

For the backscatter case, all four curves show the rapid falloff with increasing \( \sigma_s \). In both instances the effect of decreasing the permittivity is to lower the \( \sigma^\circ \) value at low \( \theta_s \) values. For forward scatter, though, once the null occurs for a particular dielectric, there are crossovers in \( \sigma^\circ \) at larger \( \theta_s \) values.

In terms of presenting scattering angle dependencies of \( \sigma^\circ \) it should be noted that Fung and Eom [12] showed the variation of \( \sigma^\circ \) as a function of \( \phi_s \) for the particular case where \( \theta_i = \theta_s \) (4.5\(^\circ\), 22.5\(^\circ\), 49.5\(^\circ\)). The case they showed is surface slope of 0.4 and dielectric constant of 81. The concern of their analysis, though, was the effect of multiple scattering, and hence the presence of nulls was not given particular attention. As would be expected, the null dependence for fixed elevation angle (\( \theta_i = \theta_s \)) as a function of azimuthal angle is not equivalent to that shown here (\( \theta_i = 75^\circ \)); however the illustrated behavior is as complex in its own right. Some of their conclusions [12] are relevant to the results of this paper. They point out that for the single scattering model there is no cross polarized component for forward or backscatter cases. In addition, they summarize the relation of multiple and single scattering results in their model. For cross polarization, multiple scattering affects only the results near back or forward scattering while for co-polarized signals, multiple scattering results differ from single scattering ones near some particular azimuthal scattering angle that depends on the configuration. Thus, for general bistatic conditions, their model's multiple scattering results would not be different from single scattering except in the neighborhood of those points.

**IV. Analysis**

In this section we will present some analyses that help to demonstrate how the various results for \( \sigma^\circ \) arise. In the initial formulations we will restrict ourselves to cases where \( \phi_s = 0^\circ \) and 180\(^\circ\). Some general forms will be derived, but for the most part the results obtained will be for either of two limiting cases. These are surfaces with extremely small conductivity (real \( \varepsilon \)) or large conductivity (perfect conductor). To carry out the analyses we ignore any shadowing aspects, and the slope statistics elements from the J-term in \( \sigma^\circ \) do not play a role. The rationale for this was introduced in Section III where it is pointed out that nulls associated with \( J \) under either geometric or physical optics assumptions, are surface condition results and do not have a bearing on angular location. Thus the angular null locations of \( \sigma^\circ \) arise from the condition \( |\beta_{pq}|^2 = 0 \).

The first analyses consider the problem of finding forward or backscatter conditions for which \( \sigma^\circ \rightarrow 0 \). If we restrict ourselves to \( \phi_s = 0^\circ \), and \( \phi_s = 180^\circ \), the two cross polarization terms \( \beta_{vH} \) and \( \beta_{Hv} \) vanish, and the \( \beta_{vv} \) and \( \beta_{hh} \) terms reduce, respectively, to terms involving \( R_\parallel (i) \) or \( R_\perp (i) \) only. We have

\[ |\beta_{vv}|^2 = 0 \quad \text{and} \quad |\beta_{hh}|^2 = 0 \]

or equivalently, then

\[ a_4^2 |R_\parallel (i)|^2 = 0 \quad \text{and} \quad a_4^2 |R_\perp (i)|^2 = 0 \]

For \( \phi_s = 0^\circ \), 180\(^\circ\) this reduces to

\[ \sin^4 (\theta_i \pm \theta_s) |R_\parallel (i)|^2 = 0 \quad \text{and} \quad \sin^4 (\theta_i \pm \theta_s) |R_\perp (i)|^2 = 0 \]

where, respectively,

\[ \cos^2 i = (1/2)[1 + \cos (\theta_i \pm \theta_s)]. \]

Thus, the minimal conditions arise from either the sine terms or the Fresnel reflection coefficient terms. We now look at individual cases.

**A. Vertical–Vertical Polarization**

We first consider the Fresnel term. This can be manipulated in terms of \( i \) for both \( \phi_s \) cases.

\[ |R_\parallel (i)|^2 = 0 \rightarrow (\varepsilon \cos i - \sqrt{\varepsilon - \sin^2 i}) \cdot (\varepsilon \cos i - \sqrt{\varepsilon - \sin^2 i})^* = 0. \]
After considerable algebraic manipulation, this reduces to
\[
\cos^2 i = \frac{(\epsilon_R - 1) \pm \sqrt{(\epsilon_R - 1)^2 + [(\epsilon_R - 1)^2 + \epsilon_i^2]}}{[(\epsilon_R^2 + \epsilon_i^2)^2 - 1]}
\]
where
\[
\epsilon_i = \frac{1 - \epsilon_r^2}{1 + \epsilon_r^2} \quad \epsilon_r = \frac{1 - \epsilon_r^2}{1 + \epsilon_r^2}
\]
\[
(\epsilon_r^2 + \epsilon_i^2)^2 - 1
\]
(9)

Then, for \( \phi_x = 0 \) we have
(a) \( \sin^4 (\theta_i + \theta_x) = 0 \Rightarrow \theta_i = \theta_x = 0 \)
(b) \( \cos (\theta_i + \theta_x) = 2 \cos^2 i - 1 \).

For the special case \( \epsilon_t = 0 \) condition (b) reduces to
\[
(\epsilon') \cos (\theta_i + \theta_x) = \left[ \frac{1 - \epsilon_R^2}{1 + \epsilon_R} \right] \Rightarrow \theta_x = \cos^{-1} \left[ \frac{1 - \epsilon_R}{1 + \epsilon_R} \right] - \theta_i.
\]

For \( \phi_x = 180^\circ \) we have
(c) \( \sin^4 (\theta_i - \theta_x) = 0 \Rightarrow \theta_x = \theta_i \)
(d) \( \cos (\theta_i - \theta_x) = 2 \cos^3 i - 1 \).

For the special case \( \epsilon_t = 0 \) condition (d) reduces to
\[
(\epsilon') \cos (\theta_i - \theta_x) = \left[ \frac{1 - \epsilon_R^2}{1 + \epsilon_R} \right] \Rightarrow \theta_x = \cos^{-1} \left[ \frac{1 - \epsilon_R}{1 + \epsilon_R} \right] + \theta_i.
\]

### B. Horizontal–Horizontal Polarization

The sine term relations are unchanged. We consider the \( R_{\perp} \) term in general
\[
|R_{\perp}(i)|^2 = 0 \Rightarrow \cos (i - \sqrt{\epsilon - \sin^2 i})
\]
\[
= \cos (i - \sqrt{\epsilon - \sin^2 i})^* = 0.
\]

This reduces to
\[
\cos^2 i = \left[ \frac{1 - (\epsilon_R - 1)^2 + \epsilon_i^2}{2(\epsilon_R - 1)} \right].
\]

This does not have a solution for \( \epsilon_R \geq 1 \). Thus, for horizontal polarization there would be no possibility for \( \phi_x = 0^\circ \) or \( \phi_x = 180^\circ \).

### C. Comparison with Results

If we make the simplifying assumption that \( \epsilon_t = 0.1 \) is approximately equivalent to \( \epsilon_t = 0 \), we have an easy form for comparing these analyses with the results of Fig. 5. For \( \phi_x = 0^\circ \), condition (b') gives the results for \( \theta_x \) shown in Table I for the dielectric values of Fig. 5. There is excellent agreement for the location of the nulls in the forward scatter direction. For \( \phi_x = 180^\circ \), condition (d') prohibits any null since \( \theta_x \) has to be less than \( 90^\circ \). Condition (c), however, says that a null is possible at \( \theta_x = \theta_i = 75^\circ \). In Fig. 5 the backscatter results show the extremely rapid falloff for \( |\beta_{vvv}| \) as \( \theta \) increases, so these results are consistent with the analyses as well. The ability of these analyses (which exclude \( J \)) to predict the \( \sigma^\circ \) nulls (including the asymptotic \( J \)) of the figure confirms that the physical optics or the geometric optics \( J \) does not affect the null location.

### D. Perfect Conductor

One final limiting case will be discussed. We will consider a perfectly conducting surface and examine the null conditions for vertical–vertical polarization and horizontal–horizontal polarization for arbitrary azimuthal scattering angle. For this case, \( R_{\perp} = +1 \) and \( R_{\parallel} = -1 \), so the relation for the nulls are identical for both cases. Hence,
\[
A_0^2 |\beta_{hh}|^2 = A_0^2 |\beta_{vv}|^2 = 0.
\]

This becomes
\[
(c \cos \theta_i \sin \theta_i \cos \theta_x \sin \theta_x)(1 + \cos^2 \phi_x)
\]
\[- \sin \theta_i \sin \theta_x \sin^2 \phi_x
\]
\[+ (\cos^2 \theta_i \sin^2 \theta_x + \sin^2 \theta_i \cos^2 \theta_x) \cos \phi_x = 0.
\]

After algebraic manipulation, (12) becomes
\[
A_0 \sin^4 \theta_x + B_0 \sin^3 \theta_x + C_0 \sin^2 \theta_x + D_0 \sin \theta_x + F_0 = 0
\]
(13)

where
\[
A_0 = C^2 + \cos^2 \theta_i \sin^2 \theta_i (1 + \cos^2 \phi_x)^2
\]
\[B_0 = 2BC
\]
\[C_0 = 2AC + B^2 + C^2 - A_0
\]
\[D_0 = 2AB
\]
\[F_0 = A^2
\]
\[A = - \sin^2 \theta_i \cos \phi_x
\]
\[B = \sin \theta_i \sin^2 \phi_x
\]
\[C = - \cos \theta_i \cos \phi_x
\]

Equation (13) can be evaluated either algebraically or numerically for \( \sin \theta_i \) given assigned values of \( \theta_i \) and \( \phi_x \). For

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**TABLE I**

<table>
<thead>
<tr>
<th>( \epsilon_r )</th>
<th>( (1 - \epsilon_r)/(1 + \epsilon_r) )</th>
<th>( \cos^{-1}[(1 - \epsilon_r) / (1 + \epsilon_r)] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>39.41</td>
<td>162°</td>
</tr>
<tr>
<td>20</td>
<td>-19.21</td>
<td>154.8°</td>
</tr>
<tr>
<td>5</td>
<td>-2/3</td>
<td>131.8°</td>
</tr>
<tr>
<td>2</td>
<td>-1/3</td>
<td>109.5°</td>
</tr>
</tbody>
</table>

Note that the numerical result at \( \phi_x = 180^\circ \) agrees with the algebraic solution.
instance, the direct algebraic solution can be determined for

$$\phi_1 = 0^\circ \Rightarrow \theta_1 = \theta_2 = 0^\circ, 90^\circ.$$  
$$\phi_2 = 90^\circ \Rightarrow \theta_1 = 0^\circ; \; \theta_2 = 0^\circ; \; \theta_1 = \theta_2 = 0^\circ.$$  
$$\phi_3 = 180^\circ \Rightarrow \theta_1 = \theta_2.$$  

Except for $\phi_3 = 180^\circ$, the solutions are not very useful cases. Numerical solutions for arbitrary $\phi_3$ are more interesting (Table II).

Although these numerical solutions show variation only of the $\theta_1$ value generating a null, we can still see similar behavior if we look at successive $\phi_3$ values and consider the $\theta_2$ solution as one angle of the $\theta_1$ profile at that value of $\phi_3$. Using this approach, we can see that for the perfect conductor we have the null location decreasing with increasing $\phi_3$ and no nulls beyond $\phi_3 = 90^\circ$ except for the backscatter case. The major difference in this limiting case is that for a perfect conductor the vertical-vertical nulls and those for horizontal-horizontal polarization are identical, which is not the case for arbitrary surface conditions. For a given set of conditions, there was always a polarization angle generating a null, and we introduce an illustration of the algorithm to locate the null value of $\theta_1$ for a given set of conditions. In the figure, the relative total scattered signal intensity is plotted as a function of the receiver polarization angle $\theta_1$. The results were obtained from (14) for a surface with dielectric constant $\epsilon = 80 + j9$, $\theta_1 = 15^\circ$, $\theta_2 = 75^\circ$, and $\phi_3 = 75^\circ$. When (16) was solved for these conditions, the null angle $\Psi_{\text{min}} = 56^\circ$.

<table>
<thead>
<tr>
<th>$\phi_3$</th>
<th>$\theta_1$</th>
<th>$\theta_2$</th>
<th>$\theta_3$</th>
<th>$\theta_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0°</td>
<td>75°</td>
<td>19°</td>
<td></td>
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</tr>
<tr>
<td>15°</td>
<td>80°</td>
<td>13°</td>
<td></td>
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</tr>
<tr>
<td>20°</td>
<td>85°</td>
<td>7°</td>
<td></td>
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</tr>
<tr>
<td>30°</td>
<td>74°</td>
<td>90°</td>
<td></td>
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<tr>
<td>45°</td>
<td>57°</td>
<td>135°</td>
<td></td>
<td></td>
</tr>
<tr>
<td>60°</td>
<td>38°</td>
<td>180°</td>
<td>75°</td>
<td></td>
</tr>
</tbody>
</table>

After algebraic manipulation, this can be solved for $\Psi$ to yield

$$\tan \Psi = \frac{-B_1 \pm \sqrt{B_1^2 - 4A_1C_1}}{2A_1}$$

where

$$A_1 = -\beta_{VV}^R\beta_{VV}^H - \beta_{VV}^B \beta_{VV}^I$$  
$$B_1 = (\beta_{VV}^R)^2 + (\beta_{VV}^H)^2 - (\beta_{VV}^B)^2 + (\beta_{VV}^I)^2$$  
$$C_1 = \beta_{VV}^R \beta_{VV}^H + \beta_{VV}^B \beta_{VV}^I.$$  

The angle $\Psi$ defines the direction of (linear) polarization the receiver should have to optimize the received power scattered from the rough surface. Note that this polarization direction depends only on the dielectric constant (complex) of the surface and the bistatic scattering angles. It is independent of the surface roughness.

In Fig. 6, we introduce an illustration of the algorithm to locate the null value of $\Psi$ for a given set of conditions. In the figure, the relative total scattered signal intensity is plotted as a function of the receiver polarization angle $\Psi$. The results were obtained from (14) for a surface with dielectric constant $\epsilon = 80 + j9$, $\theta_1 = 15^\circ$, $\theta_2 = 75^\circ$, and $\phi_3 = 75^\circ$. When (16) was solved for these conditions, the null angle $\Psi_{\text{min}} = 56^\circ$. This corresponds to the null location of the curve in Fig. 6. Similar agreement was found for a wide range of angles and dielectric constant.

The significant aspect of all the results is that, whatever the particular set of conditions, there was always a polarization angle that minimized the surface scattering effects and that the minimal angle was given by the algorithm developed in this study. It should be pointed out that a basic premise in this analysis is that the transmitted signal is vertically polarized. An equivalent relation applies in the case where the transmitted signal is horizontally polarized.

As indicated earlier, Fung and Eom [11] have addressed a somewhat different nulling problem. They are concerned with a monostatic system wherein they find the particular polarization of an incident wave which would result in no co-polarized scattered field component. For that case they find that for incident angles $\theta_1 \ll 40^\circ$, there is a roughness dependence of the polarization angle on the Poincarré sphere. In relation to our arbitrary bistatic conditions, though, we have to turn to consideration of the results of their model as described in Fung and Eom [10]. There, they have pointed out that for most
conditions, single and multiple scattering results are comparable and this suggests that, even for multiple scattering, the independence of our null location with respect to roughness would generally apply.

VI. CONCLUSION

For a given incident elevation angle and $\varepsilon$, the nulls in the $\sigma^0$ versus $\theta_2$ patterns occur at successively lower $\theta_2$ values as $\phi_2$ increases and approach the limit of $\theta_2 = 0^\circ$ as $\phi_2 \rightarrow 90^\circ$. No nulls were seen beyond that value except for the pure backscatter condition $\phi_2 = 180^\circ$. The nulls occurred at larger $\theta_2$ values for horizontal polarization than for vertical at a given set of parameters.

For our conditions, surface roughness tends to affect only the relative magnitude of $\sigma^0$, not the null location. The effect of surface dielectric is to alter the location of the null, with the null occurring at lower $\theta_2$ values as $\varepsilon\mu$ decreases.

The analytic expressions allow determination of null behavior for some limiting conditions. The overall behavior of the $\sigma^0$ dependence is complex and does not lend itself to simple prediction.

Finally, for a given incident signal polarization, this paper shows how the linear polarization of a bistatic receiving antenna may be varied to optimize (either maximize or minimize) the diffuse power cross section of a rough surface. The optimum linear polarization direction is a function only of the dielectric properties of the rough surface and the bistatic scattering angles; it does not depend upon the surface roughness.

APPENDIX

The purpose of this Appendix is to consider the several different approaches to the derivation of $\sigma^0$ in the high frequency limit and to show how these results can lead to insight into the formalisms. First, as Barrick [7] indicated, in the high frequency limit his result and those of Semyonov [4], Kodis [8], and Sancer [9] can be shown to be equivalent (provided care is taken to account for typographical inconsistencies). Then, as an example of the usefulness of these alternatives, we consider the case of $\sigma^0$ for a perfect conductor and eliminate the slope statistical aspects in the $J$-term. We find that the energy is not conserved by the $\beta_{pq}$ terms, which contain some surface statistics elements as well. Now, however, we can rewrite (2) as [7]

$$
\sigma^0 = |R_{pq}(i)|^2 \left[ \left( \frac{16\pi \cos^4 i}{\cos \theta_i + \cos \theta_j} \right) \rho(\xi_2, \xi_j) \right].
$$

The square bracketed term represents the probability distribution of all possible tilts (slopes) and $R_{pq}(i)$ is the reflection coefficient for a single tilted plane when the incident field is polarized in the $p$-direction and the scattered field is received in the $q$-direction. Barrick [8] gives expressions for $R_{pq}(i)$ in terms of $R_1(i)$, $R_2(i)$ and $\theta_1$, $\theta_2$, and $\phi_2$. $R_{pq}$ depends upon the constitutive parameters of the surface and the geometry of specular reflection from a tilted plane having slopes $\xi_2$ and $\xi_j$.

We note that because of the single scattering approximation, these geometrical optics results do not preserve conservation of energy as a general case. However, for the particular condition of a plane wave scattered from a single, perfectly conducting tilted plane, energy is conserved. For that case, $R_1(i) = 1$ and $R_2(i) = -1$. The energy conservation relations can then be expressed as

$$
|R_{VV}(i)|^2 + |R_{VH}(i)|^2 = 1
$$

and

$$
|R_{HH}(i)|^2 + |R_{HV}(i)|^2 = 1.
$$

Thus, if $R_{VV}$ becomes small, $R_{VH}$ must increase. On the other hand, no equivalent relationship exists if we restrict ourselves to using the $\beta_{VV}$ and $\beta_{HH}$ formalism.

REFERENCES

SMI Adaptive Antenna Arrays for Weak Interfering Signals

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Abstract—The performance of adaptive antenna arrays in the presence of weak interfering signals (below thermal noise) is studied. It is shown that conventional adaptive antenna arrays and matrix inversion (SMI) algorithms are unable to suppress such interfering signals. To overcome this problem, the SMI algorithm is modified. In the modified algorithm, the covariance matrix is redefined such that the effect of thermal noise on the weights of adaptive arrays is reduced. Thus, the weights are dictated by relatively weak signals. It is shown that the modified algorithm provides the desired interference protection.

I. INTRODUCTION

A MAJOR PROBLEM in satellite communications is the interference caused by transmission from adjacent satellites whose signals inadvertently enter the receiving system and interfere with the communication link. The same problem arises in the earth-to-satellite part of the link where transmission from nearby ground stations enter the satellite receiver through its antenna sidelobes. The problem has recently become more serious because of the crowding of the geostationary orbit, Indeed, this interference prevents the inclusion of additional satellites which could have been allowed if methods to suppress such interference were available. The interference can be suppressed at the originating station, either space or earth, by lowering the sidelobes of the transmitting antenna. Alternatively, the interfering signals may be suppressed at the receiving site. The latter approach is examined in this paper. The undesired signal sources (interfering signals) are assumed to be located at arbitrary angular separations from the desired signal source. The spectral characteristics and modulations of the desired and undesired signals are similar. The signal-to-noise ratio (SNR) of the desired signal is expected to be 15 dB. The undesired signals are 10-30 dB below the desired signal level (unintentional interference). Thus, the undesired signals are significantly weaker than the desired signal and in fact may even be below the noise level by several dB. Although weak, these signals, because of their coherent nature and their similarity to the desired signal, do cause objectionable interference. The interference is of the nature of “ghosts” in television or echoes in speech, which are much more annoying than “snow” or static. Very faint wavy lines across a television picture are more easily detectable by the human eye than by an instrument and this makes their suppression difficult though necessary. The efforts described in this paper aim at suppressing such interfering signals.

Adaptive antenna arrays [1]-[5] have been used to provide protection to radar and communication systems from undesired signals. Undesired signals may consist of deliberately generated electronics counter measure signals, unintentional radio frequency (RF) interference, clutter scatter returns and natural noise sources. An adaptive array automatically steers nulls onto sources of undesired signals while attempting to retain the desired main beam characteristics in the desired

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