Correction to “Equivalent Edge Currents for Arbitrary Aspects of Observation”

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In the above paper, the right side of (47) should be taken with the opposite sign, and a term \(-1/N\) should be added to it.

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MINI-REVIEW

Mini-Review of H/I Inductive Method of Conductivity Mapping

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Abstract—This is a mini-review of the H/I inductive method of conductivity mapping. Particular emphasis will be placed on the use of the H/I inductive technique in determining the effective earth conductivity beneath an existing or proposed extremely low frequency (ELF) transmitting antenna system.

INTRODUCTION

Under ideal circumstances (a homogeneous isotropic conducting half-space with frequency independent conductivity), both the power dissipated in the earth by a horizontal dipole array and the field radiated by it are determined by a single number, the conductivity \(\sigma\) of the earth [1]. However, the real earth is typically very unlike this ideal. Therefore, the concept of effective conductivity \(\sigma_E\) has arisen, which is the conductivity that a homogeneous isotropic half-space would have in order to affect the antenna in the same way as the real earth. That is, the effective conductivity of a potential antenna site is a measure of the field strength that an antenna built on the site would radiate [2]. However, only in special cases is this same measure suitable for characterizing the power that the antenna would dissipate.

Burrows [13] has shown that in the case of a current sheet antenna built on a horizontally stratified half-space, a measure that characterizes both the radiation and dissipation properties of the site is the complex surface impedance. This measure is valid for an antenna composed of separate cables and for inhomogeneous half-spaces of not too great lateral variation. The most reliable methods of measuring the surface impedance are the E/H and H/I methods, provided both amplitude and phase are measured [3].

Accordingly, the last two lines in (26) should be rewritten as

\[
\frac{2j(\mu_1 \cot \beta' - \cot \beta \cos \phi)}{k \sin \beta' \sin \alpha_1} \cdot \frac{(1/N) \sin \left[\frac{(\pi - \alpha_1)/N}{\cos \left[\frac{(\pi - \alpha_1)/N}{Nk \sin \beta'}\right]} + \frac{2j \cot \beta'}{Nk \sin \beta'}\right] + \frac{1}{N} \sin \left[\frac{(\pi - \alpha_1)/N}{Nk \sin \beta'}\right] \quad (25) 
\]

In addition, \(\mu\) should be replaced by \(\mu_1\) in (27) and (28) and in the two intermediate lines.

The final result (31) for the equivalent edge currents is unaffected by the above corrections.

In the H/I inductive method, a long insulated wire, grounded at both ends, is energized at a current \(I\) at the frequency of interest. Then, the resulting magnetic fields (\(H\)) are measured at various distances and angles from the wire.

INDUCTIVE METHODS

The bulk of electromagnetic (EM) probing methods make use of such low frequencies that radiation effects can be neglected and they can be considered as inductive or quasistatic systems. The inductive systems are designed on the concept that alternating magnetic fields generate eddy currents which can be detected in terms of the secondary magnetic fields accompanying the eddy currents. The variety of induction survey techniques is amazing (see [4]–[24] for descriptions of various techniques and general overview articles). EM field sources can be user controlled, existing man-made, or naturally occurring. Analysis can be in the frequency or time domain.

With a controlled source, fields can be generated by passing the energizing current through a long grounded cable, a short grounded cable, or an ungrounded loop. Each will produce an EM field that will behave differently in the earth. The field may be detected using a grounded wire or an ungrounded loop; or the direction of the field may be determined; or the spacial gradient may be measured. The strength of the received signal may be referred to as the strength of the current at the source, in which case the measurement is one of mutual impedance between the transmitter and receiver. Or, two components of the field may be measured at the receiving station, and their ratio taken to determine the self-impedance of the field at the receiving location [4].

Currently, methods based on EM induction are being used effectively (at depths ranging from tens of meters to several kilometers) in mineral exploration, geothermal exploration, petroleum exploration, ground water exploration, and communication system studies. The magnetotelluric method, which is based on the use of natural EM fields down to a millihertz and lower, is capable of providing information on earth properties and structures at maximum depths ranging from 10 to 100 km.

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H/I THEORY

The easiest and most intuitive method for calculating the induction fields produced by a long grounded cable is finitely conducting earth image theory. This very useful and basic concept, with appropriate modifications, turns out to be a surprisingly accurate approach to characterizing the interaction of the above-ground source antenna with the earth. The “image” is located at a complex depth that is a function of both frequency and earth electrical properties [25]-[29].

The geometry for the derivation of the magnetic induction field components is shown in Fig. 1. Ball, Maxwell, and Watt [29], [25] demonstrated that the fields of a finite length horizontal electric antenna can be calculated (employing Ampere’s law) by summing the fields produced by the elements of the equivalent loop. (Element 1 is the actual antenna; elements 2, 3, and 4 are images.) The complex distance \( d \) in Fig. 1 is equal to \( 2/\gamma_e \) and \( |d| = \sqrt{2d} \); \( \gamma_e \approx \sqrt{\omega \mu_0 \sigma_e} \) is the effective propagation constant in the earth and \( \delta_e \) is the earth skin depth. (For a horizontally stratified earth, \( d = 2Q/\gamma_e \), where \( Q \) is the familiar correction factor employed to account for the presence of stratification in the earth [12]. The resulting induction field component expressions are summarized by Bannister [25], [26].

For a cable oriented in the \( x \) direction whose length is much greater than the measurement distance \( y \), cable elevation \( h \), and earth skin depth \( \delta_e \), the broadside magnetic fields reduce to [25], [26]

\[
H_y \approx \frac{ld}{2\pi(y^2 + d^2)}
\]

and

\[
H_z \approx \frac{ld}{2\pi(y^2 + d^2)} \cdot \frac{d}{y}.
\]

whereas, the inline magnetic field reduces to [25], [26]

\[
H_y = \frac{ld}{8\pi X^2}
\]

where \( X \) is the distance from the end of the cable.

The procedure for implementing the H/I inductive method (employing a long grounded wire as the source) is as follows.

1) From (1), (2), and (3), develop theoretical curves of magnetic field strength (\( H \)) versus distance for various values of effective earth conductivity.

2) Generate the induction fields by means of a current (\( I \)) flowing in a long ground wire.

3) Measure the broadside and inline magnetic fields at various distances from the wire.

4) For each measurement distance, plot the measured values of magnetic field strength on the theoretical field strength versus distance curves.

5) Determine the effective earth conductivity by comparing the theoretical and experimental magnetic field strength versus distance curves.

Presented in Figs. 2-4 are 78 Hz theoretical plots (for various earth conductivities) of the broadside and inline magnetic fields produced by the original Sanguine/Seafarer ELF test antenna. This 175 km long, 15 m high antenna was oriented in the NE direction between Lookout Shoals, NC, and Algoma, VA. It was
used periodically from 1962 until 1970 and was referred to as Site Alpha.

From these figures we see that for earth conductivities greater than \(10^{-3}\) S/m and measurement distances greater than approximately 5 km, the horizontal magnetic fields vary as the reciprocal of the distance squared, whereas the vertical magnetic field varies as the reciprocal of the distance cubed.

When the measurements are performed at ranges where the abovementioned field strength variation is typical, horizontal magnetic field measurements are preferable to vertical magnetic field measurements because 1) the horizontal magnetic field is of greater strength, and 2) the horizontal magnetic field is less susceptible to local inhomogeneities—although successful vertical magnetic field measurements have been made at ranges up to 40 km [30]–[34].

When the cable length is much less than the measurement distance and the measurement distance is greater than an effective earth wavelength (approximately 76\(\varepsilon\)) from the cable, the appropriate expressions for the horizontal magnetic field strengths derived by Bannister and Williams [35] are

\[
H_\phi \sim \frac{ILd \cos \phi}{4\pi^3} G(t),
\]

and

\[
H_\rho \sim \frac{ILd \sin \phi}{2\pi^3} H(t),
\]

where \(G(t)\) and \(H(t)\) are correction factors employed when the measurement distance \(\rho\) is comparable to the ionospheric reflection height (50 to 75 km).

APPLICATIONS

During 1963, Westinghouse personnel made H/I measurements broadside to the Site Alpha antenna. Due to limited transmitter power and receiver bandwidth, these measurements were restricted to distances of 10 km or less. RCA personnel also made H/I measurements inline with the north end of the Site Alpha antenna, out to approximately 20 km. The 78 Hz effective conductivity inferred from the Westinghouse measurements was \(2 \times 10^{-3}\) S/m, while the RCA inferred value was approximately \(10^{-3}\) S/m.

By comparing far-field measurement interpretations of the Site Alpha and Wisconsin Test Facility (WTF) transmissions, Bannister (unpublished) has determined that the Site Alpha 78 Hz effective earth conductivity is approximately \(8 \times 10^{-4}\) S/m, which is very close to the \(10^{-3}\) S/m value inferred from the RCA H/I measurements.

In the fall of 1964 and the summers of 1965 and 1966, RCA personnel performed H/I measurements of 43 different locations in northern Wisconsin and northeast Minnesota. Most of these locations were in the area or adjacent to where the geology was indicative of low conductivity to determine if the region of low conductivity extended beyond the geological boundary. The measurements indicated a high degree of correlation between low conductivity and those areas for which the surface geology projects a shield area of Pre-Cambrian origin [2].

During the summer and autumn of 1968, RCA personnel performed H/I conductivity measurements along the rights-of-way for the NS and EW lines of the (proposed) WTF using 2 km antennas as the source. The purpose of those measurements was to obtain an estimate of the effective conductivity \(\sigma\) under each of the lines [2].

The Georesearch Laboratory of Westinghouse also measured the conductivity of the WTF area [2] using the wave impedance measurement technique (employing lightning discharges as the source) and the four-terminal array inline dipole technique [5]. Their results indicated the area to be geoelectrically complex. The factors that would influence conductivity were 1) a surface layer is highly variable in thickness and/or electrical properties, 2) possible abrupt lateral discontinuities at depth, and 3) possible dipping geoelectrical layers. The Westinghouse wave impedance data (measured from 20 to 150 Hz) were obtained with a NS dipole orientation and an EW magnetic sensor orientation.

The effective conductivity of the WTF area computed from the wave impedance measurements was \(1.5 \times 10^{-4}\) S/m at 45 Hz and \(2.1 \times 10^{-4}\) S/m at 75 Hz. The 45 Hz effective conductivity obtained from the H/I measurements was \(1.2 \times 10^{-4}\) S/m for the NS right-of-way and \(1.1 \times 10^{-4}\) for the EW right-of-way, at 78 Hz, \(\sigma_{NS}\) was \(1.7 \times 10^{-4}\) S/m along both rights-of-way. One of the RCA recommendations was that once the WTF was constructed, field strength measurements should be made at approximately 50 km from each antenna in order to determine the accuracy of the 2 km H/I and wave impedance estimates of effective conductivity.

During the summers of 1972 and 1973, we measured the effective earth conductivity at 45 and 75 Hz beneath both WTF antennas [35]–[37]. The H/I method was used with each antenna alternately employed as the source. These measurements performed mainly at distances of 25 to 75 km from the transmitter, both inline with and broadside to each antenna. To be certain that the receiving sites were acceptable, we measured 14 components of the received magnetic field at each of the 50 sites and then plotted them on a normalized cosine curve. (A site would be unacceptable if the magnetic field-strength pattern were significantly altered. Factors such as proximity to power and telephone lines, pipelines, railroads, fences, magnetic deposits, or geological faults could cause them.)

The principal results obtained from these measurements are 1) at 45 Hz, \(\sigma_{NS} = 1.7 \times 10^{-4}\) S/m and \(\sigma_{EW} = 2.8 \times 10^{-4}\) S/m; 2) at 45 Hz, the ratio of \(\sigma_{EW}\) to \(\sigma_{NS}\) is 1.65, which is in very good agreement with the pattern-measurement result of 1.74; 3) at 75 Hz, \(\sigma_{NS} = 2.2 \times 10^{-4}\) S/m and \(\sigma_{EW} = 3.2 \times 10^{-4}\) S/m; 4) at 75 Hz, the ratio of \(\sigma_{EW}\) to \(\sigma_{NS}\) is 1.45, which is in excellent agreement with the pattern-measurement result of 1.44 [38].

A comparison of the NUSC 1972/1973 whole antenna measurements with the RCA 1968 H/I estimates shows that the 2 km H/I estimates were accurate to within a factor of 1.5 except for the 2 km EW estimates that were off by factors of 2.6 at 45 Hz and 1.9 at 78 Hz.

Bannister [39] has shown that for the homogeneous, isotropic earth case, the measurement distance must be greater than an effective earth wavelength \(\lambda\) for the horizontal magnetic fields to be inversely proportional to \(1/\rho^2\). By employing a combination of 45 and 75 Hz (10 to 70 km) WTF H/I measurements, Bannister [37] has also shown that \(\lambda\) is a valid distance criterion for the geoelectrically complex WTF area. This is further illustrated in Figs. 5 and 6. Comparing the 26 to 35 km measurements at both frequencies, we see that the 76 Hz measurements (76\(\varepsilon\) ~ 26 km) are in agreement with the measurements taken at
We have also measured the effective conductivity beneath each WTF half-antenna (i.e., the N, S, E, and W legs) [37]. We then used these values to predict what the effective earth conductivity beneath each WTF whole antenna should be. The values agreed within a factor of 1.22. This factor is quite remarkable for the geoelectrically complex WTF area. Rigorously, one would expect that the effective conductivity and pattern for the entire antenna could be accurately predicted only from a detailed knowledge of the conductivity over the entire region and the use of three-dimensional numerical methods.

Bannister and Williams [35] have shown that if the values of effective conductivity determined from the 1972/1973 WTF whole antenna measurements were used in the theoretical expressions for the field strength components, there was excellent agreement between the predicted and measured field strengths at ranges from 40 km to 6.6 mm (at both 45 and 75 Hz). Since these whole antenna H/I results produce such excellent agreement over a wide range of distances, it is felt that they give the true value of effective conductivity beneath the WTF array.

When the new Michigan Transmitter Facility (MTF) is constructed, it is planned to measure the effective conductivity beneath each antenna by either the H/I or E/H method (since the two methods are essentially equivalent [1], [2]). Because the WTF antenna will be available as a controlled source, the E/H method will probably be much easier to implement.

### REFERENCES


