whose computer codes for scattering from disks were used in some of cooperation of L. Allan in the sharing of his data and S. S. Seker disk thickness on the agreement between theory and experiment. For measurements to our attention. We also wish to acknowledge the simplification appropriate for thin disks because in this data set $k_o T$ is not particularly small. Fig. 3 shows a comparison of theory and measurement for the thinnest disk available in the data set ($A/T = 5.0$) at several values of $kA$. Notice that the agreement is good for incidence angles less than about 30° at the larger values of $kA$ and is reasonable even for $kA = 0.76$. Fig. 4 illustrates the importance of disk thickness on the agreement between theory and experiment. For the thinnest disk and the largest $kA$ (Fig. 4) the agreement between theory and measurement is good for incidence angles less than about 30°; however, for thicker disks the agreement deteriorates rapidly. One would expect the agreement to improve for disks with larger values of $A/T$ (e.g. $A/T > 100$) for which the theory is more appropriate and a trend in this direction is evident. The theory ignores edge effects, as described above, and its failure to predict these effects is clear in the data (e.g. at incidence angles greater than 60°). However, for disks with much larger $A/T$, for which the theory is intended, these effects ought to be significantly less in relative importance than in this data set. (Additional examples have been presented in Le Vine et al. [7].)

## Acknowledgment

The authors would like to thank Jerome Eckerman for his interest in this work and for his assistance in bringing Dr. Carter’s measurements to our attention. We also wish to acknowledge the cooperation of L. Allan in the sharing of his data and S. S. Seker whose computer codes for scattering from disks were used in some of the numerical work.

## References


## Directional Scanning of Complex Electromagnetic Environments

J. Randa, Member, IEEE, and Motohisa Kanda, Senior Member, IEEE

Abstract—A directional scanning technique is formulated for determining characteristics of an electromagnetic (EM) environment with a relatively small number of measurements, and results of a simulated application are presented. The method relies on measurements with a directional probe to obtain information about the coefficients in a plane-wave expansion of the field within a large volume. The simulation indicates that although the spatial variation of the field can not be accurately determined, the spatial average of the field intensity can.

## Introduction

As RF and microwave sources multiply, the electromagnetic (EM) environment in which electronic devices (and people) must function becomes increasingly complicated, while at the same time its characterization becomes more important. In order to completely characterize an EM environment without knowledge of the radiating sources, the sampling theorem requires that systematic measurements of the amplitude and phase of the field be made throughout the volume at spacings of no more than one half-wavelength (of the highest frequency present). This is often impossible and seldom convenient. There is a need for practical techniques which would determine useful properties of an EM environment from relatively few measurements [1]. One recent suggestion for such a technique [2]...
is to use directional measurements at a single point in conjunction with a plane-wave expansion of the field. In [1] this procedure was outlined for the case of a scalar field. We have now completed the formulation and performed simulations for the (vector) electric field and we report the results in this communication.

The general idea of the technique is as follows. The electric field within a large source-free volume is written as an integral over plane-wave directions. A directional antenna is used to make measurements in many directions at one point. The antenna response for each direction is a measure of the plane-wave components around that direction, convoluted with the antenna acceptance. Deconvolution of the expression, after appropriate discretization, yields the coefficients in the plane-wave expansion of the field. In principle these coefficients allow one to reconstruct the electric field throughout the volume of interest (the "lab" coordinate system), and one fixed with respect to the source. In practice it will prove more useful to use them to compute bounds on the average of the electric-field intensity. The technique is much like an inverse of the usual near-field spherical scanning using a plane-wave expansion [3]-[8]. In the present case, the probe is located at the origin, and one can think of equivalent plane-wave sources distributed on the surface of the sphere.

The next section details the mathematical formulation of this approach, from the plane-wave expansion of the electric field through the approximate expressions for the plane-wave coefficients in terms of the measurement results. In Section III, we describe simulated applications of the directional scanning technique, present bounds on the results of the simulations and dependence of the results on the variable parameters. Finally, in Section IV, we comment on the significance and note some limitations of this work.

II. FORMULATION

We first establish the notational conventions for this communication, since various coordinate transformations will be required. There are two coordinate systems of interest, one fixed with respect to the volume of interest (the "lab" coordinate system), and one fixed with respect to the probe (the probe coordinate system), cf. Fig. 1. The two coordinate systems have a common origin, the location of the probe. (The probe is assumed small.) The z axis of the probe system is described by angles θ₀, φ₀ in the lab system; the z axis of the probe system is at an angle of x₀ to the z × z plane. In the course of the scanning, θ₀, φ₀, and x₀ vary. Vectors and angles referred to the probe coordinate system will be denoted by a prime. The transformation of vectors between coordinate systems is effected by a rotation with Euler angles (in our convention) θ₀, φ₀, x₀, with

\[ V' = R(\theta_0, \phi_0, x_0) \cdot V \]  

(1)

The direction of incidence of a plane wave with respect to the probe system will be denoted θ', φ'; the same direction seen in the lab system is called θ, φ. The angles θ, φ depend not only on θ', φ', but also on θ₀, φ₀, x₀ And conversely θ' = θ'(θ, φ; θ₀, φ₀, x₀), φ' = φ'(θ, φ; θ₀, φ₀, x₀)

Having dispensed with these preliminaries, we can begin the actual formulation of the approach. We assume an electric field of a single frequency, and require that the volume under consideration be free of sources (both primary and induced) and very large (every dimension >> λ), so that it can be considered to be effectively infinite. The volume need not be physically enclosed. We can then write

\[ E(x) = \frac{1}{\lambda^2} \int d\Omega e^{i\mathbf{k} \cdot \mathbf{x}} e(\theta, \phi) \]  

(2a)

\[ \mathbf{k} \cdot \mathbf{e}(\theta, \phi) = 0, \quad \mathbf{k}^2 = \omega^2 \mu_0 \epsilon \]  

(2b)

which is the desired plane-wave expansion of the electric field. For multiple-frequency fields, the f would be a function of k as well as of θ, φ. For finite volumes the relations of (2b) fail for technical reasons, and so the present analysis must be restricted to volumes each of whose dimensions is much larger than the wavelength. Consequently the applications anticipated for this method are to large, relatively open environments.

Returning to (2), we note that with a perfectly directional probe the response for a given probe orientation θ₀, φ₀ would directly measure a component of the plane-wave coefficient \( \delta(\theta_0, \phi_0) \). Real antennas integrate over a finite solid angle - allowing us to cover the entire 4π solid angle with a finite number of measurements, but requiring a deconvolution to obtain the plane-wave coefficients. The response of the probe to a single plane wave incident at angles θ', φ' in the probe coordinate system will be the product of the probe acceptance at that angle, the amplitude per solid angle of the wave, and the increment of amplitude.

\[ dR = e^{i\mathbf{k} \cdot \mathbf{x'}} A'(\theta', \phi') \cdot \delta(\theta', \phi') \]  

(3)

where \( x' \) is the location of the active part of the probe (e.g., the mouth of the horn) in the probe coordinate system. Consequently the total response is

\[ R(\theta_0, \phi_0, x_0) = \int d\Omega e^{i\mathbf{k} \cdot \mathbf{x'}} A'(\theta', \phi') \cdot \delta(\theta', \phi') \]  

\[ = \int d\Omega e^{i\mathbf{k} \cdot \mathbf{A}(\theta, \phi; \theta_0, \phi_0, x_0) \cdot \delta(\theta, \phi)} \]  

(4)

The second form is an integral over lab angles, and follows from the rotational invariance of dΩ and the dot products. It is more convenient to use because it is in terms of the lab plane-wave expansion coefficients \( \delta(\theta, \phi) \), which are what we wish to determine. A component of \( A \) is the acceptance of the probe for that component of the electric field when the probe is at angles \( \theta_0, \phi_0, x_0 \) in the lab and the plane wave is incident from direction \( \theta, \phi \) in the lab. It is obtained from \( A' \) by an exercise in rotations.

Equation (4) can be simplified somewhat by choosing to make all measurements with the sensitive part of the probe at the origin, so that \( |x| = 0 \). In addition, \( \delta(\theta, \phi) \cdot k = 0 \), and so the radial (i.e., \( r, \phi \) direction, not \( \theta_0, \phi_0 \)) component of \( A \) is immaterial. One then makes two measurements \( x = 0, \pm \pi/2 \) in each of various directions \( \theta_0, \phi_0 \) (\( l = 1, N \)), obtaining 2N equations of the form

\[ R(\theta_0, \phi_0, x_0) = \int d\Omega[A_0(\theta, \phi; \theta_0, \phi_0, x_0)\delta(\theta, \phi) \]  

\[ + A_1(\theta, \phi; \theta_0, \phi_0, x_0)\delta(\theta, \phi)] \]  

(5)

This set of equations can then be discretized and solved for \( \delta \) and
\( e_\phi \) in any of a number of ways. The method we choose is to expand \( e_\theta \) and \( e_\phi \) in angular pulses, and to choose the measurement angles \( \theta_i, \phi_i \) to coincide with centers of the pulses. The surface of the unit sphere centered at the origin is divided into patches as illustrated in Fig. 2. Excluding the two polar caps, let there be \( N_p \) different polar measurement angles \( \theta_i \), for each of which there are \( N_\phi \) azimuthal measurement angles \( \phi_i \). Thus \( N = N_p N_\phi + 2 \), and we shall number the patches 0 through \( N - 1 \) as indicated in Fig. 2. Once \( N_p \) and \( N_\phi \) are chosen, the dimensions of the patches are determined. Each subdents solid angle \( \Delta \Omega = 4\pi/N \), and consequently the polar patches extend out to an angle of \( \cos \theta_{\text{ap}} = 2/N \). All other patches have dimensions \( \Delta \phi \pi/N, \Delta \cos \theta = 2(1 - \cos \theta_{\text{ap}})/N \).

The plane wave coefficients are approximated by

\[
e_\alpha(\cos \theta, \phi) = \sum_{j=0}^{N-1} e_{\alpha,j} \Pi^j(\cos \theta, \phi),
\]

where the pulse functions \( \Pi(\cos \theta, \phi) \) are equal to one within the solid angle defined by patch \( j \), and zero elsewhere. Inserting (6) into (5) we obtain the \( 2N \) equations (\( i = 0, N - 1; \alpha = 1, 2 \))

\[
R(\theta_i, \phi_i, x_o) = \sum_{j=0}^{N-1} e_{\alpha,j} A_\alpha^j(\theta_i, \phi_i, x_o) + e_{\alpha,j} A_\alpha^j(\theta_i, \phi_i, x_o),
\]

\[
A_\alpha^j(\theta_i, \phi_i, x_o) = \int d\Omega \Pi^j(\cos \theta, \phi) A_\alpha(\theta, \phi; \theta_i, \phi_i, x_o). \tag{7}
\]

Equation (7) can be cast in the form of a simple matrix equation by combining \( e_{\alpha,j}^j \) and \( e_{\alpha,j} \) into one vector of length \( 2N \), which also puts \( A_{\alpha,j}^j \) into the form of a \( 2N \times 2N \) matrix.

\[
R = \bar{\mathbf{e}} \mathbf{A}, \quad \bar{\mathbf{e}} = \mathbf{A}^{-1} R. \tag{8}
\]

Solution of this matrix equation yields the quantities \( e_{\alpha,j} \). These determine the approximation to the plane-wave coefficients (6), which in turn determine the approximate electric field throughout the volume (2).

III. SIMULATION

To test this method we have simulated the measurement process by taking a known incident field and feeding it into (4) to produce sets of measurements which were then analyzed. The antenna was assumed to be sensitive to only one component of polarization and to have a Gaussian acceptance,

\[
A_\alpha^j(\theta, \phi) = e^{-\theta^2/j^2} \cos \phi, \quad A_\alpha^j(\theta, \phi) = e^{-\theta^2/j^2} \sin \phi, \tag{9}
\]

in the probe coordinate system. The incident field was composed of a specified number of plane waves (one through 15), whose directions, phases, and relative amplitudes were usually generated randomly. We investigated the dependence of the results on the width \( W \) of the acceptance pattern of the probe, the number of measurements, and the complexity of the incident field (the number of plane waves).

For a crude, relatively simple comparison of results for different sets of parameters, we used the reconstructed field at the origin. These preliminary tests indicated that for a given width \( W \) there is an optimal number of measurements one should make. Not surprisingly, the optimal number of measurements for a given width \( W \) occurs for patch dimensions comparable to \( W \). If \( N_p \) or \( N_\phi \) is too small, it is easy to see that there will be dead spots between measurement angles. If \( N_p \) or \( N_\phi \) is too large, instabilities occur in the inversion process in solving (8). This happens because for patches very small compared to \( W \), the acceptance for a given probe orientation does not change much from one patch to the next. As a result, adjacent columns in the matrix \( \bar{\mathbf{e}} \) in (8) are very nearly equal, and \( \mathbf{A} \) is very nearly singular.

Assuming one uses the number of measurements appropriate to the probe width, the quality of the results is insensitive to the number of incident plane waves, even for a width as large as 1.0, from which we infer that the method does not require simple field configurations. The results improve for smaller \( W \) (larger \( N \)), but are quite acceptable even for \( W = 1.0, N = 10 \), as we shall see below.

The reconstruction of the spatial dependence of the field is not very successful, except for the case of a single plane wave with incidence angles which coincide with a measurement direction. In other cases with one incident plane wave, the magnitude of the field is approximately correct, but spatial variation of magnitude and phase is rather poor. This is no surprise—in order to reconstruct the field throughout the volume, we must measure systematically at the rate required by the sampling theorem. Nevertheless, it is possible to extract useful information from the measurements. Provided the volume is large, different plane waves are approximately orthogonal; and we can derive expressions for the rms electric field within the volume. From (2) and (6) we obtain

\[
E_{\text{rms}} = \left( \frac{\langle |E|^2 \rangle_{\text{vol}}^{1/2} \Delta \Omega}{N} \right)^{1/2} \Delta \Omega \tag{10}
\]

where \( \Delta \Omega = 4\pi/N \). It is also possible to calculate an upper bound on the maximum electric-field intensity. In principle, having measured all the \( e^j \) we can reconstruct the field and search the entire volume for the maximum \( |E| \). In practice this is a slow and awkward process, and we seek a method which uses the \( e^j \) directly. Although we do not have an expression for the maximum \( |E| \) in terms of the \( e^j \), it is not hard to derive an upper bound on \( |E|_{\text{max}} \). The upper bound follows from the obvious observation that any component of \( |E| \) can be no larger than \( \Delta \Omega \) times the sum of the magnitudes of that component of \( e^j \). And of course \( |E|^2 \) can be no larger than the sum of the maximum squared values of the three components. Thus,

\[
|E(\chi)|_{\text{max}} \leq E(\text{ub}),
\]

\[
E(\text{ub}) \equiv \left( \sum_j |e_{j1}|^2 \right)^{1/2} \Delta \Omega. \tag{11}
\]

Equations (10) and (11) refer to the measured quantities of course. To test the accuracy of the technique the results from (10), (11) must be compared to the true results for \( E_{\text{rms}} \) and \( E(\text{ub}) \), which are obtained from (10) and (11) by converting the sums over measurement directions to sums over incident plane waves. We have done this, and the results for \( W = 1.0 \) and \( W = 0.5 \) with \( N = 10 \) and 52, respectively, are shown in Table 1, for various numbers of plane
waves. For each number of plane waves, 25 or 50 different random configurations were generated, and that sample was used to compute the rms fractional errors in $E_{\text{rms}}$ and $E(\theta, \phi)$ for that number of incident plane waves.

The results are very encouraging, particularly for $E_{\text{rms}}$. Even for $W = 1.0$, with ten measurement directions (a total of 20 measurements), the error in $E_{\text{rms}}$ is around 14 percent, which corresponds to a 1.2 dB error in the average intensity in the volume. The results for $W = 0.5$ are better still, but the total number of measurements ($2N = 104$) is excessive if they are done manually. The results for the upper bound on $|E|_{\text{max}}$ are not quite as good, but are still good enough to be useful. Bear in mind, however, that the error in $E(\theta, \phi)$ measures its deviation from the true $E(\theta, \phi)$, computed from (11) using the correct incident fields, and not the deviation from the true $|E|_{\text{max}}$. The reason that the error in $E(\theta, \phi)$ is greater for $W = 0.5$ than for $W = 1.0$ is that the upper bound is a very loose bound, and having more plane wave coefficients whose magnitudes must be summed increases the bound artificially. The fewer the incident plane waves, the greater is the effect. The error is virtually always such that $E(\theta, \phi)$ is overestimated, so that it is still an upper bound, but is not as good as it could be.

### IV. Comments and Conclusion

The method of directional scanning appears to hold great promise as a practical technique for assessing complicated electromagnetic environments using a manageable number of measurements. The actual reconstruction of the spatial dependence of the field does not appear feasible; but the rms electric field (where the average is over configurations were generated, and that sample was used to compute the wavelength for which this method will usually be applied. In addition there are possible refinements of the technique—it is unlikely that all the details of the preceding application are optimal. For example, there may be better ways to divide the sphere into patches and/or to solve for the plane-wave coefficients in terms of the measurements; it is possible that small improvements could be realized by using a different acceptance pattern for the receiving antenna; it may be possible to obtain reasonably good results for $E_{\text{rms}}$ using even fewer measurements. It may also be possible to actually reconstruct $E(\theta, \phi)$ by using a more sophisticated "probe" such as an array with adaptive processing [9], [10] although such an improvement would come at the expense of ease and economy.

More work on such problems and refinements is clearly warranted. The restriction to large volumes, in order to use the plane-wave expansion and to obtain a simple form for $E_{\text{rms}}$, means that this method will be most useful for large, relatively open environments. For such applications, our results indicate that the directional scanning technique could be very valuable in measuring field intensities and assessing hazards.

### ACKNOWLEDGMENT

We thank D. Hill of NBS for helpful criticism and discussion.

### REFERENCES


