**Abstract**—A bicollimated Gregorian reflector is structurally similar to a classical confocal Gregorian reflector, but its surfaces are shaped to have better scan capability. A geometrical optics procedure is used in designing the reflector surfaces. A three-dimensional ray tracing procedure is used in analyzing the aperture phase errors as the beam is scanned to different angles. The results show that the bicollimated configuration has about 45 percent greater angular scanning range than the equivalent confocal Gregorian reflector antenna.

**INTRODUCTION**

Antennas which are required to scan only over relatively small angles are called limited scan antennas. Such antennas are required in some radar systems which must acquire and track a number of targets within a limited angular range. Further applications involve satellite communication systems in which highly directional antennas are used to cover a limited angular range.

For a given aperture antenna, a reflector antenna is much less expensive than a phased array. However, unless the reflector antenna is mechanically rotatable, its performance is limited to a very small angular range. Conversely, a phased array of the same aperture has very wide angular coverage but is expensive and difficult to build. Recently, the near-field Gregorian antenna [1], [2] which uses confocal paraboloids and small phased array feed was shown to have some attractive features in limited scan applications. The system combines the low cost of a reflector antenna with the performance capabilities of an electronically scanned array over a limited scan coverage. However, one problem with this type of antenna is that its angular coverage is still limited.

It is well-known that bifocal reflector antenna [3] and bifocal dielectric lens antennas [4], [5] have wider angle scan capability than their single focus counterparts. We propose a bicollimated Gregorian reflector antenna [6], [7] which has better scan capability compared to the confocal Gregorian reflector antenna (collimated in one direction only). The design presented applies to both symmetric or offset configurations. However, only the offset configuration, which eliminates feed blockage, will be discussed here.

The bicollimated reflector is obtained by first designing a bicollimated cylindrical reflector system using geometrical optics techniques and then revolving the cross section curves to form surfaces of revolution. Selected parts of these surfaces will form an offset reflector configuration.

Fig. 1 is a central cross section illustrating the principle of a bicollimated dual reflector antenna. The antenna comprises a main reflector, a subreflector and a phased array feed. The phased array can radiate a plane wave beam in any direction over a large scanning range. For instance, a beam can be radiated at an angle $\beta$ with respect to the Z-axis. This beam is represented by the two solid lines emanating from the feed array. The phased array is also capable of radiating a beam at an angle $-\beta$ as represented by the two dotted lines emanating from the feed array. The cross sections of the main and subreflector are shaped such that the antenna system is bicollimated. This means that all the rays in an initial beam either at $\beta$ or $-\beta$ direction be reflected between the subreflector and main reflector into a final beam at $-\alpha$ with respect to the antenna axis. It is shown later in this paper that only a series of points and slopes (or tangents) at those points on the cross sections of the reflector can be obtained. Using these data, the reflected cross sections are represented completely by best fit polynomials. This polynomial representation is used in computing the aperture phase errors when the antenna beam is scanned to different angles by scanning the feed array. Three-dimensional ray tracing procedure is used in analyzing the aperture phase errors. The results showed that the bicollimated configuration has about 45 percent more scanning range than an equivalent confocal configuration.

**DESIGN PROCEDURE**

Fig. 2 is a central cross-section of a dual reflector antenna including the rays used to calculate the discrete points on both reflectors. The Z-axis is the antenna axis of rotational symmetry. The subreflector is chosen to be perpendicular to the Z-axis at $Z = Z_1 = P$, where $P$ is the separation between the origin and the subreflector. In addition, the path length between the reflected phase front $B$ and the corresponding incident phase front $A$ is assumed to be $L$. Similarly, the path length between the phase fronts $C$ and $D$ is also equal to $L$.

Knowing the initial point $(X_1, Z_1)$, and the slope at that point on the subreflector, one can determine the main reflector point $(X'_1, Z'_1)$ and the slope at that point by applying geometrical optics principles to the initial ray incident on the subreflector at the point $(X_1, Z_1)$. Once the slope and location of the point $(X'_1, Z'_1)$ on the main reflector is determined, the procedure is reversed to find point $(X_2, Z_2)$ and the corresponding slope on the subreflector.

Thereafter the procedure is repeated but using the subreflector’s already determined second point $(X'_2, Z'_2)$ as a starting point to determine a second point on the main reflector and a third point on the subreflector. Further repetitions will determine the locations and slopes of a finite sequence of points $(X_K, Z_K)$ on the subreflector and points $(X'_K, Z'_K)$ on the main reflector.

The mathematical formalism required for this iterative process is hereafter derived on generalized points on each reflector as shown in central cross section in Fig. 2. An initial ray (solid line) oriented at an angle $\beta$ is incident on the subreflector at a generalized point $(X_K, Z_K)$ and slope that were previously determined. The orientation $\gamma_K$ of the reflected intermediate ray relative to Z-axis is determined by applying Snell’s law. The intermediate ray is reflected at the point $(X'_K, Z'_K)$ on the main reflector into a final ray oriented at $-\alpha$ relative to the Z-axis. The total ray-pathlength and Snell’s law requirement determines the coordinates.
Fig. 1. Central cross section of a bicollimated Gregorian reflector antenna.

Fig. 2. Geometry of an offset bicollimated Gregorian reflector antenna.

\( (X'_k, Z'_k) \) and the slope at that point. The pertinent relations can be shown to be the following first set of formulas:

\[
Z'_k = \frac{R_k - L + Z_k W' - X_k \sin \alpha}{W' + \cos \alpha}.
\]

\[
X'_k = X_k + (Z'_k - Z_k) \tan \gamma_k.
\]

\[
\left( \frac{dz'}{dx'} \right)_{Z'_k, X'_k} = \tan \left[ \frac{\gamma_k + \alpha}{2} \right].
\]

where

\[
R'_k = R_k \cos \beta + X_k \sin \beta,
\]

\[
\gamma'_k = \gamma_k + 2\beta.
\]

and

\[
W = \frac{1 + \sin \gamma_k \sin \alpha}{\cos \gamma_k}.
\]

In addition, \( Z_k, X_k, \) and \( \gamma'_k \) are given by (7), (8), and (11) for \( k \gg 2 \). The initial values, for \( k = 1 \), are \( Z_1 = P \) and \( X_1 = 0 \). From Fig. 2 it can be noted that \( \gamma_1 = \beta \). Therefore, \( \gamma'_1 \) is not needed in (5) to find \( \gamma_1 \).

Reversing the procedure for the final ray (dotted line) at \( \alpha \) produces an intermediate ray at an orientation \( \gamma'_{k+1} \), reflecting from the point \( (X'_{k+1}, Z'_{k+1}) \) on the subreflector and corresponding to an initial ray oriented at \( -\beta \). The location \( (X'_{k+1}, Z'_{k+1}) \) and the slope at that point are thus determined. The mathematical relationships can be shown to be the following second set of formulas:

\[
Z_{k+1} = \frac{L - R_k + Z_k W' + X_k \sin \beta}{W' + \cos \beta}.
\]

\[
X_{k+1} = X'_k + (Z'_k - Z'_{k+1}) \tan \gamma'_{k+1}
\]

\[
\left( \frac{dz}{dx} \right)_{Z_{k+1}, X_{k+1}} = \tan \left[ \frac{\gamma'_{k+1} + \beta}{2} \right],
\]

where

\[
R'_k = -Z'_k \cos \alpha - X'_k \sin \alpha,
\]

\[
\gamma'_{k+1} = \gamma_k + 2\beta,
\]

and

\[
W' = \frac{1 + \sin \gamma'_{k+1} \sin \beta}{\cos \gamma'_{k+1}}.
\]

Starting with the initial point \( (X_1, Z_1) \) and the angle \( \gamma_1 (= \beta) \), and making use of the first and then the second set of formulas and continuing the process, a series of points and slopes on each reflector surface can be found in succession.

POLYNOMIAL APPROXIMATION

The design procedure discussed in the previous section gives a finite number of points and an equal number of slopes on the reflector surfaces. In order to define the reflector surfaces completely, it is necessary to use an approximation. It is convenient to approximate the reflector cross sections by best fit polynomials. Since the reflectors are axially symmetric, only even powers are required. The reflector cross sections are represented by the polynomials.

\[
Z_s = B_0 + B_1 X_s^2 + B_2 X_s^4 + \cdots
\]

\[
Z_m = A_0 + A_1 X_m^2 + A_2 X_m^4 + \cdots
\]

where \( X_s \) and \( Z_s \) are the subreflector coordinates, and \( X_m \) and \( Z_m \) are the main reflector coordinates.

If the number of data points available limits the degree of the polynomial, the known slopes on the reflector curves can be used to improve the accuracy.

EQUIVALENT GREGORIAN CONFOCAL REFLECTOR

To compare the reflector shapes and performance of the bicollimated reflector system, it is necessary to define an equivalent confocal reflector system. The equivalence is established here by first choosing the main reflector shape of the confocal
system as an equivalent parabola by selecting the first two terms of the polynomial representation of the bicollimated reflector and then choosing the spacing (measured along the reflector axis) between the confocal reflectors equal to that of the bicollimated configuration. The former is justified because, when specific examples are considered, the main reflector cross section of the bicollimated configuration approximates a parabola (the higher order terms in (14) are very small). Therefore, an equivalent main reflector is defined by

\[ Z_{mp} = A_0 + A_1 X_{mp}^2, \]  

(15)

where \( Z_{mp} \) and \( X_{mp} \) are the coordinates of the equivalent main reflector.

Then, the equivalent subreflector can be expressed as [7]

\[ Z_{sp} = B_0 + \frac{X_{sp}^2}{4F_s}, \]

(16)

where \( F_s = A_0 + B_0 - A_1/4 \) is the focal length of the equivalent parabolic subreflector.

There are other ways of defining an equivalent confocal reflector. However, the one defined in this section is more appropriate for comparing the reflector shapes. In the next section we will consider the limiting confocal reflector which may also be considered as an equivalent confocal reflector and will be used in comparing scanning performance. However, there is no need to find precise equivalence (if there is such a thing) because small changes in the Gregorian antenna parameters do not appreciably influence its scanning performance as long as the confocal condition is not violated. Another point which should be noted is that at the outset it may appear that an equivalent confocal reflector can be obtained simply by taking the first two terms in the polynomial representation of the bicollimated reflector. However, the main and subreflectors so obtained will not form a confocal set.

LIMITING GREGORIAN CONFOCAL REFLECTOR

Consider a set of bicollimated reflectors which are designed by keeping \( L, P \) and \( M = \beta/\alpha \) the same but change \( \alpha \) and \( \beta \). As \( \alpha \) approaches zero, the limiting bicollimated reflector degenerates to a confocal reflector. This limiting case (\( \alpha = 0 \)) is called Limiting Gregorian confocal reflector. This equivalent confocal reflector configuration will be used, later on, to compare the scanning performance of a set of bicollimated reflectors which are designed using the same values for \( L, P, \) and \( M \). It is not difficult to see that \( M \) represent the magnification of the limiting Gregorian confocal reflector, \( L \) (the path length between the incident and the radiating wavefronts for an on axis beam for the confocal reflector) is equal to twice the spacing between the main and subreflector and \( P \) is the distance between the origin and the subreflector. Then, it can be shown [7] that the limiting parabolic subreflector is given by the equation

\[ Z_{s1} = P - \left( \frac{1 + M}{2L} \right) X_{s1}^2, \]

(17)

and the limiting parabolic main reflector is given by

\[ Z_{m1} = (L/2) - P - \left( \frac{1 + M}{2LM} \right) X_{m1}^2. \]

(18)

PHASE ERROR ANALYSIS

In the classical near-field Gregorian system [1], it is known that the amplitude distribution applied to the feed array is reproduced over the main aperture without alteration. For values of \( \alpha \) which are of practical interest, the bicollimated reflector system does not deviate much from an equivalent classical near-field Gregorian system. Therefore, it is reasonable to assume that the main aperture amplitude distribution is the same as that of the feed array. However, the aperture phase errors are different in the two systems. The purpose of this section is to analyze the aperture phase errors and show the advantages of the bicollimated reflector system. Fig. 3 shows the geometry used in analyzing the aperture phase errors. The aperture phase errors are found by assuming that a plane wave is incident on the main reflector at an angle \( \theta \) and \( \phi \), which also corresponds to the main beam direction. Path-length errors on the aperture are determined from the path length between the incident wavefront and the corresponding feed array wavefront, as discussed in the Appendix. Equation (39) gives the path-length error on the aperture. The procedure given in the Appendix applies to both bicollimated and, confocal reflector antennas.

NUMERICAL EXAMPLES

As an example, a bicollimated reflector antenna is designed with \( \alpha = 3^\circ, \beta = 9^\circ, \) and \( L/P = 2.5 \). Table 1 gives computed data points.

By use of the data points shown in Table I, the reflector cross sections are approximated by the following best fit polynomial representation:

\[ z_s = 0.999998 - 0.8018732 x_s^2 - 0.01234972 x_s^4, \]

(19)

\[ z_m = 0.253768 + 0.26682 x_m^2 + 0.00025741 x_m^4, \]

(20)

where \( z_s = Z_s/L, x_s = X_s/L, z_m = Z_m/L \) and \( x_m = X_m/L \).

Reflector surfaces are obtained by rotating the above cross sections about the Z-axis and choosing only selected parts. Fig. 4 shows the geometry and the antenna parameters of the offset bicollimated reflector which is chosen as an example. The main reflector surface is chosen so that it is circular when projected into the \( X \)Y-plane. The main aperture is assumed to be completely utilized over the scanning range of interest. The corresponding illuminated areas of the feed array and the subreflector surface may change with scan angle. For the example under consideration, the main reflector diameter \( D = 1.6P \) and the main reflector is offset from the Z-axis by 0.3P to eliminate blockage due to the subreflector when the beam is scanned below the Z-axis.

By use of the three-dimensional ray tracing method developed in the Appendix, a computer program is devised to determine the aperture phase errors when the beam is scanned to different scan angles. Using that program, path-length errors are obtained over the whole aperture. However, only phase errors in the XZ-plane are shown in Fig. 5, as the errors are similar in other planes.
TABLE I
COMPUTED POINTS ON THE REFLECTOR CROSS SECTIONS

<table>
<thead>
<tr>
<th>( \frac{z}{\rho} )</th>
<th>( \frac{x}{\rho} )</th>
<th>( \frac{y}{\rho} )</th>
<th>( \frac{z}{\rho} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.000000</td>
<td>0</td>
<td>-0.24342</td>
<td>0.196928</td>
</tr>
<tr>
<td>0.985926</td>
<td>-0.132464</td>
<td>-0.154958</td>
<td>0.608431</td>
</tr>
<tr>
<td>0.938416</td>
<td>-0.276962</td>
<td>0.057515</td>
<td>1.079356</td>
</tr>
<tr>
<td>0.836951</td>
<td>-0.459222</td>
<td>0.49982</td>
<td>0.679324</td>
</tr>
</tbody>
</table>

Fig. 4. Cross section of the bicollimated reflector designed with \( \alpha = 3^\circ \), \( \beta = 9^\circ \) and \( L = 2.5P \).

Fig. 5. Aperture phase errors in XZ-plane.

Fig. 6. Maximum error on the aperture when scanned in \( \phi = 0 \) plane.

Curves belong to the bicollimated reflector. As expected, the aperture errors for \( \theta_z = 3^\circ \) (\( \alpha = 3^\circ \)) are zero. However, for \( \theta_z = 0^\circ \) (on axis beam) the bicollimated reflector has aperture errors whose magnitude increases toward the edges of the aperture. For comparison, aperture errors are computed for an equivalent confocal reflector and are shown as dashed curves in Fig. 5. For the confocal reflector, the aperture errors are zero for \( \theta_z = 0^\circ \) and increases with \( \theta_z \). The maximum errors appear toward the edges of aperture.

For each scan angle, the maximum error on the full aperture is determined by using the data shown in Fig. 5 and similar data obtained for other planes. This maximum path-length error normalized to aperture diameter is plotted in Fig. 6 as a function of the scan angle when the main beam is scanned in \( \phi = 0 \) plane. For the confocal reflector, the maximum path-length error increases monotonically with the scan angle whereas the path-length error for the bicollimated reflector decreases with the scan angle and becomes zero for \( \theta_z = \alpha (3^\circ \) in the example) and then increases monotonically with the scan angle. For a maximum normalized path-length error of 0.0011, Fig. 6 shows that the confocal reflector can be scanned up to 2.7° and the bicollimated reflector can be scanned up to 4°. Therefore, the bicollimated reflector has about 48 percent more scanning range than the confocal reflector. Figs. 7 and 8 show similar results for scanning in \( \phi = 90^\circ \) and \( 180^\circ \) planes. Fig. 9 shows the complete scanning ranges, for a maximum normalized path-length error of 0.0011, for both confocal and bicollimated reflector antennas. The results show that the bicollimated reflector has about 45 percent more scanning range than an equivalent confocal reflector.

Keeping \( L = 2.5P \), \( D = 1.6P \), and \( \beta/\alpha = 3 \), aperture phase errors are calculated for different values of \( \alpha \) (and hence \( \beta \)) including the limiting case of \( \alpha = 0 \) (which corresponds to an equivalent confocal configuration). Fig. 10 shows the maximum path-length error on the aperture versus scan angle when the beam is scanned in \( \phi = 0 \) plane. Using these results, one can obtain the curves shown in Fig. 11 which relates maximum scan angle for a specified maximum path-length error on the aperture or vice versa. These curves can be used to choose an optimum value of \( \alpha \) for a specified maximum path-length error on the aperture by noting the maximum scan angle \( \theta_m \) for specified phase error and then choosing \( \alpha \approx \theta_m/1.45 \). Figs. 12 and 13 show the maximum aperture errors versus scan angles when the beam is scanned in the xz-plane and yz-plane, respectively. The curves are not symmetric.
Fig. 7. Maximum error on the aperture when scanned in $\phi = 90^\circ$ plane.

Fig. 8. Maximum error on the aperture when scanned in $\phi = 180^\circ$ plane.

Fig. 9. Comparison of scanning ranges for confocal and bicollimated reflectors for maximum path-length error of 0.00110.

Fig. 10. Maximum path-length error on the aperture when scanned in $\phi = 0$ plane for bicollimated reflectors designed with different values of $\alpha$. 
Fig. 11. Relation between maximum scan angle and maximum path-length error on the aperture when bicollimated antenna is scanned in $\phi = 0$ plane.

Fig. 12. Maximum path-length error on the aperture when scanned in XZ-plane for different offset bicollimated reflectors.

Because of the large blockage expected due to the feed and subreflector, a symmetric bicollimated reflector cannot be used in practice. However, one can calculate the aperture phase errors (ignoring the blockage effects) and compare them with those of the offset configurations. Fig. 14 shows the maximum aperture error as a function of scan angle, when scanned in XZ-plane. Because of the symmetry, the results will be the same when scanned in other planes. Comparing these results with the results of offset configurations given in Figs. 12 and 13, one may conclude that an offset configuration does not result in any noticeable degradation in scanning performance when compared to a symmetric configuration.

CONCLUSION

A bicollimated dual reflector antenna, which can collimate a beam in two different directions, is proposed. A design procedure is presented for determining the reflector surfaces. Aperture phase errors are analyzed for different scan angles. The results show that the bicollimated reflector has about 45 percent more scanning range compared to an equivalent near-field Gregorian reflector. In addition, the scanning performance of an offset bicollimated reflector is comparable to the performance of its symmetric counterpart.
APPENDIX
THREE-DIMENSIONAL RAY TRACING PROCEDURE
A ray tracing procedure is used to compute aperture phase errors for the bicollimated reflector antenna. Fig. 3 of this paper is used to illustrate the procedure. First we compute the path length of a general ray from a point $A$ on the incident wave front through a point $B(X_m, Y_m, Z_m)$ on the main reflector and a point $C(X_s, Y_s, Z_s)$ on the subreflector, and finally to a corresponding point $D(X', Y')$ in the feed plane. The equation of the subreflector is assumed to be a sixth degree polynomial and is given as

$$Z_s = B_0 + B_1 Y_s^2 + B_2 Y_s^2 + B_3 Y_s^2,$$  \hspace{1cm} (21)

where $B_0$, $B_1$, $B_2$, $B_3$ are known constants.

Equation (30) is used to find $R$ by using a standard routine for solving the roots of a polynomial. In general there will be six roots for a sixth order polynomial; some are complex and some are real, with only one correct real root. A procedure is devised, using physical constraints, to select the correct root, and hence the value of $R$. Equation (29) is then used to find the point $(X_s, Y_s, Z_s)$ on the subreflector.

Next, the direction of the reflected ray $CD$ and the length $|CD|$ will be determined. Snell’s law of reflection at the point $(X_s, Y_s, Z_s)$ on the subreflector is given as

$$\vec{cd} = \vec{bc} - 2n_s(\vec{n}_s \cdot \vec{ab}),$$  \hspace{1cm} (31)

where $n_s$ is the unit normal at the point of reflection on the subreflector.

A ray $\vec{CD}$ can also be expressed as

$$\vec{CD} = \vec{k}(X' - X_s) + \vec{j}(Y' - Y_s) + \vec{i}(-Z_s),$$  \hspace{1cm} (32)

where the array aperture plane is defined as the plane $Z' = 0$ and $(X', Y')$ is the point of intersection $D$ in the feed array plane.

The equation $\vec{cd} = \vec{CD}/|\vec{CD}|$ yields three equations; only two of them are independent, which are sufficient to determine the point of intersection $(X', Y')$ in the aperture plane $Z' = 0$ and are given as

$$X' = X_s - \frac{L_x Z_s}{L_z},$$  \hspace{1cm} (34)

$$Y' = Y_s - \frac{L_y Z_s}{L_z},$$  \hspace{1cm} (35)

and

$$Z' = 0.$$  \hspace{1cm} (36)

The total path length is simply the sum of the component path lengths. Hence,

$$L_0 = |\vec{AB}| + |\vec{BC}| + |\vec{CD}|.$$  \hspace{1cm} (38)

For an assumed direction $(\theta, \phi)$ for the mainbeam it was noted that the direction of the rays incident on the feed array are not perfectly parallel to each other. However, an optimum phase front which minimizes aperture phase errors can be found. Let this phase front be defined by the feed steering angles $\theta_f$ and $\phi_f$ which are necessary to steer the mainbeam in the direction of $\theta$ and $\phi$. Then, one can show that the aperture path length errors are given by

$$\Delta L = L_0 - X' \sin \theta_f \phi_f - Y' \sin \phi_f \sin \phi_f - L.$$  \hspace{1cm} (39)

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Analysis of an Antenna Composed of Arbitrarily Located Slots and Wires

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Abstract—A technique of analyzing an antenna composed of arbitrarily oriented thin wires and slots in a planar conducting screen is presented. The analysis is based on the generalized Hallen's type integral equation with closed form kernel. A parametric evaluation is done for a configuration of slot and two bending wires (Clavin element). The results obtained are shown to be in close agreement with the experimental ones obtained by Clavin et al.

I. INTRODUCTION

THE RADIATION pattern of a single electric or magnetic dipole element is fixed for a given frequency of excitation. In practical applications there is quite often a need for either improving the directive properties or controlling the radiation pattern. For this purpose, an antenna system consisting of some slots and wires properly arranged in space is available. The rather unique complementary properties of slot and wire antennas [61-[81.]

In practical applications there is quite often a need for either improving the directive properties or controlling the radiation pattern. For this purpose, an antenna system consisting of some oriented thin wires and slots with closed form kernel. A parametric evaluation is done for a configuration of slot and two bending wires (Clavin element). The results obtained are shown to be in close agreement with the experimental ones obtained by Clavin et al.

In this paper, a general method of solution is developed for determining the current distributions and far-zone radiation fields due to some arbitrarily oriented slots and wires. The method is based upon the generalized Hallen's type coupled integral equation for the electric currents on the wires and magnetic currents on the slots. The kernels of these integral equations are expressed in closed forms [11] which allow relatively simple numerical solution.

The validity of the proposed theory is checked by comparing calculated results with experimental data [3] for the Clavin element (slot with two parasitic bending wires). The agreement between theoretical and experimental results is quite satisfactory.

II. SUMMARY OF APPROACH

The radiating system to be studied here consists of $N_s$ slot segments and $N_w$ wire segments as shown in Fig. 1. The ideal slots are cut in a perfectly conducting screen of infinite size and vanishing thickness. These slots are arbitrarily oriented in the screen. The wires are made of perfect conductors, and arbitrarily located in the upper half-space. The antenna then consists of $N_s + N_w$ segments. The slot segments carry the indices $1, 2, \ldots, N_s$, the wire segments the indices $N_s + 1, N_s + 2, \ldots, N_s + N_w$. The $i$th segment has the length $h_i$, and the width $w_i(1 \leq i \leq N_s)$ or the radius $a_i(N_s + 1 \leq i \leq N_s + N_w)$. Let a Cartesian coordinate $(x_i, y_i, z_i)$ be defined with the origin at one end of the $i$th segment and with the $x_i$ axis coincident with the axis of the segment. The $i$th segment is identified by its starting point $r_i$ and unit vector $\mathbf{n}_{x_i}$ directed along the $x_i$ axis. It is assumed that the slot width $w_i$ or wire radius $a_i$ is very small compared to the segment length $h_i$ and the wavelength $\lambda$.

As discussed by Butler [9], the equivalent radius of the narrow slot in a perfectly conducting planar screen of infinite extent is one-fourth its width, and the equivalent radius may be used in computations employing approximate formulas that have been derived for thin circular wires. The authors have previously formulated the simplified Hallen's equations with closed form kernels for a thin wire antennas of arbitrary shape [10], [11]. These methods are very useful for the analysis of the junction problems.

For the case of an antenna composed of slots and wires, let $\mathbf{E}_i(1 \leq i \leq N_w)$ be the equivalent radius $w_i/4$ for the slot segment or the actual radius $a_i$ for the wire segment. Then, the current distribution on each segment...