creases, and it should be considered as an alternative to the conventional narrow-band horn.

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REFERENCES


Penetrating Electromagnetic Wave Applicators

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Abstract—It is demonstrated that in the near field of suitably designed applicators, the penetration depth of the electromagnetic wave can greatly exceed the usual skin depth. The application of this principle in therapy is suggested.

I. INTRODUCTION

Radio-frequency and microwave heating of lossy dielectric materials, particularly of biological origin, is widely employed in industry and medicine. In general, at microwave frequencies the usual mechanism of this interaction is the rotation of the polar water molecules in the electromagnetic field; at lower frequencies protein and other macromolecular structures also contribute to the absorption process. Animal tissues, especially the soft, high-water tissue of muscles and organs are efficient absorbers of microwave and RF energy, as illustrated by the phenomena observed in microwave cooking. Medically, microwave and RF energy has long been employed in diathermy, in which heating is used to obtain the beneficial effects of the resulting increase of the flow of blood and lymph to the tissue. Recently, numerous laboratory and clinical tests have supported the use of hyperthermia, alone or in combination with ionizing radiation or drugs, in the treatment of cancer. In this treatment, the cancer is treated by raising its temperature for a few degrees centigrade for periods of several hours. Electromagnetic heating, using frequencies in the range ~0.3 to 3.0 GHz, offers one of the most promising and widely employed methods for hyperthermia treatment since it includes the possibility of selective, controlled heating of specific tissue volumes. At lower frequencies, conduction currents begin to dominate the absorption process, and the volume being heated cannot be as closely controlled because of the complicated geometry of the body. On the other hand, the small penetration depths of high frequency electromagnetic waves in lossy media is an obstacle in the application of RF and microwave fields in many instances. For example, the use of microwave frequencies to heat tissue in diathermy or hyperthermia treatment has been restricted to surface layers (when the possibilities for invasive applicators or applicators inserted into some body orifice are not available). The reason for the lack of penetration is well-known: a plane wave field is exponentially attenuated in a conducting medium. The "skin depth" δ, the distance in which the field amplitude is reduced to e^-1 of its initial value, is determined by the frequency and by the electrical parameters of the medium. At S-band frequencies for example, δ is of the order of a centimeter in normal muscle and organ tissues. Moreover, because any wave field can be regarded as the sum of an angular spectrum of plane waves, the common conclusion is, apparently, that no high frequency wave field can achieve greater penetration depths in a lossy medium. This analysis, however, is flawed.

The expression for the skin depth usually given refers to penetration of the conducting medium by a homogeneous plane wave. Homogeneous plane waves are the traveling planar fields which are the usual result of distant sources. However, an arbitrary wave field may contain a spectrum of inhomogeneous waves. Thus the penetrating properties of an electromagnetic field in a conducting medium cannot be judged only on the basis of the skin depth for homogenous plane waves. The penetrating properties of inhomogenous waves may be quite different and must be included.

Inhomogenous waves are represented by a variety of wave and field types, such as surface waves and antenna near fields. Typically, they are bound, though often loosely, to an interface or antenna structure. A significant property of surface wave types is that unlike ordinary plane waves, the penetration depths of these waves into a conducting medium is not solely determined by the properties of the conducting medium, but also depends upon the parameters of the medium in which the wave is launched.

In the following sections of this communication we present some examples to illustrate this phenomenon and conclude that an inhomogenous wave field launched from a region of high dielectric permittivity can penetrate a lossy dielectric to significantly greater depths (in the vicinity of the applicator) than the usual skin depth estimate.

II. MATHEMATICAL DESCRIPTION

Consider the planar interface (z = 0) between media with (possibly) complex permittivities \( \varepsilon_1(z > 0) \) and \( \varepsilon_2(z < 0) \). Setting \( \mu = \mu_0 \) in the entire space, and assuming no \( \gamma \)-dependence,
the wave fields

\[ H_y = H_0 e^{-\gamma_1 z - j k_1 x + j \omega t}, \quad z > 0 \]
\[ H_y = H_0 e^{\gamma_2 z - j k_2 x + j \omega t}, \quad z < 0 \]

are continuous across the interface. Here \( \gamma_1, \gamma_2 > 0 \).

The tangential electric field, \( E_x \), will be continuous at \( x = 0 \) provided \( \gamma_1/e_1 = -\gamma_2/e_2 \). To satisfy the wave equation, \( \lambda^2 = \gamma_1^2 + e_1 \mu_0 \omega^2 = \gamma_2^2 + e_2 \mu_0 \omega^2 \). Consequently \( \gamma_1 = \gamma(1), \gamma_2 = \gamma(2), \lambda = \lambda(e) \) where

\[ \gamma(1) = -\frac{e_1^2}{e_1 + e_2} \mu_0 \omega^2 \]
\[ \gamma(2) = -\frac{e_2^2}{e_1 + e_2} \mu_0 \omega^2 \]
\[ \lambda^2 = \frac{e_1 e_2}{e_1 + e_2} \mu_0 \omega^2. \]

For present purposes, we may consider the region \( z < 0 \) to be lossy with \( e_2 = e_w - j \sigma_w/\omega \); the medium \( z > 0 \) is the wave launching volume, and \( \epsilon_1 \) is real. For \( \epsilon_1 \leq \epsilon_2 \), it is readily seen that the propagation factors correspond to the well-known Zenneck solution which may be called a "penetrating wave" appears

\[ \gamma(1) \approx \sqrt{\epsilon_1 \mu_0 \omega} \]
\[ \gamma(2) \approx \sqrt{\epsilon_2 \mu_0 / \epsilon_1 \omega}. \]

To our knowledge, this wave type has never been discussed in the literature. It has the unusual feature that the propagation factors correspond to the well-known Zenneck surface wave [5]. For \( \epsilon_1 \gg \epsilon_2 \), however, a limiting form of the Zenneck solution which may be called a "penetrating wave" appears

\[ \gamma(1) \equiv f(\sqrt{\mu_0} \omega) \]
\[ \gamma(2) \equiv f(\sqrt{\mu_0 / \epsilon_1 \omega}). \]

In order to study the excitation of such waves we next consider a geometry in which planar interface at \( z = 0 \) exists between two media with permittivities and permeability \( \epsilon_1, \mu_1(z > 0) \) and \( \epsilon_2, \mu_2(z < 0) \). In what follows, \( \epsilon_1, \mu_1, \mu_2 \) may be regarded as real and \( \epsilon_2 \) is complex. In the upper space a y-directed magnetic current \( m_y(x, z) \) is located in \( z > 0 \). This is a simple generalization of the geometry considered by Hill and Wait [2], and this analysis closely follows theirs, except that our attention is directed at the fields in \( z < 0 \). We regard the magnetic current as a sum of magnetic line sources \( m_y(x, z) = m_y e_y (x - x' \delta(z - z')) \) of strength \( m_yz \) and, slightly modifying Wait's procedure for obtaining the Green's function, write the x-spectral transform

\[ M(\lambda, z) = \int_{-\infty}^{\infty} m_y(x, z) e^{j \lambda x} dx = m_\delta e^{j \lambda x} \delta(z - z') \]
\[ H_y(x, z) = \int_{-\infty}^{\infty} f(\lambda) [e^{j \lambda x} + R(\lambda) e^{-\gamma_1 z}] e^{-j \lambda x} d\lambda, \quad 0 < z < z' \]

Here \( \gamma_1, \gamma_2 > 0 \). Again from the wave equation \( \gamma_1^2 = \lambda^2 - k_1^2 \), \( \gamma_2^2 = \lambda^2 - k_2^2 \) where as usual, \( k_1^2 = \epsilon_1 \mu_1 \omega^2, k_2^2 \approx \epsilon_2 \mu_2 \omega^2 \). The continuity of \( H_y \) at \( z = 0 \) leads to \( g(\lambda) = f(\lambda) \). We also find \( g(\lambda) = f(\lambda) e^{j \gamma_1 z} + R(\lambda) \) and \( h(\lambda) = f(\lambda) [1 + R(\lambda)] \). Substituting and using \( E_x(0+) = (j/\epsilon_1 \omega) \partial H_y/\partial z \), we find

\[ R(\lambda) = \frac{(\gamma_1/e_1) - (\gamma_2/e_2)}{(\gamma_1/e_1) + (\gamma_2/e_2)}. \]

Finally, using

\[ E_x(x, z = z') - E_x(x, z = 0) = m_y(x, z') \]
\[ = \frac{1}{2\pi} \int_{-\infty}^{\infty} M(\lambda, z') e^{-j \lambda x} d\lambda \]

we find

\[ \frac{2j}{\epsilon_1 \omega} \int_{-\infty}^{\infty} \gamma_1 f(\lambda) e^{-j \lambda x} \gamma_1 z' \lambda = \frac{1}{2\pi} \int_{-\infty}^{\infty} M(\lambda z') e^{-j \lambda x} d\lambda \]

whence

\[ f(\lambda) = \frac{j \epsilon_1 \omega}{4\pi \gamma_1} M(\lambda, z') e^{-\gamma_1 z'}. \]

Combining we obtain for \( z < 0 \) the Sommerfeld integral

\[ H_y(x, z) = -\frac{j \omega}{2\pi} m_y e_x \int_{-\infty}^{\infty} \gamma_1^{-1} \left[ e^{-\gamma_1 (z - z')} \right] \left( \frac{\gamma_1}{\epsilon_1} \epsilon_1 \gamma_2 \gamma_2' \lambda \epsilon_2 \gamma_1 \epsilon_1 \epsilon_2 \gamma_1' \epsilon_2' \lambda \epsilon_2' \gamma_1' \epsilon_2' \lambda \right) e^{-j \lambda (x - z')} d\lambda. \]

For an aperture \( m_y = m_y(x', z') \) we must carry out an integration over \( x', z' \). For an infinite vertical aperture with \( z \)-variation corresponding to that of the \( z \)-variation of the Zenneck surface wave, Hill and Wait [2] have shown that by considering the problem of the field above an impedance plane, we are led to a pure Zenneck wave in the region \( z > 0 \); a finite aperture excites a wave which is a Zenneck-like in the near field but becomes the usual ground wave at large distances, decaying as \( x^{-3/2} \). The "penetrating" wave in \( z < 0 \) is due to the contribution from the Sommerfeld pole in the denominator of the integrand (in the case \( \epsilon_1 \gg \epsilon_2 \) which yields the Zenneck wave. With this example in mind, we consider another configuration for launching the wave.

A more realistic configuration for a generating mechanism is a magnetic ring source, since it may be regarded as a simple model for an excited circumferential slot on a cylinder, or for a circular stripline patch antenna. We therefore consider a ring source of radius \( \rho' \) at height \( z' \) above the planar interface. The magnetic
current density is

\[ m_\phi(\rho, z) = m_\theta(\rho - \rho') \delta(z - z') \].

This problem has been discussed by Wait [5]. Again, with a slight modification in order to obtain the field in the lossy medium \( z < 0 \), and using azimuthal symmetry we write, paralleling the previous development

\[ H_\phi(\rho, z) = \int_0^\infty f(\lambda)[e^{\gamma_1 z} - R(\lambda)e^{-\gamma_2 z}] J_1(\lambda \rho) \lambda d\lambda, \quad 0 < z < z' \]

\[ H_\phi(\rho, z) = \int_0^\infty f(\lambda) e^{-\gamma_2 z} J_1(\lambda \rho) \lambda d\rho, \quad z > z' \]

\[ H_\phi(\rho, z) = \int_0^\infty h(\lambda) e^{\gamma_2 z} J_1(\lambda \rho) \lambda d\rho, \quad z < 0 \).

We have again \( \text{Re} \{\gamma_1 \} > 0 \), \( \lambda^2 = \kappa^2 - k_1^2 \), \( \gamma_2^2 = \lambda^2 - k_2^2 \). Also, the continuity of \( H_\phi \) at \( z = 0, z' \) again leads to \( g(\lambda) = f(\lambda)[e^{\gamma_2 z} + R(\lambda)] \) and \( h(\lambda) = f(\lambda)[1 + R(\lambda)] \). Now using

\[ E_\rho(\rho, z = 0^+) = (j/e_1 \omega) \left( \frac{\partial H_\theta}{\partial z} \right)_{z=0} \]

\[ = E_\rho(\rho, z = 0^-) = (j/e_2 \omega) \left( \frac{\partial H_\theta}{\partial z} \right)_{z=0} \]

we again find \( R(\lambda) = [(\gamma_1/e_1) - (\gamma_2/e_2)] / [(\gamma_1/e_1) + (\gamma_2/e_2)] \). The discontinuity in \( E_\rho \) across the \( z = z' \) plane leads to

\[ (j/e_1 \omega) \left[ \left( \frac{\partial H_\theta}{\partial z} \right)_{z=z'} - \left( \frac{\partial H_\theta}{\partial z} \right)_{z=z'+0} \right] \]

\[ = m_\phi \delta(\rho - \rho') = m_\phi \rho' \int_0^\infty J_1(\lambda \rho) J_1(\lambda \rho') \lambda d\lambda. \]

Combining these results

\[ (j/e_1 \omega) \int_0^\infty \left\{ -[e^{2\gamma_1 z'} + R(\lambda)] e^{-\gamma_1 z'} - [e^{\gamma_2 z'} - R(\lambda)e^{-\gamma_2 z'}] \gamma_1 f(\lambda) J_1(\lambda \rho) \lambda d\lambda \]

\[ = m_\phi \rho' \int_0^\infty J_1(\lambda \rho) J_1(\lambda \rho') \lambda d\lambda. \]

Thus

\[ f(\lambda) = \frac{j e_1 \omega}{2\gamma_1} e^{-\gamma_1 z'} m_\phi \rho' J_1(\lambda \rho') \]

and the field in the lossy medium \( z < 0 \) is

\[ H_\phi(\rho, z) = j \omega m_\phi \rho' \int_0^\infty e^{-\gamma_1 z'} e^{\gamma_2 z'} J_1(\lambda \rho) J_1(\lambda \rho') \lambda d\lambda \]

\[ = [(\gamma_1/e_1) + (\gamma_2/e_2)] \]

For a radial distribution of magnetic current \( m_\phi(\rho') \), the field is obtained by integration over \( \rho' \). For present purposes we shall take \( m_\phi = m_\phi \rho', \ a < \rho' < b \), at \( z' = 0 \). This distribution is chosen as a model for a "coaxial" excitation. We obtain

\[ H_\phi(\rho, z) \]

\[ = \frac{j \omega m_\phi}{\gamma_1/e_1 + (\gamma_2/e_2)} \{ J_0(\lambda \rho) - J_0(\lambda a) \} \lambda d\lambda. \]

The electric field component \( E_z \) in the region \( z < 0 \) is obtained from

\[ E_z(\rho, z) = (j/\omega e_2 \rho) \frac{\partial}{\partial \rho} (\rho H_\theta) \]

\[ = \frac{m_\phi}{e_2} \int_0^\infty \frac{e^{\gamma_2 z} J_0(\lambda \rho)[J_0(\lambda b) - J_0(\lambda a)]}{(\gamma_1/e_1) + (\gamma_2/e_2)} \lambda d\lambda. \]

We can consider only the on-axis field behavior to study the penetration distance

\[ E_z(0, z) = \frac{m_\phi}{e_2} \int_0^\infty \frac{e^{\gamma_2 z} \{ J_0(\lambda b) - J_0(\lambda a) \}}{(\gamma_1/e_1) + (\gamma_2/e_2)} \lambda d\lambda. \]

The reader may observe at once that

\[ \frac{dE_z(0, z)}{dz} \neq \gamma_2 E_z(0, z). \]

The penetration distance is not the plane-wave skin depth.

Using \( J_0(\xi) = H_0(1)(\xi) - H_0(1)(-\xi) \) reduces the expression to

\[ E_z(0, z) = \frac{m_\phi}{e_2} \int_0^\infty \frac{e^{\gamma_2 z} H_0(1)(\lambda b) - H_0(1)(\lambda a)}{(\gamma_1/e_1) + (\gamma_2/e_2)} \lambda d\lambda. \]

To cast the integral into one of the forms considered at length in the book by Tyras [6], many other, extensive, treatments (Ban\

\[ = \frac{e^{-j\pi/4}}{\gamma_1/e_1 + (\gamma_2/e_2)} \]

Here \( H_0^{(1)}(\lambda b) \) is the asymptotic form of the magnitude of \( H_0^{(1)}(\lambda b) \). Viz. \( H_0^{(1)}(\lambda b) \rightarrow (2/\pi b)^{1/2} \text{as} \lambda b \rightarrow \infty \). We have taken some pains to cast the integral into one of the forms considered at length in the book by Tyras [6]. Many other, extensive, treatments (Ban\

\[ = e^{-j\pi/4} \int_0^\infty \frac{e^{\gamma_2 z} H_0(1)(\lambda b)}{(\gamma_1/e_1) + (\gamma_2/e_2)} \lambda d\lambda. \]

Here \( H_0(\xi) = H_0(1)(\xi) - H_0(1)(-\xi) \) and \( \Gamma_1 \) are available. Briefly, the transformation \( \lambda = k_2 \sin \beta, b = r \sin \theta, \]

\[ z = r \cos \theta \text{ reduces } I(b) \text{ to the form} \]

\[ I(b) = \int_1^\infty \frac{e^{\gamma_2 z} H_0(1)(\lambda b)}{(\gamma_1/e_1) + (\gamma_2/e_2)} \lambda d\lambda. \]

Thus

\[ f(\lambda) = \frac{j e_1 \omega}{2\gamma_1} e^{-\gamma_1 z'} m_\phi \rho' J_1(\lambda \rho'). \]

and the path \( \Gamma_1 \) in the complex \( \beta = \beta - j\beta' \) plane is shown in the figure. We assume \( e_1 \) is real and \( \Im e_2 \). Thus \( \lambda = |\lambda| e^{j\theta} \text{ with } |\lambda| > 1 \) and \( \phi = 1/2 \tan^{-1}(a/\omega e_2) \), \( 0 \leq \phi \leq \pi/4\). Consequently the branch points in \( \beta_B = \pm \lambda \) are given, approximately by \( \beta_B = t_f \ln (2/|n| e^{j(\pi/2)|n|/2}) \) and are located at \( (\pi/2 - \phi, j \ln |n| e^{j(\pi/2)n/2}) \), \( (-\pi/2 + \phi, j \ln |n| e^{j(\pi/2)|n|/2}) \), \( (+\pi/2 - \phi, j \ln |n| e^{j(\pi/2)n/2}) \), \( (-\pi/2 + j \ln |n|) \). The poles are located at \( \sin \beta = \pm (n^2 - \sin^2 \beta)^{1/2} \). So \( \beta_B \approx \pm (n^2/2 \pm 1/n) \) and the poles are located as shown in Fig. 1.

The modified method of steepest descents was employed by

\[ m_\phi = m_\phi \rho', \ a < \rho' < b, \text{ at } z' = 0. \]
Fig. 1. Integration path in complex \( \beta \)-plane.

Fig. 2. Bolus concept.

Tyras [6] to obtain the above integral. We use this result and write

\[
I(b) = -2e_2 \frac{e^{ikr}}{r} \sum_{m=1}^{\infty} \frac{1}{n(\theta - \beta_p)} \left( \frac{1 \cdot 3 \cdot 5 \cdots (2m-1)}{[jk_r(\theta - \beta_p)^2]^{m/2}} \right).
\]

For points such that \( (z/b)^2 < 1 \), we have \( \sin \theta \approx 1, \theta - \beta_p \approx -1/n \), \( \cos \theta \approx z/b \), \( b \approx r \) and obtain, retaining only the first term in the sum,

\[
I(b) \approx \frac{2e_1}{k_r^2} e^{ikr}[1 - jk_rz/n] \approx \frac{2e_1}{k_r^2} e^{ikr} e^{-jk_zz/n}.
\]

Consequently,

\[
E_z(0, z) \approx \frac{2e_1\mu_0}{k_2e_2r^2} e^{ikr} e^{-jk_zz/n}.
\]

and the \( z \)-dependence of the field is reduced. The effective skin depth is again seen to be increased by the factor \( n = \sqrt{\varepsilon_1/\varepsilon_2} \).

III. SIGNIFICANCE OF MATHEMATICAL RESULTS

The significance of this result and also that of Hill and Wait, in the present application is that a wave of the Zenneck-type does exist in the vicinity of a finite antenna and that it can be enhanced by proper design of the exciting elements. We have shown that by proper choice of dielectric it can provide increased penetration capabilities into the lossy (tissue medium). In the models described, it is clear that some special and (admittedly inconvenient) arrangements must be applied to obtain this effect. First, a very high dielectric material must be employed in the wave launcher, and second, the launcher must be large compared with the penetration skin depth. However, high dielectric materials with \( \varepsilon \gg \varepsilon_{\text{tissue}} \) are commercially available and the antenna dimensions required do not appear unreasonably large. For medical purposes, we can easily imagine employing an applicator with dimensions large compared to the few centimeters of penetration distance required; the problems of geometry of the object to be heated can be resolved by suitable bolusing using tissue-equivalent materials. In Fig. 2 we illustrate this concept. The bolus can be cooled to reduce conduction effects. At 915 MHz, for example, the permittivity of muscle and organ tissue is \( \sim 50 + j30 \) and the electromagnetic skin depth is \( \sim 2.3 \) cm. Using waves launched in a very high permittivity material such as barium titanate, this depth could be increased to \( \sim 7 \) cm, which combined with surface cooling would permit selective heating of deep-lying organs. Finally, it may be pointed out that surface waves generated by transient excitations (Vander Pol, [3]) display behavior similar to those described here and could be employed in a similar manner.

REFERENCES


Comments on "Plane-Wave Diffraction by a Wedge—A Spectral Domain Approach"

A. MICHAELI

In the above paper, the authors manipulate the Fourier transforms of the currents flowing on the two faces of a wedge with exterior angle \( \phi \) illuminated by a plane wave normally incident from the \( \phi' \) direction (see Fig. 1). For the Fourier transform

\[
J_0(w) = \int_0^\infty dx e^{iwx} \tilde{j}_0(x)
\]

of the current \( j_0(x) \) on the face \( \phi = 0 \), they arrive at (22). 

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