The cross polarized fields and principal polarization fields are typically measured over a 360° interval in turntable angle (−180° < θ ≤ 180°) with pitch angle γ and bistatic angle β (β ≠ 0) held fixed [5]. Since the linear polarization patterns are either symmetric or antisymmetric in γ, it is sufficient to make only nonnegative (or nonpositive) pitch measurements. The cross polarized fields VH and HV are antisymmetric in γ. Therefore, for fixed β, it is sufficient to make only four measurements when γ = 0: the cross sections and phases of the principal polarization fields (HH and VV). When γ ≠ 0 VH patterns can be obtained by reflecting the HV patterns about τ = 0 (or vice versa). Therefore, for fixed β and γ (γ ≠ 0), only six measurements are necessary. Consequently, for fixed β (β ≠ 0), three sets of 360° static patterns (e.g., HH and VV for 0 ≤ γ ≤ 90° and HV for 0 < γ < 90°) are sufficient for the complete characterization of scattering from a body of revolution. However since the HH and VV 360° patterns are symmetric about τ = 0 measurement redundancy is not completely eliminated.

The measurement redundancy inherent in 360° experimental static range patterns can be avoided when making theoretical or computer calculations. For fixed β four sets of 180° (0 ≤ τ ≤ 180°) static patterns (e.g., HH and VV for 0 ≤ γ ≤ 90°; HV and VH for 0 < γ < 90°) are sufficient.

For a target whose physical shape is a body of revolution but whose electrical properties vary in roll angle ρ (modulo 2π) about the physical symmetry axis, Table II is not applicable (the roll angle is the angle between the h − z plane and an arbitrary reference plane containing the z axis which is fixed with respect to the body). Consequently unless the target has roll angle symmetries, all eight static patterns over a 360° range of roll angles are needed for the complete characterization of the target when β and |γ| are fixed. Measurement redundancy can be avoided by using the reciprocity relations of (2) in the form

\[ a_{HH}(−τ, β, γ, ρ) = a_{HH}(τ, β, γ, ρ) \]  

(9)

to obtain negative pitch patterns from positive pitch patterns (or vice versa).

### REFERENCES


### HF Ground Wave Propagation over Sea Ice for a Spherical Earth Model

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Abstract—Calculations of the field strength at 10 MHz are reported for a ground wave signal where the path is sea water covered by a uniform layer of sea ice. It is shown that a surface wave type mode may dominate the conventional ground wave modes at the shorter ranges (i.e., < 10 km). At larger ranges the field strength due to the ice layer may be seriously degraded.

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There have been numerous reports of anomalous radio wave propagation over ice-covered seas at high latitudes. Only one study at medium frequency has been documented [1]. The actual electrical properties of sea ice, however, have been investigated [2]–[4], and there is evidence that the ice itself is anisotropic [5]. The actual situation is obviously rather complex because the lateral variations of the ice packs, including intervals of open sea, require an elaborate theory if all effects are to be included. A basic aspect, however, is to understand the transmission characteristics of a uniform layer of sea ice. Then we should be in the position to better cope with the mixed path aspect. This is the purpose of the present communication.

We adopt an idealized spherical earth model. The normal atmospheric refraction is accounted for by taking the effective earth’s radius to be 4/3 times the actual radius [6]. The source of the vertically polarized signals is a vertical electric dipole that is effectively at zero elevation. The interpathing is a uniform ice layer of thickness \( h \) overlaying sea water of conductivity \( \sigma_w \) and permittivity \( \varepsilon_w \).

Two sea-ice models are considered. In the first case the ice is taken to be isotropic in its electrical properties with conductivity \( \sigma_i \) and permittivity \( \varepsilon_i \). To simplify matters, we describe the upper surface of the ice layer by a surface impedance \( Z \) that is given by

\[
Z \approx K_i \frac{K_w + K_i \tanh u_i h_i}{K_i + K_w \tanh u_i h_i},
\]

where

\[
K_i = u_i/(\sigma_i + i\varepsilon_i \omega), \quad u_i = (\gamma_i^2 + \lambda_s^2)^{1/2},
\]

\[
K_w = u_w/(\sigma_w + i\varepsilon_w \omega), \quad u_w = (\gamma_w^2 + \lambda_s^2)^{1/2},
\]

\[
\gamma_i^2 = i\mu_0 \omega(\sigma_i + i\varepsilon_i \omega), \quad \gamma_w^2 = i\mu_0 \omega(\sigma_w + i\varepsilon_w \omega),
\]

and

\[
\lambda_s = k + (ka/2)^{1/3} t_s/a.
\]

Here \( k = 2\pi/(\text{free-space wavelength}), a \) is the effective earth radius, and \( t_s \) are roots of the ground wave mode equation

\[
w'(t) = q w(t) = 0,
\]

where \( q = -i(ka/2)^{1/3} Z/120 \pi \) and \( w(t) = \pi^{1/2} [\text{Bi}(t) - i\text{Ai}(t)] \) in terms of Airy functions. The numerical solution of (2) for an arbitrary value of \( Z \) has been discussed previously [6], where the notation and conventions are described more fully.

The actual radial electric field strength \( E \), at the great circle distance \( d \), on the sea-surface is written in the form

\[
E = E_0 W,
\]

where \( E_0 \) is the electric field of the same dipole (i.e., same current moment) located on a flat perfect conductor and \( W \) is the attenuation function [6]. The "residue series" representation for the latter is

\[
W = \left( \frac{\pi x}{i} \right)^{1/2} \sum_{s=1}^{\infty} \frac{\exp(-i\pi t_s)}{t_s - q^2},
\]

where \( x = (ka/2)^{1/3} (d/a) \). Alternate forms of \( (4) \), more convenient for shorter distances (i.e., smaller \( x \), are also available [7].

For a generally anisotropic sea layer, the formulation is a great deal more complicated because azimuthal symmetry would no longer prevail. However, often the orientation of the brine pockets in the sea ice is vertical, in which case symmetry holds. Furthermore, because of the relatively high conductivity of the brine inclusion in the form of vertical stringers, the wave normal within the sea is approximately vertical. In effect, this means that surface impedance is essentially the same as for a normally incident wave. Clearly, the appropriate form of the surface impedance \( Z \) to use is given by (1) when \( \lambda_s \) is set equal to zero. A more general anisotropic model would allow for the finite ratio of the vertical to the horizontal conductivity of the sea ice using an earlier formulation [8]. Here we will just consider the two extreme cases, which will be dubbed the isotropic and the anisotropic model.

To consider a concrete case, we chose the following typical values of the parameters: frequency, \( \omega/2\pi = 10 \text{ MHz} \); sea-water conductivity, \( \sigma_w = 1 \text{ mho/m} \); sea-water dielectric constant, \( \varepsilon_w/\varepsilon_0 = 80 \); sea-ice permittivity, \( \varepsilon_i/\varepsilon_0 = 6 \); sea-ice loss tangent, \( \sigma_i/\varepsilon_i \omega = 0.1 \). For such cases the surface impedance \( Z \) for the isotropic model is approximately independent of the mode number \( s \). In fact, we can replace \( \lambda_s \) in the expression (1) for \( Z \) by \( k \). To give some idea of the magnitude of the quantities involved, we list in Table I the complex values of the first two roots, \( t_1 \) and \( t_2 \), for various values of ice thickness. The corresponding values for perfectly conducting sea ice (i.e., \( \sigma_w = \infty \)) are also shown. Of course the limit \( h_i \to 0 \) corresponds to having an ice-free surface. In that case, the two dominant modes have the expected "creeping wave" character when the phase angles of \( \gamma_i \) and \( \gamma_w \) are near \(-\pi/3\) rad. However, for nonzero values of \( h_i \), the dominant mode has a "surface wave" character; it has a lower attenuation and the phase velocity is considerably reduced. The other roots are of the creeping wave type. The same remarks apply to the results shown in Table II for the anisotropic model.

For the same parameter values, the attenuation function \( W \), in magnitude and phase, is shown in Figs. 1 and 2, respectively, for ranges \( d \) varying from 100 m to 100 km. The values of the sea-ice thickness \( h_i \) are indicated. The curves for the nonzero \( h_i \) values exhibit the expected interference between the surface wave and the creeping wave modes. At the shortest ranges the surface wave dominates, while at the larger ranges the creeping waves dominate. In the intermediate distance range the behavior is rather complicated. To obtain results such as illustrated in Figs. 1 and 2, as many as 50 terms in the residue series representation are required in order that the curves blend into the curvature-corrected flat earth formulas at the shorter ranges [7].

In general we see that the existence of a uniform sea-ice layer over the propagation path may either enhance or degrade the field strength. The enhancement is usually associated with the thinner ice layers at the shorter ranges. The general situation is not changed appreciably for other values of the parameters. For example, a higher sea-water conductivity (e.g., \( \sigma_w = 4 \text{ mho/m} \)) would extend the field strength enhancement to greater ranges, but the general character of the curves would not be changed. Also, the calculations for higher loss tangents (e.g., 0.5) of the sea ice do not change the results in any signifi-
TABLE I
ISOTROPIC SEA-ICE MODEL

| $\sigma_w$ (mho/m) | $h_i$ (m) | $|\Delta|$ | Phase ($\Delta$) (degree) | $t_1$ | $t_2$ |
|-------------------|----------|----------|--------------------------|------|------|
| $\infty$         | 0        | 0        | 0.0                      | 0.509 - 10.882 | 1.624 - 12.813 |
| 0.25             | 0.0439   | 88.84    | 17.90                    | $t_1$ | 1.412 - 12.030 |
| 0.5              | 0.0890   | 88.74    | 73.18                    | $t_2$ | 1.287 - 12.023 |
| 1.0              | 0.1887   | 88.31    | 328.6                    | 1.194 - 12.023 |
| 2.0              | 0.5060   | 85.36    | 239.9                    | 1.190 - 12.023 |
| 0                | 0.0236   | 43.71    | 1.460                    | 1.224 - 13.077 |
| 0.25             | 0.0630   | 73.07    | 30.59                    | 1.331 - 11.977 |
| 0.5              | 0.1079   | 79.08    | 100.0                    | 1.264 - 12.007 |
| 1                | 0.2101   | 82.21    | 393.4                    | 1.218 - 12.018 |
| 1                | 0.5493   | 79.58    | 2608.0 - 1993.0          | 1.118 - 12.021 |

Note: $\Delta = Z/120 \pi$ is the normalized surface impedance.

TABLE II
ANISOTROPIC SEA-ICE MODEL

| $\sigma_w$ (mho/m) | $h_i$ (m) | $|\Delta|$ | Phase ($\Delta$) (degree) | $t_1$ | $t_2$ |
|-------------------|----------|----------|--------------------------|------|------|
| $\infty$         | 0        | 0        | 0.0                      | 0.509 - 10.882 | 1.624 - 12.813 |
| 0.25             | 0.0526   | 89.84    | 25.66                    | 0.509 - 10.141 | 1.370 - 12.030 |
| 0.5              | 0.1069   | 89.35    | 105.7                    | 105.7 - 12.391 | 1.267 - 12.024 |
| 1                | 0.2293   | 87.14    | 483.8                    | 483.7 - 14.838 | 1.214 - 12.023 |
| 1                | 0.6316   | 72.07    | 2989.0                   | 2989.0 - 12.611 | 1.185 - 12.020 |
| 0.25             | 0.0236   | 43.71    | 1.460                    | 1.460 - 11.630 | 2.294 - 13.077 |
| 0.5              | 0.1256   | 81.33    | 413.3                    | 413.3 - 12.339 | 1.313 - 11.990 |
| 1                | 0.2516   | 83.99    | 572.5                    | 572.5 - 11.219 | 1.210 - 12.021 |
| 1                | 0.7252   | 80.14    | 4578.0                   | 4578.0 - 11.641 | 1.183 - 12.022 |

Note: $\Delta = Z/120 \pi$ is the normalized surface impedance.

significant fashion, although the field strength enhancements are then slightly weakened. We would also expect some further modifications if the vertical nonuniformity and the sea-ice thickness variations were accounted for.

The results shown here are a small example of extensive calculations of ground wave field strength for nonstandard paths such as may occur at high latitudes. The inevitable mixed path conditions such as exist at coast lines have also been included in the general formulation, and computer programs have been written that encompass these more complex situations. The results of this research, supported by the Rome Air Development Center (USAF), will be reported elsewhere.

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