EM Scattering by an Array of Perfectly Conducting Strips by a Physical Optics Approximation

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Abstract—The scattering of electromagnetic waves by planar arrays of perfectly conducting strips is analyzed by a simple method based on physical optics. The induced current as determined by physical optics is used in simple hand computation to obtain the amplitudes of various propagating space harmonics. Results are compared against some exact results available in the literature to show the accuracy of the proposed approximate method.

I. INTRODUCTION

The diffraction of electromagnetic waves by a planar array of perfectly conducting strips has been of great interest, and a number of rigorous, semirigorous, and numerical solution techniques have been developed for the analysis of the problem [11], [21]. These methods are normally mathematically involved and require an extensive programming effort and the use of a digital computer. The object of this communication is to present a simple approximate method based on physical optics which only requires very little hand computation. In this method a periodic Green’s function is first found for the problem [2]. The induced surface current density is taken as twice the magnetic field intensity of the incident wave. A simple integration operation which is easily performed by hand computation yields the amplitudes of various waves. Some typical results obtained are compared against rigorous results available in the literature [11], which indicate the accuracy of the present approximate technique.

II. METHOD OF ANALYSIS AND RESULTS

The structure under consideration is a periodic array of perfectly conducting strips. The period of the structure is \( d \), the spacing between the strips is \( a \), and the width of each strip is equal to \( b \). The incident wave is \( E \)-polarized as shown in Fig. 1. The electric field of the incident wave after suppression of the \( \exp(j\omega t) \) time variation takes the form

\[
E_i = \gamma_0 \exp(-j\beta_0 x + j\gamma_0 z)
\]

where \( \beta_0 = k \sin \theta_i \), \( \gamma_0 = k \cos \theta_i \), \( k \) is the wavenumber, and \( \theta_i \) is the incidence angle.

The electric fields of the reflected and the transmitted waves will have a \( y \) component only and the problem is, therefore, treated as a scalar problem (hereafter we write \( E_y \) as \( \phi \)). To solve the problem a periodic Green’s function, which is the electric field due to an infinite periodic array of line currents on the \( x \) axis, is sought. The currents flow along the \( y \) axis, and its density is approximated using physical optics as twice the tangential component of the incident magnetic field:

\[
J_s(x_0) = \frac{2}{120\pi} \cos \theta_i \exp(-j\beta_0 x_0).
\]

Substituting (3) into (4) we obtain

\[
\phi_{\text{scat}}(x, z) = \int_{-b/2}^{b/2} J_s(x_0) G_p(x, z, x_0) dx_0.
\]

In view of the periodic nature of the problem, the scattered field can be alternatively expressed in terms of space harmonics, namely,

\[
\phi_{\text{scat}}(x, z) = \sum_{n=-\infty}^{\infty} \left\{ \begin{array}{l}
A_n e^{-j\beta_n x - j\gamma_n |z|}, \quad \text{for } z \geq 0 \\
B_n e^{-j\beta_n x + j\gamma_n |z|}, \quad \text{for } z < 0
\end{array} \right. + \sum_{n=-\infty}^{\infty} \left\{ \begin{array}{l}
A_n e^{j\beta_n x + j\gamma_n |z|}, \quad \text{for } z \geq 0 \\
B_n e^{j\beta_n x - j\gamma_n |z|}, \quad \text{for } z < 0
\end{array} \right.
\]
where \( \beta_n = \beta_0 + (2\pi n/d) \), and \( \{ A_n, B_n \} \) are the unknown amplitudes to be determined. Equating (5a) and (5b) term-wise we get

\[
B_n = \frac{2\pi}{d\gamma_n} \int_{-b/2}^{+b/2} \exp(\beta_n x_0) \, dx_0
\]
or

\[
B_n = \frac{2\pi}{d\gamma_n} \frac{\exp(\beta_n b/2) - \exp(-\beta_n b/2)}{j\beta_n} \cos \theta_i. \tag{6}
\]

The reflected wave amplitudes \( A_n \) follow from the relations

\[
A_n = B_n, \quad \text{for } n \neq 0
\]

\[
A_0 = -1 + B_0. \tag{7}
\]

Notice the simplicity of the final solution presented in (6) and (7).

Formulation for the \( H \)-polarized problem is obtained by a similar mathematical procedure. Instead of an infinite array of electric line currents, we take in this case an infinite array of magnetic line currents as the source of the Green’s function. The surface magnetic current in the opening of the reference cell \(-a/2 < x < a/2\) with the \( z \) axis shifted to the center of the opening is taken as twice the tangential component of the incident electric field intensity. The final result for the magnetic field amplitudes of the transmitted waves \( B_n \) are as follows:

\[
B_n = \frac{2\pi}{d\gamma_n} \frac{\exp(j\beta_n a/2) - \exp(-j\beta_n a/2)}{j\beta_n} \cos \theta_i. \tag{8}
\]

Comparison of (6) and (8) indicates that for the self-complementary structure \( a = b = d/2 \) the solutions are identical.

A self-complementary strip grating \( a = b = d/2 \) was considered at normal incidence for different periods. This structure was previously analyzed exactly by Baldwin and Heins [1]. Their results for the amplitudes of the waves of orders zero and one, together with the results based on the present method, are shown in Table I. Both magnitudes and phases in degrees are compared and indicate good agreement. The exact magnitude of the order zero wave has an oscillatory behavior which settles to the physical optics value of 0.5 as the structure period is made larger. The phase of the order zero wave goes similarly through oscillations with peaks occurring at periods which are whole multiples of the wavelength at which strong resonances take place. The magnitude and phase of the order one wave agree much more favorably with the physical optics values as the structure period is increased.

### REFERENCES


### Measurement of the Microwave Structure Constant Profile

**MOODY C. THOMPSON, JR., F. E. MARLER, AND K. C. ALLEN**

**Abstract**—Measurements of the microstructure of refractivity at 9.4 GHz were made in Colorado and Florida up to altitudes of about 29 000 ft. The structure function parameter \( C_n^2 \) was calculated from these data. Examples of the resulting profiles are presented with corresponding profiles of refractivity and temperature. Values of \( C_n^2 \) varied from about \( 10^{-13} \) to \( 10^{-17} \) m \(^{-2/3}\). The heights of occurrence of maximum and minimum values are summarized.

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