The rms error \( \Delta \sigma \) of the deviations \( \sigma \) are for the suggested two-feed method the same as for the method of Scott and Ryle [6] who derive

\[
\Delta \sigma = \frac{n \lambda}{4\pi R},
\]

(21)

if \( n^2 \) surface points are to be measured and where \( \lambda \) is the wavelength, \( R \) is the signal-to-noise ratio at the reference feed for an integration time \( t \) equal to the duration of a scan sample, or \( 1/n^2 \) of the total observing time for a given telescope pointing which is limited to one or two hours because of the changing gravitational deformations. The signal/noise is proportional to

\[
R \sim \frac{S\eta/\sqrt{2t}}{T},
\]

(22)

where \( S \) is the flux of the source, \( \eta \sim \exp\left[-(4\pi \sigma_0/\lambda)^2\right] \) is the aperture efficiency, \( b \) is the bandwidth, \( r \sim n^{-2} \) is the sample integration time, and \( T \) is the system noise temperature. Thus

\[
\Delta \sigma \sim n^2 \frac{7\lambda}{b} e^{(4\pi \sigma_0/\lambda)^2},
\]

(23)

where \( \sigma_0 = \text{rms } \sigma \).

We see that \( \Delta \sigma \sim n^2 \) for a limited total time. The best wavelength \( \lambda \) should be chosen such that (23) is minimized, which depends on the available receivers (\( T \) and \( b \)) and radio sources (\( S \)). For many cases, observations at the strong waterline may be best, at \( \lambda = 1.35 \text{ cm} \).

In case of continuum observations, we use \( S \sim v^{-k} \sim \lambda^k \), where \( v \) is the frequency and \( k \) the spectrum index; we shall assume \( b \sim v \sim \lambda^{-1} \) and \( T = \text{constant for simplicity}. \) Then

\[
\Delta \sigma \sim n^2 \lambda^{1.5-k} e^{(4\pi \sigma_0/\lambda)^2}
\]

(24)

which has its minimum at

\[
\lambda = \frac{8\pi}{\sqrt{3-2k}} \sigma_0.
\]

(25)

For example, \( \lambda = 14\sigma_0 \) for \( k = 0 \) (flat spectrum), and \( \lambda = 21\sigma_0 \) for \( k = 0.8 \) (normal spectrum). We see that the best wavelength is about the same as the shortest wavelength of observation, usually adopted as \( 16\sigma_0 \), where the efficiency is degraded by a factor of two.

V. NOTES REGARDING APPLICATION

Up to now we considered feed 1 as staying always at the focus. However, then the finite feed sizes prevent feed 2 from scanning the important center region of the beam. Thus we need two locations for feed 1; the second location so far off axis that feed 2 now can scan whatever region it missed in the first case. Conversion from one case to the other can be done if the second location of feed 1 had formerly been occupied by feed 2; then the phase difference and the amplitude ratio between the two locations are both known.

For the feed illumination pattern \( I(\theta, \phi) \), we have neglected any phase errors which actually may amount up to \( 20^\circ \text{ to } 30^\circ \). They should be measured before the feed goes on the telescope and can then be corrected. Mutual coupling between the two feeds was estimated to be about 40 dB down and was regarded negligible.

Maintaining phase calibration as one of the feeds is moved could impose a problem. If so, one should mount a transmitter with a constant-wave signal at the telescope vertex.

ACKNOWLEDGMENT

It is a pleasure to thank R. Fisher for valuable help.

NOTE ADDED IN PROOF

The possibility of a two-feed was already mentioned by Shenton and Hills in "A Proposal for a United Kingdom Millimeter Astronomy Facility," Science Research Council, November, 1976.

REFERENCES


Microwave Delay Characteristics of Cassegrainian Antennas

ALAN G. CHA, MEMBER, IEEE, W. V. T. RUSCH, FELLOW, IEEE, AND TOM Y. OTOSHI, SENIOR MEMBER, IEEE

Abstract—An approach is presented in which the time an RF signal is delayed in propagating through a Cassegrainian antenna can be determined. The approach is based on the recognition that both the group delay (time) and the envelope delay (time) can be obtained from the antenna transfer function. Two antenna transfer functions are derived. The delay time under various antenna operating conditions is discussed.

I. INTRODUCTION

Cassegrainian reflector antennas are used as the ground station terminal in most space communication systems. At some stations the same antenna is also used in a microwave ranging system to determine the distance of the spacecraft.

Manuscript received October 11, 1977; revised July 3, 1978. This work was supported by the National Aeronautics and Space Administration under contract NAS 7-100.

A. G. Cha and T. Y. Otoshi are with the Jet Propulsion Laboratory, Pasadena, CA 91103.

W. V. T. Rusch is with the Department of Electrical Engineering, University of Southern California, Los Angeles, CA 90007.
from the station. In the latter application, the signal delay time through the antenna system must be properly accounted for in order to accurately determine the range of the spacecraft [1]. Accurate determination of the signal delay time through the antenna system is also critical to the success of very long baseline interferometer (VLBI) systems useful in many disciplines of science [2].

There are two common concepts of the signal delay time: those of the group delay (time) and the envelope delay (time). All RF signals have a finite bandwidth and can be considered to be a wave packet consisting of different frequency components. The group delay is the time it takes a particular frequency component to propagate through a microwave device or medium. The envelope delay is the time it takes the envelope of the RF signal to propagate through the device or medium. The domain of application of each concept is largely dictated by the propagating medium or device. Generally speaking, an envelope delay time can be defined when the signal waveform emerging from the propagating medium or device is nearly identical to that entering the medium or device. If the two waveforms are identical, the propagating medium or device is said to be distortionless. Examples of distortionless media are the free space and TEM transmission lines. Signal distortion in propagating through a medium or device arises if either the amplitude of each frequency component is unevenly attenuated (or amplified) or the group delay time varies with each frequency component. In the latter case, the medium or device is said to be dispersive. A common dispersive device is a piece of waveguide. The group delay time assumes special significances and is often discussed in the context of a narrow-band signal propagating in a dispersive medium. In this case the group delay time has been shown to be also the envelope delay time of the RF signal as well as the delay time of the propagation of electromagnetic energy [3]. In practice there are a large number of microwave ranging and VLBI systems operating on narrow-band signals. For these systems the group delay time is the only delay time that needs to be considered. For systems operating on wider band signals, it should be recognized that the two delay concepts are not equivalent. Furthermore, the envelope delay is not always clearly definable but is dependent on the condition that the output waveform is not significantly distorted from the input waveform of the antenna. Under this condition the envelope delay can be established by comparing the time dependence of the input and output signals.

A theoretical approach to determine both the signal group delay and envelope delay through the Cassegrainian antenna is presented herein. In this approach an antenna transfer function is first derived. From the transfer function, the group delay can be computed from a differentiation operation. Determination of the envelope delay requires first determining the time-dependent waveform of the RF signal leaving the antenna. This is accomplished by performing a Fourier transform using the transfer function and the (frequency domain) input signal. The envelope delay can then be determined by comparing the time dependence of the RF signal leaving the antenna with that entering the antenna.

With the understanding of the relations between the antenna delay characteristics and its transfer function, the problem of finding the group delay time through the antenna becomes largely one of finding the antenna transfer function. Since the accuracy of the theoretically determined delay time is totally dependent on the adequacy of the antenna transfer function, two different methods are used to derive the transfer function. Numerical computations of the group delay using each transfer function are presented. It is seen that the Cassegrainian antenna is essentially a nondispersive device over a narrow band when only the primary path (nonmultipath) signal is considered.

II. THE RANGING PROBLEM

The ranging problem can be explained using the Cassegrainian antenna in Fig. 1 as an example. For the sake of simplicity in explanation, it is assumed that the antenna has separate elevation-axis and azimuth-axis drive mechanisms. It is also assumed that the two axes cross at point A, which can be taken as the antenna center of rotation. Let H be the phase center of the feed horn, F1 and F2 be the two foci of the hyperboloidal subreflector, and F be the focus of the paraboloid. The operating principle of the Cassegrainian antenna requires that the phase center H of the feed horn be placed at F1 and that the two points F2 and F coincide. For a large practical reflector antenna, these conditions are sometimes violated as the surface of the reflector deforms under changing gravity load conditions. In this communication axial displacement of the feed and the subreflector from their proper locations will be allowed. Let us assume that the antenna is tracking a point target B in the far field of the antenna. The ranging problem is to determine the distance of the point target B from a stationary reference point on the ground. Without loss of generality, the antenna center of rotation A can be taken as the stationary reference point on the ground, and the distance R in Fig. 1(a) is then the range of the target to be determined. Assuming the time $t_{AB}$ it takes the signal to travel from point A to point B has been determined experimentally, we must deduct from $t_{AB}$ the time that the signal is delayed in going through the waveguide (between points A and H) and the antenna system and then translate the balance into the range R.

Both the envelope delay (time) and the group delay (time) will be examined. It is noted that an unambiguous determination of the envelope delay is possible only if the waveform of the signal $E_B$ at the point B is identical to that of the signal $E_A$ at the point A. This situation is shown in Fig. 1(b). In the event that $E_B$ differs from $E_A$ in shape as is shown in Fig. 1(c), the envelope delay can at best be approximately defined. For example, one might arbitrarily define the envelope delay as the time between the peak of $E_A$ and the peak of $E_B$.

III. THE ANTENNA TRANSFER FUNCTION

We will now derive the transfer function of the Cassegrainian antenna. To this end the scattered field from the subreflector and the radiation field from the main reflector would have to be worked out. The transfer function is dependent on the technique one uses to calculate the diffraction from the two reflectors. In this communication the main reflector radiation is computed from physical optics (PO) while the subreflector diffraction can be computed from either geometric optics (GO) or geometric theory of diffraction (GTD). The latter technique has been demonstrated to yield results comparable to those computed from physical optics in the context of far-field approximations [4].

A time dependent of $e^{i\omega t}$ will be assumed in this communication. In addition, we define the following symbols.

- $f_r$ Frequency.
- $\omega$ Angular frequency.
- $k$ Free space wavenumber.
- $\mu_0$ Free space permeability.
- $\varepsilon_0$ Free space permittivity.
- $\lambda$ Wavelength.
Using the coordinates in Fig. 2, the feed radiation is assumed to be

\[ E_F = E_{in} e^{-j\beta l} \frac{e^{-jkR_F}}{R_F} \]

where \( a_1(\theta_F) \) and \( d_1(\theta_F) \) are two scalar functions determining the feed illumination taper in the planes \( \phi = 90^\circ \) and \( \phi = 0^\circ \), respectively, and \( \theta_F \) and \( \phi_F \) are unit vectors in \( \theta_F \) and \( \phi_F \), respectively. The real constant \( E_{in} \) is to be determined from the condition that the total radiated power from the feed is equal to the input power at point \( A \). The factor \( e^{-jkR} \) gives the phase delay due to the waveguide run from the time reference point \( A \) to the feed phase center \( H \), where \( \beta \) and \( l \) are the propagation constant and the physical length of the waveguide, respectively.

In the following derivation of the antenna transfer function, we will restrict ourselves to the case where the point target \( B \) in Fig. 1 is on the z-axis. In this case the unit radial vector \( \hat{R} \) is identical to the unit vector \( \hat{z} \). It is further assumed that in (1)

\[ a_1(\theta_F) = d_1(\theta_F) = \cos N \theta_F, \quad N \geq 0 \]  

The physical meaning of (1) and (2) is that the feed has a symmetric tapered illumination pattern and is polarized in \( y \). Under these conditions the \( y \) component of the far field at the point \( B \) in Fig. 1 represents the output field \( E_B \). Following the procedures outlined above for obtaining the subreflector scattered field and the antenna far field, it can be shown that

\[ E_{sub} = \frac{E_{in} e^{-j\beta l} e^{-jkR_F}}{R_F} \frac{a_1 \text{GTD}(\psi) \sin \psi + 2 \text{GTD}(\psi) \cos \psi}{\rho} \]

where \( a_1 \text{GTD}(\psi) \) and \( d_1 \text{GTD}(\psi) \) are scalar functions determining the subreflector illumination pattern. The functions \( a_1 \text{GTD}(\psi) \) and \( d_1 \text{GTD}(\psi) \) can be readily found by applying GTD on the subreflector [5]. Alternatively, (4) can be written as

\[ E_B = S(\omega) E_{in} \]  

The function \( S(\omega) \) is clearly the transfer function between the input field \( E_{in} \) and the output field \( E_B \). Rigorously, \( S(\omega) \) should be defined between \( E_A \) and \( E_B \). However, it is noted that \( E_A \) and \( E_{in} \) can only differ by a real constant as required by the law of energy conservation. Defining the transfer function \( S(\omega) \) by (5) thus will not affect the determination of range. The function \( S(\omega) \) in (5) can be written as

\[ S(\omega) = S_{WG}(\omega) \cdot S_{ANT}(\omega) \cdot S_{BIAS}(\omega) \cdot S_{FS}(\omega) \]  

where

\[ S_{WG} = e^{-j\beta l} \]  

\[ S_{ANT} = -j\pi kD \cot (\Psi/2) e^{-j2l} \]  

\[ \cdot \int_0^{\Psi} \left( a_1 \text{GTD} - d_1 \text{GTD} \right) \tan \frac{\psi}{2} d\psi \]  

\[ S_{BIAS} = e^{j2l} \]  

\[ S_{FS} = e^{-j2l} \frac{R_s}{4\pi R} \]  

The breakdown of \( S(\omega) \) into four transfer functions may appear arbitrary at this point, but it allows us to divide the total delay time into three parts attributable to the waveguide, the antenna, and the free space, respectively. This will become obvious in the next section. We will now consider an important special case where the subreflector scattered field is given by \( \Psi \) and there is no defocusing in the feed and subreflector positions. In this case it can be shown that (8) reduces to

\[ S_{ANT} = -j\pi kD \cot \frac{\Psi}{2} e^{-j2l} \]  

\[ \cdot \int_0^{\Psi} \left( a_1(\theta_F) + d_1(\theta_F) \right) \frac{R_s}{R_F} \tan \frac{\psi}{2} d\psi \]  

where \( 2a \) is the distance between the two vertices of the hyper-
boloid and $R_s$ is the distance from $F_2$ to $Q$, the specular point of GO.

Note for the type of illumination assumed in (2), the integral in (8a) is a real constant independent of the frequency. The frequency dependence of the phase of $S_{\text{ANT}}$ in (8a) is thus given by the factor $e^{-j\theta(2\pi+2\alpha)}$. For the more accurate $S_{\text{ANT}}$ given in (8), the frequency dependence of the phase factor will be somewhat different from $e^{-j\theta(2\pi+2\alpha)}$ due to the diffraction from the edge of the subreflector. These observations on the frequency dependence of the phase of $S_{\text{ANT}}$ will help the reader understand the antenna delay characteristics presented in the following sections.

IV. GROUP DELAY TIME IN THE ANTENNA

Equating the phase on both sides of (6), we obtain

$$\Phi_{AB}(\omega) = -\beta l + \Phi_{\text{ANT}}(\omega) + kL - kR$$

(11)

where $\Phi_{AB}$ is the signal phase delay between points $A$ and $B$ and $\Phi_{\text{ANT}}(\omega)$ is the phase of the antenna transfer function $S_{\text{ANT}}(\omega)$. The total group delay time is given by

$$t_{\text{AB}} = -\frac{d\Phi_{AB}}{d\omega} = \frac{l}{V_g} - \frac{d\Phi_{\text{ANT}}}{d\omega} + \frac{R - L}{V_c}$$

(12)

where $V_g$ is the signal group velocity in the waveguide and $V_c$ is the velocity of light in free space. Equation (12) gives the one-way ranging equation of the antenna. The meaning of each term on the right side of (12) is clear. The first term is the group delay in the waveguide between points $A$ and $H$ in Fig. 1. The last term gives the delay time in the free space between points $F$ and $B$ in Fig. 1. Thus the second term must be the delay time in the antenna system

$$t_{\text{ANT}} = -\frac{d\Phi_{\text{ANT}}}{d\omega}$$

(13)

In the special case that the transfer function in (8a) is used to obtain $\Phi_{\text{ANT}}$, (13) reduces to the form expected from optics

$$t_{\text{ANT}} = \frac{2f + 2\alpha}{V_c}$$

(13a)

In this case the group delay in the antenna is a constant independent of the frequency and is equal to the time required of a signal to travel the optical path length of the antenna at the speed of light. It is of great interest to see whether the extremely simplified optics formula in (13a) can be used as a practical substitute for the rigorous but complicated microwave formula given in (13). This question is answered next by numerical computations.

V. NUMERICAL RESULTS

In this section we will present the numerical results on the subreflector scattered field, the transfer function, and the group delay time of a Cassegrainian antenna. The Cassegrainian antenna used in the computation has $D = 25$ m and $F/D = 0.43$. The bandwidth of computation is 0.99 $f_0$ to 1.01 $f_0$, where $f_0 = 2.295$ GHz. At the center frequency $f_0$, $D = 191\lambda$. For the subreflector, $a = 13.981\lambda$, $e = 1.545$, and $D_{\text{sub}} = 19.1\lambda$, where $e$ and $D_{\text{sub}}$ are, respectively, the eccentricity and diameter of the hyperboloid. The feed illumination is given by $\cos^{45} \theta_F$, which provides a 11.8-dB illumination taper at the subreflector edge where $\theta_F = 14^\circ$.

A. Subreflector Scattered Field

With both the feed and the subreflector at focused positions, the subreflector scattered field at frequencies 0.99 $f_0$ is shown in Fig. 3. The scattered field computed from GO is characterized by a smoothly tapered amplitude which drops to zero at the subreflector edge angle $\psi = \Psi = 61^\circ$ and by a constant phase corresponding to a spherical wave emanating from the paraboloid focus $F$. The scattered field computed from GTD shows an oscillatory variation from the GO field in both amplitude and phase. In addition, the field amplitude does not drop to zero at the subreflector edge.

B. Antenna Transfer Function

The amplitude and phase of the antenna transfer functions in (8) and (8a) have been computed and are shown in Fig. 4. The two transfer functions are labeled in Fig. 4 according to the method used in obtaining the subreflector scattered field in the derivation of each transfer function. The amplitude of the antenna transfer function is also the relative gain of the antenna over an isotropic radiator. It is noted that the gain computed includes the effect of nonuniform aperture illumination but not spillover losses. The antenna gain is slightly higher when the subreflector scattered field is computed from GO rather than the more accurate GTD. At the center frequency $f_0 = 2.295$ GHz, for example, the relative gain of the two cases are 54.84 and 54.63 dB, corresponding to 84.6 and 80.7 percent illumination efficiency, respectively. There is also a difference of about 2$^\circ$ in the phase of the transfer functions which is not noticeable in Fig. 4(b) because of the scale factor.

C. The Group Delay Time—Focused Antenna

The group delay time as computed from (13), using the transfer function in (8), is shown in Table I. Alternatively, the group delay time as computed from (13a) is 83.757 ns. Since any delay error of the magnitude of 0.01 ns (corresponding to a range error of 3 mm) is not of concern at present, the Cassegrainian antenna can be considered a nondispersive device over the 1.5 percent frequency band of interest. The small frequency variations of antenna group delay shown in Table I are due to the edge diffraction from the subreflector. It is expected that as subreflector diffraction becomes more significant (as the subreflector gets smaller in terms of wavelengths) frequency variation of antenna group delay will become more significant.

D. The Group Delay Time—Defocused Subreflector

While the optics formula in (13a) can only be applied in dealing with an ideal Cassegrainian antenna, the microwave formula in (13) remains valid when there are imperfections in the antenna system due to varying antenna operating environments. As an example, the delay time in the same antenna with a defocused subreflector has been computed using (13) and is shown in Table II. The change in delay time for small displacements of the subreflector can amount to tenths of a nanosecond. This is considered significant in many ranging and interferometer applications and should be calibrated accordingly.

VI. CONCLUSIONS

An approach has been presented in which the time an RF signal is delayed in propagating through a Cassegrainian antenna...
Fig. 3. Subreflector scattered field at $f_r = 0.99 f_0$. (a) Amplitude in $E$ plane. (b) Amplitude in $H$ plane. (c) Phase in $E$ plane. (d) Phase in $H$ plane.

Fig. 4. Transfer function of Cassegrainian antenna. (a) Amplitude. (b) Phase.
can be determined. The approach is based on the recognition that both the group delay (time) and the envelope delay (time) can be obtained from the antenna transfer function.

Numerical computations reveal two important findings. First, a focused antenna is essentially a nondispersive device whose delay time can be accurately predicted from a simple optics formula. Second, even small subreflector displacements lead to significant delay changes that require calibrations in precision applications.

The above results are obtained based on an ideal antenna model which takes into account the primary optical path propagation of the signal only. It is recognized that the signal could propagate to the far field via many secondary paths which exist by virtue of multiple reflections within the antenna. Radiation from the struts supporting the subreflector. To the extent that the secondary path signals could be ignored, the Cassagrainian antenna has been shown to be an essentially nondispersive device. At present the effects of the secondary path signals on the antenna ranging performance are not fully understood and are the subject of investigation by theoretical and experimental means [6], [7].

REFERENCES

Broad-Band VSWR Measurements of a Cassegrainian Antenna

W. M. Nunn, JR., Senior Member, IEEE, and M. Figueroa

Abstract—Broadband VSWR measurements are reported on a small F-number Cassegrainian antenna. Cylindrical variations of VSWR between 1.00 and 1.25 with a 350-MHz frequency interval between unity VSWR values were observed over the 8.5-12.4 GHz frequency range. These variations correspond to a change in the power transmission ratio of 1.231 percent between the unity and 1.25 values of VSWR.

INTRODUCTION

It is well-known [1] that the reflector of a reflector-type antenna scatters a portion of its energy back into the primary feed giving rise to standing waves in the space between the feed and reflector, and a reflection coefficient observed from the transmission line feed. Silver derived the reflection coefficient assuming that the reflector lies in the far field of a point source feed, neglecting multiple scattering between the feed and reflector. An important consequence of this feed-reflector interaction is the frequency dependence it produces, e.g., if the antenna is receiving a band of frequencies whose signal characteristics are unknown prior to reception at the detector, a correction must be made for this interaction at each frequency of interest.

BROADBAND VSWR MEASUREMENTS

An experimental study was recently made on a Cassegrainian antenna with feed-line standing-wave measurements taken at 50-MHz intervals over the 8.2-12.4 GHz frequency range. The diameter of the antenna parabolic main reflector is 4 ft and that of the hyperbolic subreflector is 6.5 in. The main reflector F-number 0.3704 lies within the recommended limits of 0.25, which minimizes spillover, and 0.42, which provides low-noise antenna performance [2]. The subreflector diameter yielding minimum blockage, according to Hannan [3], is (2Fm)(λ/k) 1/2 where Fm is the main reflector focal length (17.8 in for this antenna), k is the ratio of the effective feed-aperture diameter to its blocking diameter, and λ is the radiation wavelength. Since k is nearly unity for this antenna, the minimum blocking aperture is

$$B_{\text{min}} = \sqrt{\frac{2F_m D_m}{D_m/\lambda}} \quad (1)$$

where Dm is the main reflector diameter. From Henry [4] the gain reduction factor, G_{r,f}, due to blockage by the 6.5-in subreflector is -0.32 dB at 9.93 GHz for a B_{\text{min}} of 0.1354.

The primary feed horn has an aperture 5-in wide by 2.5-in high that exponentially tapers from standard X-band waveguide over a 5-in length. A TE_{10} waveguide mode propagates through the horn input port, and the radiated wave is polarized predominantly in the vertical direction. The horn aperture is 9.8 in from the subreflector and 5.81 in inside the main reflector; the axes of the horn, subreflector, and main reflector are coincident.

Manuscript received August 3, 1977; revised December 2, 1977. The authors are with the Department of Electrical Engineering, Florida Institute of Technology, Melbourne, FL 32901.