USING SPECTRAL CORRELATION FOR NON-COOPERATIVE RSS-BASED POSITIONING

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ABSTRACT

Typically, methods for received signal strength (RSS) based source localization assume that the signal can be separated from the noise and interference. However, in non-cooperative applications where the details of the modulation format are not known a priori, this is not possible. In a high SNR environment, the RSS of the signal can be estimated from the power spectral density (PSD) via energy detection, but the PSD becomes dominated by noise at low SNR, leading to a loss of positioning information. This paper proposes the use of the spectral correlation function (SCF) for RSS-based localization, as the SCF has been shown to outperform simple energy detection for detecting the presence of a signal.

Index Terms—Source localization, received signal strength, spectral correlation function, noise floor

1. INTRODUCTION

In source localization (also called geolocation), a Wireless Sensor Network (WSN) is used to locate the source of an Radio Frequency (RF) transmission [1], [2], [3]. In Cognitive Radio (CR), geolocation can assist in finding the spatial RF footprint of primary users, so that they may be avoided. In military and law enforcement applications, geolocation may be used to find the source of jamming or illegal interference. In these types of applications, many properties of the emitter are unknown, requiring a non-cooperative method of geolocation.

Geolocation may be accomplished through Angle of Arrival (AOA), Received Signal Strength (RSS), Time of Arrival (TOA), and/or Time Difference of Arrival (TDOA) measurements. Though each measurement type has its own merits, this paper focuses on RSS, since RSS sensors are cheap, enabling a large, dense network of sensors that are individually expendable. In a non-cooperative scenario, the RSS may be determined by integrating the observed Power Spectral Density (PSD). However, the observed PSD is dominated by noise at low Signal to Noise Ratio (SNR) values. Thus, if the signal is low-power or far from a given receiver, the PSD may contain little information about the location of the emitter.

Integrating the PSD is a form of energy detection. Energy detection is simple, but has been shown to perform poorly for signal detection in CR applications [4], [5], [6]. In contrast, the Spectral Correlation Function (SCF) [7], [8], [9] has been shown to perform well for signal detection and modulation recognition, even at low SNR [4], [5], [6], [10]. Thus, a natural extension is to use the SCF for signal geolocation. To the best of our knowledge, no one has ever used the SCF for geolocation. The nearest case is [11], wherein the SCF was used to provide supplemental information for signal selectivity in a multi-signal environment, but TDOA was used as the method of geolocation. In this paper, we propose and analyze the use of features of the SCF as a method of obtaining RSS values. We then use these RSS measurements for geolocation, and show that the SCF outperforms energy detection for non-cooperative geolocation.

2. SYSTEM MODEL

Throughout, (·)*, (·)T, and E {·} denote complex conjugate, matrix transpose, and statistical expectation, respectively. The matrix I is the identity matrix. A hat (e.g. ẑ) indicates an estimate of its argument.

Given a transmitted signal x(t) with carrier frequency fc and bandwidth fc, the received signal is given by

\[ y(t) = x(t) \ast h(t) + n(t), \]

where h(t) and n(t) are the multipath channel and additive noise, respectively. Define the T-windowed Fourier transform as

\[ Y_T(t, f) = \frac{1}{\sqrt{T}} \int_{-T/2}^{T/2} y(u) e^{j2\pi fu} du. \]

The SCF can then be defined as

\[ S_y(f, \alpha) = \lim_{T, \Delta t \to \infty} \int w(t, \Delta t) Y_T\left(t, f + \frac{\alpha}{2}\right) Y_T^*\left(t, f - \frac{\alpha}{2}\right) dt \]

where w(t, Δt) is a window of width Δt and unit area, centered at the origin. Note that the PSD is the SCF evaluated at \( \alpha = 0 \). An example SCF of a Binary Phase Shift Keying (BPSK) signal is shown in Fig. 1, where the signal has been processed in discrete time using a sampling frequency of \( f_s = 8 f_c \).

Many papers use the Spectral Coherence Function (SOF), which is a normalized version of the SCF,

\[ C_y(f, \alpha) = S_y(f, \alpha) / \sqrt{S_y(f + \alpha/2, 0) S_y(f - \alpha/2, 0)}. \]

However, the effects of multipath have been derived explicitly in terms of the SCF [12], [13], and the SCF is sufficient for the purposes of this paper, hence we will not discuss the SOF further.

Of particular note is that the SCF is additive. Ignoring the channel for the moment, if x and n are independent and ergodic, then

\[ S_y(f, \alpha) = S_x(f, \alpha) + S_n(f, \alpha), \]
where $S_s(f, \alpha) = 0$ for $\alpha \neq 0$. Thus, features of the SCF away from the $\alpha = 0$ line (the PSD) should be detectable, even at low SNR. Instead, the limiting factor will be the length of time $T$ used in the observation and processing intervals.

Typically, cooperatively observed RSS is modelled as having log-normal fading. With $S$ sensors at known positions $(x_s, y_s)$, for $s = 1, 2, \ldots, S$ and a transmitter at unknown position $(x_0, y_0)$, the transmitter-to-receiver distances are

$$d_s = \sqrt{(x_s-x_0)^2 + (y_s-y_0)^2}. \quad (6)$$

The log-normal model for RSS observations $\{p_s\}$ is

$$p = [p_1, \ldots, p_S]^T \sim \mathcal{N}(\mathbf{m}, \sigma^2 \mathbf{I}), \quad (7)$$

where the mean is given by

$$\mathbf{m} = [m_1, \ldots, m_S]^T,$$

$$m_s = P_0 - \eta \cdot 10 \log_{10}(d_s/d_0), \quad (8)$$

and $\sigma$ ranges from 4 dB to 12 dB [14], corresponding to uncluttered environments to environments rich in shadowing and multipath. The value of $\sigma^2$ can be approximated from controlled measurements in a given environment. The remaining parameters to define are the path loss exponent $\eta$, which typically ranges from 2 to 4; the reference transmit power $P_0$, in dB; and the reference distance $d_0 = 1$ m, which is the distance at which an RSS of $P_0$ is expected. $P_0$ and $\eta$ may be pre-characterized or estimated as nuisance parameters.

In a non-cooperative situation, the mean $\mathbf{m}$ does not drop arbitrarily low with distance. Instead, it will exhibit a noise floor, as in Fig. 2. A full investigation of this effect, including modelling and algorithm derivation was submitted in [15]. Due to space limitations and to avoid publication redundancy, in this paper we do not deal with the subtleties of such effects. Instead, we assume that the noise background level is known and simply discard RSS values approaching this level. However, as will be shown, the noise floor of the SCF-based features is much lower than that of PSD-based (energy detection) features.

Now we need to related the signal model of (1)-(3) to the RSS model of (7)-(9). The baseline energy detection will use the power in the PSD as a feature,

$$P_{\text{PSD}} = \int_{f_c-f_b}^{f_c+f_b} |Y(f)|^2 df \quad (10)$$

$$= \int_{f_c-f_b}^{f_c+f_b} |H(f)X(f)|^2 df + \int_{f_c-f_b}^{f_c+f_b} |N(f)|^2 df,$$

where $f_c$ is the carrier frequency and $2f_b$ is the signal bandwidth. Assuming that the signal and noise PSDs are flat,

$$P_{\text{PSD}} = \sigma_x^2 \int_{f_c-f_b}^{f_c+f_b} |H(f)|^2 df + \sigma_n^2. \quad (11)$$

To reconcile (11) with the log-normal model, we assume that the total multipath channel energy $\int |H(f)|^2 df$ has a log-normal distribution.

The SCF-based RSS measurement will assess the power in one of the signal-dependent features. In modulations with a purely real time-domain baseband signal, such as BPSK, there will be a strong feature at $f = 0$ and $\alpha = \pm 2f_c$, where $f_c$ is the carrier frequency, as shown in Fig. 1. In the absence of noise, this feature is comparable to the PSD feature. In the presence of noise, the PSD is washed out, but this feature (and all other SCF points for $|\alpha| > 0$) is still clearly defined.

In the presence of multipath, the SCF is given by [12],[13]

$$S_{\phi}(f, \alpha) = H \left(f + \frac{\alpha}{2}\right) H^* \left(f - \frac{\alpha}{2}\right) S_s(f, \alpha) + S_n(f, \alpha), \quad (12)$$

where we have assumed a time-invariant channel for simplicity. We propose using an SCF feature to assess the RSS, by integrating the feature across the signal bandwidth,

$$P_{\text{SCF}} = \int_{-f_c}^{f_c} S_{\phi}(f, \alpha)_{|\alpha=2f_c} df \quad (13)$$

$$= \int_{-f_c}^{f_c} H \left(f + \frac{\alpha}{2}\right) H^* \left(f - \frac{\alpha}{2}\right) S_s(f, \alpha)_{|\alpha=2f_c} df + \int_{-f_c}^{f_c} S_n(f, \alpha)_{|\alpha=2f_c} df$$

$$= \int_{-f_c}^{f_c} H \left(f + f_b\right) H^* \left(f - f_b\right) S_s(f, 2f_c) df \quad (14)$$

If the signal PSD is flat, the SCF will have the same shape across the $2f_b$ bandwidth. Again assuming a real baseband time-domain
signal, it can be shown that (14) simplifies to
\[ P_{\text{SCF}} = \sigma_n^2 \int_{f_c-f_s}^{f_c+f_s} |H(f)|^2 df, \]  
which is identical to the PSD feature, except without the noise. This is a key feature of the proposed approach. Note that we have implicitly assumed that the carrier frequency is known, in order to select \( \alpha = 2f_c \). The SCF is known to have very narrow features with respect to \( \alpha \), hence this assumption is non-trivial. In practice, in a geolocation system, the sensor with the strongest signal can compute the full SCF with high resolution, and pass the value of \( f_c \) to the other sensors.

3. GEOLOCATION

This section briefly reviews Maximum Likelihood (ML) RSS geolocation. Though this material is well-understood, it is necessary to briefly cover it here to place Section 4 in context.

The log-likelihood of the observations given unknowns \( z = [P_0, \eta, x_0, y_0]^T \) is
\[ L = \ln f(p|z) = -\frac{1}{2\sigma^2} (p - m(z))^T (p - m(z)), \]  
ignoring an additive constant which does not affect the subsequent maximization process. The ML estimate maximizes \( L \) by choice of \( z \),
\[ \left( \hat{P}_0, \hat{\eta}, \hat{x}_0, \hat{y}_0 \right) = \arg \max_{P_0, \eta, x_0, y_0} L \left( P_0, \eta, x_0, y_0 \right) \]  
\[ = \arg \min_{P_0, \eta, x_0, y_0} \sum_{s=1}^{S} \left[ p_s - P_0 + \eta T \log_{10} d_s (x_0, y_0) \right]^2. \]  
This is a non-linear least squares problem. A variety of methods have been proposed for efficient near-optimal solutions; however, as our focus is on the new method of RSS assessment rather than algorithm optimization, we will use a global search over \((x_0, y_0)\) for simplicity of presentation. The parameters \( P_0 \) and \( \eta \) appear linearly, hence they will be explicitly optimized for each possible \((x_0, y_0)\).

4. SIMULATIONS

This section presents a numerical characterization of the PSD and SCF features as the SNR changes, as well as the use of these features for geolocation. For simplicity, a BPSK transmitter is used. The pulse shape is a raised cosine with 60% excess bandwidth. The received signal is oversampled by a factor of 8 relative to the symbol rate, \( F_s = 8f_s \), and the carrier frequency is \( f_c = 0.34F_s = 2.72f_s \). The resulting SCF is shown in Fig. 1. Since we are interested in geolocation, wherein the signal power drops with distance, the SNR value is determined by a constant background noise power and a variable signal power. For each sensor, the BPSK signal had a different power level, determined by realization of a log-normal random variable with mean based on distance. Each sensor also had a different realization of additive noise, which was added after the BPSK signal was scaled to the appropriate power level.

Figures 3 and 4 show the levels of the effective noise floor for the PSD and SCF features. For the SCF, the noise floor is due to numerical effects caused by the use of a finite amount of data, hence the floor drops as the observation time (and computation budget) is increased. The location of each “knee” was found via intersecting two linear fits: a constant fit at low SNR and a fit with unit slope at high SNR. The overall fit was found by adding the two fits in the linear domain, then converting back to the log domain.

Fig. 5 shows the geolocation performance of the old and new methods, for the case when the transmitter is inside and outside of the convex hull of the WSN, respectively. The number of BPSK symbols was \( 2^{13} \). For the usual PSD method, the bias dominates the Root Mean Squared Error (RMSE), and for the SCF feature, the bias is much reduced. The overall RMSE is improved by a factor of 2 to 5. The PSD-based errors would actually be even larger, but the finite search space truncated many of the large errors.

The bubbles around the sensors give a rough idea of the observed feature values (with a non-linear scaling used for visibility). This can be used to give intuition regarding the poor performance of the PSD-based method. When the source is outside the WSN, the
bias(PSD,SCF) = (9.94, 1.33) m
stddev(PSD,SCF) = (29.1, 6.57) m
rmse(PSD,SCF) = (30.7, 6.7) m

bias(PSD,SCF) = (58.8, 6.3) m
stddev(PSD,SCF) = (49.3, 36.3) m
rmse(PSD,SCF) = (76.7, 36.8) m

Fig. 5. Geolocation performance with the source inside (top) and outside (bottom) the WSN. The error ellipses would yield 86% confidence intervals if the position estimates were Gaussian.

PSD features are large everywhere, hence the Maximum Likelihood Estimate (MLE) judges that the receiver is far away and the path loss exponent is small.

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6. REFERENCES