Abstract

There is a need to develop automated methods to analyze images. A method that relies on the Helmholtz principle to discover edge features that is weakly dependent on the parameters is presented. An application of the Helmholtz principle requires a method of finding groups of independent and identically distributed random variables, and the fast level set transform was used to find such groups. The ability of the method to locate contrasted sections of curves is reviewed, and an original technique to find sections of curves with uniform curvature is demonstrated.

1. Introduction

A large number of images taken for surveillance, reconnaissance, and diagnostics, are only evaluated in a precursory manner or not evaluated at all due to lack of automated imaging techniques. The Helmholtz principle is a method of finding features that is very weakly dependent on the parameters, and has been used in Gestalt theory to find visually perceptual structures such as alignments, edges, and shapes in digital images [6, 8, 7, 5, 4]. The Helmholtz principle, combined with the fast level set transform can be used to robustly and accurately find edge features in images.

The Helmholtz principle describes a general methodology to find features in an image. An implementation of the Helmholtz principle requires a method to find groups of independent and identically distributed (IID) random variables, and an a-contrario hypothesis model. The a-contrario model describes how the groups of features will behave in the absence of the feature of interest. For example, to find modes of a histogram, the a-contrario model is a uniform distribution, and sets in the domain of the histogram are searched to determine if there are more points in that set than would be expected in a uniform histogram.

If there are more points then is expected, the set, which consists of a group of bins, is labeled as meaningful. A mapping, the number of false alarms (NFA), from the group of elements to the real numbers is used to determine if the group is a feature of interest.

The NFA has two properties that make it a desirable function to use for feature determination. The NFA correctly scales with the image size, and it can be interpreted as the expectation value of the number of false alarms. A group of elements create a feature, and are defined as meaningful, if the NFA of the group are less than a predetermined threshold. The interpretation of the NFA as the expected number of false alarms is only possible by correctly scaling the NFA to the image size, thus the threshold does not need to be modified between images. In most applications the threshold is set to one; therefore all meaningful objects should not appear in a random image.

In most applications of the Helmholtz principle, the dependence of the method on the threshold is further reduced through the application of a maximal principle [2, 7]. A group of objects typically retains the meaningful property if an element is added to the group, or if an element is taken away from the group. A maximal principle finds groups that are not contained in or do not contain groups with a lower NFA. The application of the maximal principle finds disjoint groups that have a minimal NFA with respect to the grouping procedure.

The rest of the paper is organized as follows. A brief orientation to the FLST given in section 2. This section is provided to maintain coherency, and any details about the FLST can be found in the references. The next section describes the necessary components to apply the Helmholtz principle to groups of objects. The Desolneux et al edge detection method is shown along with a novel method to detect regions with uniform curvature. The edge features these methods detect are shown in a image of peppers.
2. The FLST

A level set decomposition, which has been used in imaging science to obtain a parameter free contrast invariant representation of an image, is used to find closed contours in the image. It has been shown by Monasse et. al. that a level set decomposition can be used to build a tree of shapes such that the shapes are oriented by set inclusion, and he provides a fast algorithm to compute the decomposition [1,9,10]. The level set structure provides the curves and a procedure to group elements that can be used in a Helmholtz formulation of feature extraction.

Consider the level sets of a continuous real valued function \( h: \mathbb{R}^2 \rightarrow \mathbb{R} \). Let \( \lambda \in \mathbb{R} \) and define the upper level set \( \lambda \) on \( h \) as
\[
\lambda \rightarrow \{ x \in \mathbb{Z} | h(z) \geq \lambda \},
\]
\[
\lambda \rightarrow \{ x \in \mathbb{Z} | h(z) < \lambda \}. \tag{1}
\]
Since \( h \) is a function it is clear that the level sets have a natural inclusion structure
\[
\lambda_1 \geq \lambda_2 \iff \lambda \lambda_1 \subseteq \lambda \lambda_2,
\]
\[
\lambda_1 < \lambda_2 \iff \lambda \lambda_1 \subseteq \lambda \lambda_2. \tag{2}
\]
The idea is to decompose each level set into a collection of maximally connected components [9]. Consider a bounded component of a level set, and define a shape as the set obtained after including any holes interior to the bounded component [9]. A tree structure can be built from the shapes such that a parent is determined by set containment, i.e. the parent of the shape is the smallest shape that contains the given shape. The root of the tree is therefore \( \mathbb{R}^2 \).

3. The Helmholtz Principle

An application of the Helmholtz principle requires a method of grouping independent and identically distributed (IID) random variables, an \textit{a-contrario} probability model for the groups of data, and a function, the NFA, that maps the groups of random variables to the real numbers. The NFA scales correctly with the number of groups such that it maintains its interpretation as the expected number of false alarms independent of the image chosen. The three requirements for an implementation of the Helmholtz principle will be discussed in more detail below.

A method of determining groups of IID random variables must be given before the Helmholtz principle can be applied. The random variables are IID if they are drawn from the \textit{a-contrario} background mode. In general, the random variables should not be expected to be IID in real images. The FLST provides a fast and complete representation of the image that readily lends itself to finding edge features.

The procedure to find groups of edge features consists of finding the borders of the shapes found in the level set decomposition. Shapes of length \( l \) can be partitioned into a family of subsets of the border such that each element is adjacent to another element. The total number of sets, \( N \), is given by the formula
\[
N = \sum_{i=1}^{q} l_i(l_i - 1) \over 2 \tag{3}
\]
where \( l_i \) is the length of the border of the \( i^{th} \) shape and \( q \) is the number of shapes.

The \textit{a-contrario} probability model can be given \textit{a-priori} or it can be learned from the data, and an example of both will be given. Any probability model that describes groups of IID random variables is appropriate to use with the Helmholtz principle as the \textit{a-contrario} probability model. Desolneux et al. searched the level lines of the image to find contrasted sections of these lines [5]. Use the notation \( |I| \) to equal the number of elements in the finite set \( I \). Denote the contrast of the region by
\[
c(x) = |\nabla I|(x), \tag{4}
\]
where \( c(x) \) is calculated using a \( 2 \times 2 \) neighborhood difference operator. The empirical distribution \( H(\mu) \) given by
\[
H(\mu) = \frac{1}{M}|\{x : c(x) \geq \mu \}|, \tag{5}
\]
where \( M \) is the total number of points in the image, will be used to determine if a line segment is contrasted. Consider a level line segment of length \( l \) found using the FLST and define \( \mu \) as the minimal contrast along this segment. Define the number of false alarms of the edge segment \( I \) as
\[
NFA(I) = N[H(\mu)]^l. \tag{6}
\]
Any segment \( I \) that satisfies \( NFA(I) < \epsilon \) will be called a \( \epsilon \)-meaningful contrasted line segment. Using this definition of \( \epsilon \)-meaningful segments, it is not difficult to show that the expected number of false alarms using the \textit{a-contrario} model for a contrasted edge segment is less than or equal to \( \epsilon \); therefore to find contrasted edge segments that should not appear in random images set \( \epsilon \) equal to one [5]. When \( \epsilon \) is one the \( \epsilon \)-meaningful edge segments will be called simply meaningful edge segments.

The work of Desolneux et al. introduced a maximal principle to find disjoint sets of features found using the Helmholtz principle. A set of IID random variables is defined to be maximal meaningful set if it is not contained in or it does not contain a more meaningful set. The notion of a maximal principle greatly reduces the dependence on the parameter \( \epsilon \), and using the maximal principle, the \( NFA \) of a maximal meaningful set is often several orders of magnitude less than one. The maximal principle can also be used
to decrease the width of the edge, but this procedure will not be discussed due to length constraints [5].

The trivial observation that $H(\mu)^l < H(\mu)^q$ for $l < q$ simplifies the algorithm to find maximal edge features. The input to the edge detection algorithm is a gray-scale image and the output is a list of edge features. An outline of the algorithm is given in the following steps.

1. Calculate the FLST on the gray-scale image. The output is a tree of level lines.
2. Set the current line segment to the current level line and perform the following steps.
   (a) Find the maximum contrast along the line segment and calculate the NFA of the line segment.
   (b) If the NFA of the current segment is less than or equal to $\epsilon$, save the level line as a meaningful edge segment. If there are more edge segments along the level line continue, else move on to the next level line.
   (c) If the NFA of the current segment is greater than $\epsilon$, divide the level line into two more segments by splitting this segment at the location of the minimum contrast.
   (d) If the level line is decomposed into meaningful segments or segments of length less than 3, then exit loop, otherwise continue with (a).

Examples of contrasted edge features are shown in figures 1 and 2, where the black lines in the image represent the edges. This 512x512 image took 2.4 seconds to analyze on a 2.8 Ghz G5 processor.

The distribution $H(\mu)$ used by Desolneux et al. was inferred from the image because the contrast in the images changes as the range of pixel values changes [5]. The second, original, application of the Helmholtz principle to edge feature detection locates sections of level line edges in which the curvature does not change much along the entire section. The change in curvature will be measured using an a-contrario probability model that is the same among all images.

The $a$-priori probability distribution chosen for uniform curvature edge features is a uniform random model where the change in curvature is a uniform random variable between 0 and $2\pi$. A few comments on the calculation of the curvature along level line edges are appropriate. Cao et al. found the curvature of a line segment using the difference between angles along the line segment. He was able to find sections of curves in which the curvature varied little, and he was able to find junctions in the image. His technique to find the curvature does not give a very precise estimate of the curvature along the curve due to pixel effects. The pixel effects were reduced in this work by using an affine smoothing method by Cao et al. [3], and the software can be obtained using the freely available Megawave software.

After applying the affine smoothing to all the level lines, a three point technique was used to find the curvature of the level line. Consider three points in a plane where $a$, $b$, $c$ are the sides of the triangle described by these three points. The curvature of a circle that runs closest to these three points is given by the formula

$$k(x) = 4 \sqrt{s(s-a)(s-b)(s-c)} / abc,$$

where $s = .5 \ast (a + b + c)$. 
Assume that the curvature is a random walk with each step length is a uniform random variable between 0 and $2\pi$. The curvature can take on any value, but only sections of curvature that do not change significantly are important; therefore it is appropriate to set all changes of curvature greater than $2\pi$ equal to $2\pi$. Using this model, all sections of curves that contain a point with changes of curvature greater than $2\pi$ will not be considered to have uniform curvature. This definition is appropriate since such changes of curvature represent corners along the level lines.

The a-priori distribution function, $p(k)$, is given by

$$p(k) = \frac{k}{2\pi}. \quad (8)$$

Consider a level line segment of length $l$ found using the FLST and define $\kappa$ as the maximum curvature along this segment. Define the number of false alarms of the edge segment $l$ as

$$NFA(I) = N[p(\kappa)]^l. \quad (9)$$

Any segment $I$ that satisfies $NFA(I) < \epsilon$ will be called an $\epsilon$-meaningful uniform curvature edge segment. Using the definition of $\epsilon$-meaningful segments, it is not difficult to show that the expected number of false alarms for a uniform curvature edge segment are less than or equal to $\epsilon$. The proof is identical to the $\epsilon$-meaningful contrasted edge proof, and due to lack of space it will be omitted.

The algorithm to locate the maximal meaningful uniform edge segments is the same as the contrasted edge segments, excepts the level lines are divided using the maximum of the curvature. An example of meaningful curvature segments is given in Figure 2. The image is the familiar peppers image, and the places of uniform curvature is denoted with dark black lines. Most of the regions of uniform curvature are along places where the edges are also contrasted, but a close comparison of this Figure with Figure 1 shows some areas that have uniform curvature, but are not well contrasted. Note that the features in Figure 2 are smoother than in Figure 1 due to the affine curve smoothing used in the algorithm.

4. Conclusions

The Helmholtz principle applied to edge feature extraction was reviewed, and a new method to locate sections of the image in which the curvature does not change significantly was demonstrated. The edge detection method was based on one parameter, $\epsilon$. A maximal principle was then implemented to reduce the dependence on the parameter. The uniform curvature feature detector required the use of an affine smoothing technique that introduces many more parameters [3]. These parameters were chosen to perform a minimal amount of smoothing before the Helmholtz principle was applied.

Edge features are used in numerous image processing applications, and the Helmholtz principle combined with the FLST provides a method to find edges without using arbitrarily assigned parameters. Locating sections of edges with uniform curvature is a new application of the Helmholtz principle. Regions of uniform curvature are areas of the image in which an oscillating circle is tangent to the curve; therefore regions of uniform curvature are objects in the image in which there is a robust estimate of the object’s scale. Regions of uniform curvature should be usable in applications where a estimate of the scale is required. Such applications include, but is not restricted to, image registration, change detection, and content based image database searches.

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References


