Optimal Placement of Distributed Antennas in Cellular Systems

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Abstract—We investigate the optimal placement of transmit antennas in distributed antenna systems. Our optimization framework imposes no constraints on the location of the antennas. Based on stochastic approximation theory, we adopt a formulation that is suitable for node placement optimization in various wireless network scenarios. We show that optimal placement of antennas inside the coverage region can significantly improve the power efficiency of wireless networks. We obtain the optimal placement topologies for different numbers of antennas and illustrate that the circular deployment is not optimum in general. Finally, we show via simulations that the optimal placement solution does not depend on the underlying shadowing model.

I. INTRODUCTION

Next generation cellular systems aim to increase their network capacity and coverage, as well as mitigate the adverse effects of interference. One possible strategy to alleviate the interference, both in the uplink and the downlink of cellular networks, is to reduce the overall transmit power by using distributed antenna systems (DAS). These systems have the additional advantage of improving capacity and coverage [10]. Moreover, by reducing the access distance between the transmitter and the receiver, distributed antenna systems have direct impact on the energy efficiency of the cellular network, which may lead to greener architectures for cellular networks in the future. However, the capacity increase (or equivalently the power saving) of DAS is largely influenced by antenna locations.

Optimal placement of antennas has received significant attention in wireless networks [2], [11]. Optimizing the location of the antennas for obtaining the minimum bit error rate in linear cells [13], maximizing the coverage of a sensor network [1] or finding the optimum radius for antenna deployment of distributed antenna systems (DAS) in circular-layout [5] are some examples of the benefits of antenna placement optimization in wireless systems. In [8], it was shown that the placement of distributed antennas can significantly influence system performance and that this optimal placement magnifies the advantages of using DAS over the traditional systems with centrally-located antennas [12]. In other words, the capacity increase (or equivalently the power saving) of DAS is largely influenced by antenna locations. However, many of the studies on the placement optimization of antennas [11], [13] impose restrictions on the topology of the network, such as linear cells or antennas deployed along a circle. To address the problem without imposing these topological constraints, this paper investigates the optimal location of transmit antennas in distributed antenna systems in a general framework. While our framework can incorporate a broad set of performance metrics, our results will focus on the capacity and power efficiency gains obtained through optimal placement.

The remainder of this paper is organized as follows. First in Section II, we provide a basic description of our wireless system. In Section III, we formulate our placement optimization problem and introduce our problem framework. We introduce the stochastic gradient algorithm for solving the placement optimization problem in Section IV. Finally, Section V illustrates the performance of the proposed algorithms in different settings.

II. SYSTEM MODEL

We now describe the system model for analyzing the downlink performance of distributed antenna systems in a cellular setting. Figure 1 is the general architecture of the distributed antenna system $DAS(N, L)$ in the multicell environment considered in this paper. We assume that each cell is covered with a total number of $N$ distributed antenna ports and each port has $L$ microdiversity antennas. We assume that...
the antenna ports are all connected to the central antenna port via a dedicated link (e.g. fiber) with unlimited bandwidth and zero delay. We consider a one-tier cellular structure in which a given cell is surrounded by a continuous tier of six cells that can cause interference. The central cell in Figure 1 is indexed by \( j = 0 \), while the surrounding cells are indexed by \( j = 1, \ldots, 6 \). We denote the positions of the ports in the central cell with a \( 1 \times 2N \) vector \( \mathbf{p} \) which is defined as

\[
\mathbf{p} = [\mathbf{p}_1, \mathbf{p}_2, \ldots, \mathbf{p}_N],
\]

where each of the \( \mathbf{p}_i \)'s indicate the position of the \( i \)th port in the Cartesian plane as \( \mathbf{p}_i \triangleq [x_i, y_i] \) for \( i \in \{1, 2, \ldots, N\} \). Denoting the centroid of the \( j \)th cell in the Cartesian plane with \( \mathbf{o}^j \triangleq [p_x^j, p_y^j] \), we assume that the relative positions of the ports in the \( j \)th cell with respect to \( \mathbf{o}^j \) is identical for all \( j = 1, 2, \ldots, 6 \). In other words, if we denote the location of the \( i \)th port in the \( j \)th cell by \( \mathbf{p}_i^j \), we assume that for all antenna ports \( \{i = 1, 2, \ldots, N\} \) in the neighboring cells \( \{j = 1, 2, \ldots, 6\} \) we have

\[
\mathbf{p}_i^j = \mathbf{p}_i + \mathbf{o}^j.
\]

All of the \( NL \) antennas in our \( N \) distributed antenna ports together construct a macroscopic MISO downlink channel with the \( NL \times 1 \) channel vector

\[
\mathbf{h}(j) = [h_1(j), h_2(j), \ldots, h_N(j)],
\]

where \( h_n(j) \) denotes the channel vector gain from the \( n \)th antenna of port \( n \) to the user at location \( \mathbf{u} \), which is a function of the distance \( r \) between the \( n \)th transmitter port and the user. Because of the different distances between the antenna ports and the user, our channel vector gain should encompass not only the small scale multipath fading but also the large scale shadowing together with the path loss as

\[
h_n(j) = \frac{g_n^{(j)}}{\mathcal{L}^{(j)}(r(\mathbf{p}_n, \mathbf{u}))},
\]

where \( f_n^{(j)} \) denotes multipath fading in the channel at the \( i \)th port \( n \) and is an i.i.d. complex Gaussian random variable distributed as \( \mathcal{CN}(0,1) \). Also \( g_n^{(j)} \) is a log-normal random variable representing shadow fading. In other words, \( 10 \log_{10} g_n^{(j)} \) is a zero-mean Gaussian random variable with standard deviation \( \sigma_{sh} \). \( \mathcal{L}^{(j)}(r) \) in (4) is the power path loss function which is a function of distance \( r \) and also depends on the propagation frequency, as we will discuss in more detail later. In this paper we consider an exponential path loss function of the form \( \mathcal{L}(r) = \beta \times r^\alpha \), where \( \alpha \) is the path loss exponent whose value is normally in the range of two to six. We assume independent log-normal shadow fading between each port\(^1\) and also independent Rayleigh fading at each antenna per port. Here, the distance function \( r(\cdot) \) can be calculated as

\[
r(\mathbf{p}_n, \mathbf{u}) = \max\{D(\mathbf{p}_n, \mathbf{u}), r_0\} \quad i \in \{1, 2, \ldots, N\},
\]

where \( D(\mathbf{p}_n, \mathbf{u}) \) is the Cartesian distance between the user at location \( \mathbf{u} \in \Pi \) and the \( i \)th antenna port's location \( \mathbf{p}_n \), and can be written as

\[
D(\mathbf{p}_n, \mathbf{u}) = \sqrt{(p_x - u_x)^2 + (p_y - u_y)^2}.
\]

Also, \( r_0 \) in (5) is the minimum allowable value of \( d_i \) for which the far field approximation of the antenna propagation model is valid.

Let the transmit signal vector of the \( j \)th cell be \( x(j) = [x_1(j), x_2(j), \ldots, x_N(j)] \), in which \( x_n(j) \) denotes the \( 1 \times L \) transmission vector of the \( n \)th port of the \( j \)th cell. Then the received signal of the user in the central cell can be written as

\[
y = \mathbf{h}(0)^T x(0) + \sum_{j=1}^{6} \sqrt{\gamma_j} \mathbf{h}(j)^T x(j) + n,
\]

where \( n \) is an additive Gaussian noise with variance \( \sigma_n^2 \) and \( \gamma_j \) is a positive real spreading code gain that quantifies the fact that the signals from the \( j \)th cell are not completely orthogonal to the signals of the central cell\(^2\). In deriving (8) from (7), since the neighboring cells are sending independent messages, we approximate the sum of interfering signals plus noise as a complex Gaussian random variable \( z \) with variance \( \sigma_z^2 \). We also assume that each port has a transmit power budget of \( \bar{S} \), i.e.

\[
E[x_n(j)x_n^H(j)] = \bar{S},
\]

for all ports \( 1 \leq n \leq N \), and in each of the cells \( 0 \leq j \leq 6 \). Note that in (9), the \( n \)th transmission port can allocate its own power budget \( \bar{S} \) among its antennas, but different ports cannot allocate power among themselves.

### III. Problem Formulation

Given the channel model in Section II, for fixed port location matrix \( \mathbf{P} \) and fixed user’s location \( \mathbf{u} \), we can write the average ergodic rate of a single user under different circumstances. We consider two cases of channel information at the transmitter: the channel coefficient vector \( \mathbf{h}(j) \) is known to the transmitters of the \( j \)th cell (channel with CSIT) or it is not (channel with only CSIR). In both cases, we assume that the channel coefficient vector is known at the receiver (i.e. CSIR is available in both cases), which is a good approximation for practical systems with sufficient receiver channel estimation. The capacity of this channel depends on the power constraint on the input signal vector \( x \) in (8) and also the availability of the channel side information at the transmitter. Because of the Gaussian noise and known channel at the receiver, the optimal input signal for both cases of having CSIT or not, is Gaussian with zero mean \( [4], [3] \). The main problem though is to find the covariance matrix of this Gaussian vector that satisfies the power constraints of the antenna ports. For the case of having only CSIR, for a symmetric fading channel, \( 1 \)

\[\text{Note that we put the origin of the Cartesian plane at the centroid of the central cell (} j = 0 \).

\[\text{2Justified because of the large distance between the ports.}\]
e.g. Rayleigh fading with log-normal shadowing, we can show that (see [7] for the proof) the capacity is

\[ C(u, P, S) = E_f \left[ \log_2 \left( 1 + \frac{1}{L \sigma^2_n} \sum_{n=1}^{N} ||h_n(0)||^2 S \right) \right] , \]

(10)

where \( ||.|| \) denotes the \( L^2 \)-Norm. For the case of having CSIT, the average ergodic rate of a DAS(N,L) system with CSIT can then be written as (proof is in [7])

\[ C(u, P, S) = E_f \left[ \log_2 \left( 1 + \frac{1}{\sigma^2_n} \left( \sum_{n=1}^{N} ||h_n(0)|| \sqrt{S} \right)^2 \right) \right] , \]

(11)

where \( E_f [.] \) denotes the expectation with respect to both multipath and shadow fading. Comparing (11) and (10), we can conclude that under the per-port power constraint, the presence or lack of channel phase information at the transmitter has a significant impact on the optimal transmit strategy and hence on the channel capacity. Depending on whether (10) or (11) is used for the capacity, the cell averaged ergodic capacity \( C \) for a particular placement of the antennas can be calculated by averaging \( C(u, P, S) \) over user positions \( u \) inside the cell region \( \Pi \) as

\[ \bar{C}(P, S) = E_u C(u, P, S) , \]

(12)

where \( E_u \{ \} \) denotes the expectation with respect to the location of the user and in our model we assume that it has a uniform distribution inside the cell region.

Now we can describe the placement optimization problem for DAS that we consider in this section as

\[ P^* = \arg \min \bar{C}, \]

(13)

subject to : 

\[ \bar{C}(P, S) \geq C_t, \quad p_i \in \Pi \quad \forall i \in \{1, 2, ..., N\} , \]

(14)

where \( C_t \) is the target for the expected cell average ergodic rate. Here \( P \), as defined in Section II, is the matrix containing the collection of locations for all the ports in the system.

IV. ANTENNA PLACEMENT

In this section, based on stochastic approximation theory, we introduce a framework for solving the placement optimization paradigm. For the sake of clarity, rather than solving (13) directly, we will first focus on the equivalent problem of maximizing the average ergodic rate with the given power budget \( S \). We touch upon the generalization of the proposed algorithm for more sophisticated optimization metrics and also the direct solution to (13) afterwards.

Assume we want to find the optimal placement vector \( P \) in order to maximize

\[ \bar{C}(P, S) = E_u E_f C(u, P, g, f, S) , \]

(15)

where \( g \overset{df}{=} [g_1, g_2, ..., g_N] \) is defined to be a random vector of size \( N \), indicating the shadowing and \( f \overset{df}{=} [f_1, f_2, ..., f_N] \) denotes the random vector for modeling the multipath fading in all of the \( NL \) channels in our model\(^4\). Also \( C(u, P, g, f, S) \) is the instantaneous ergodic rate for a given realization of the user position \( u \), fading vectors \( g, f \), and given location matrix \( P \), depending on which information we assume to be available at the transmitter. In particular, when we assume that CSI is only available to the receiver but not the transmitter, from (10) we can write the capacity as

\[ C(u, P, g, f, S) \overset{df}{=} \log_2 \left( 1 + \frac{S}{\sigma^2_n} \sum_{n=1}^{N} g_n \| h_n(0) \| f_n \right)^2 , \]

(16)

and when CSI is assumed to be known both at the transmitter and the receiver as in (11), the instantaneous capacity can be written as

\[ C(u, P, f, S) \overset{df}{=} \log_2 \left( 1 + \frac{S}{\sigma^2_n} \left( \sum_{n=1}^{N} \sqrt{\frac{g_n}{L_n(r_p, u)}} \right)^2 \right) . \]

(17)

Now, in order to find the optimal location matrix \( P^* \) that maximizes (15), we use the Robbins-Monro procedure [9] from stochastic approximation theory. Before getting into the algorithm that finds \( P^* \), we will briefly describe the Robbins-Monro procedure. In stochastic approximation, maximizing a utility function \( U(x) \) can be done via an iterative update of the optimization variable \( x \) in the following way

\[ x_{t+1} = x_t + \sigma^t \left( U_t(x_t) + M_t \right) , \]

(18)

where \( U_t(x) \overset{df}{=} \frac{\partial U}{\partial x} \), \( M_t \) is some zero mean noise in the estimate of \( U_t(x) \) and \( \{\sigma^t\}_{t=1}^{\infty} \) is a square summable but not summable sequence.\(^5\) In our specific placement optimization problem, we can look at the expected ergodic capacity as the utility function that we want to maximize and use the Robbins-Monro procedure to update the location of the ports at each iteration.

We can now specify the following iterative algorithm for solving (15).

1) Initialize the location of the ports randomly inside the coverage region and set \( t = 1 \)
2) Generate one realization of the shadowing vector \( g \) and multipath fading vector \( f \), based on the probabilistic model that we have for the fading of the channel.
3) Generate a random location \( u \) for the position of the user based on the distribution of the users in the coverage area.
4) Update the location vector as

\[ P_{t+1} = P_t + \sigma^t \frac{\partial C(u, P, g, f, S)}{\partial P} |_{P_t} , \]

(19)

5) let \( t = t + 1 \) and go to step 2 until convergence.

In order to provide some intuition about this algorithm, we will try to put it in the form of the Robbins-Monro recursion\(^5\) each element \( f_n \) in \( f \) is a vector of size \( L \) itself that indicates the multipath fading for each of the antennas in the array, i.e \( f_n = [f_{n,1}, f_{n,2}, ..., f_{n,L}] \)

\(^{4}\)The convergence of this Robbins-Monro iteration is shown under broad conditions in [9, chapter 6]. In particular, the iterations converge to a local optimum when the noise terms \( M_t \) have bounded variance.
As we can see, \( \frac{\partial C(u, P, g, f, S)}{\partial P} \) is an unbiased estimator of \( E_u E_f \left[ \frac{\partial C(u, P, g, f, S)}{\partial P} \right] \). Hence by defining

\[
M_t = \frac{\partial C(u, P, g, f, S)}{\partial P} - E_u E_f \left[ \frac{\partial C(u, P, g, f, S)}{\partial P} \right],
\]

we can rewrite the position update (19) of step 4 in the form of the Robbins-Monro recursion by noticing that in our placement problem the utility function \( U(\cdot) = E_u E_f C(\cdot) \) and the zero mean noise term \( M_t \) is defined in (21).

Now we describe how we can solve (13) directly, rather than solving the equivalent problem of maximizing the average ergodic rate with given power budget \( S \). Similar to [14], a primal-dual algorithm can be used to solve (13) with the same stochastic update mechanism. Briefly, the dual problem and the Lagrange multiplier can be updated in an iterative fashion with a stochastic gradient method [14]. Unfortunately, due to the non-convexity of the objective function, there is no guarantee that the algorithm converges to the global optimum solution, and it’s possible that stochastic gradient steps may converge to a local optimum. This problem has been addressed for the special case where the path loss function has an exponential fall off with distance, where it was shown that \( \frac{\partial C}{\partial P} \) converges to a local optimum. This problem has been addressed by the networks. In order to quantify this amount, we compare DAS(6,1) with the case where we have all of the antenna ports colocated in the center. In other words, for the case where we have the CSI only at the receiver and all antenna ports are co-located in the center of the cell, we set the per-antenna transmit power \( S \) so that the average SNR at the edge of the coverage region is 10 dB. According to this transmit power for each value of \( \alpha \), we calculate the average ergodic rate \( C_t(\alpha) \) when all of the three antennas are co-located at the center. Then for each value of \( \alpha \), we solved (13) in order to find the minimum power required to achieve the same average ergodic rate \( C_t(\alpha) \), with the optimal placement of six ports in the cell. Figure 3 illustrates the ratio of the minimum powers required to satisfy \( C_t(\alpha) \) in two cases. This gain is only due to the location optimization and, as the figure indicates, the power gain increases with increased \( \alpha \). The same procedure can be repeated for the case where the CSI is available at the transmitter as well. Figure 3 also shows the gain that we get from optimally placing the ports when we have CSIT. As we can see, the power gain is considerably smaller when channel side information is not available to the transmitter. In other words, optimally distributing the antenna ports in the case of having CSIT does not help much, whereas the gain is noticeable when channel side information is not available to the transmitter, but only at the receiver.

We also studied the effect of interference in the optimal placement of antenna ports, as described in Section IV. For \( N = 7, L = 1 \), we observe that the optimal layout almost always has one port in the center and six other ports in a circle of radius \( r \) around it (see Figure 1), where \( r \) depends on the interference coefficients \( \gamma_j \) as well as the path-loss coefficient \( \alpha \). In this part we set all the interference coefficients from neighboring cells \( j = 1, 2, \ldots, 6 \) to \( \gamma_j = \gamma \). Figure 4 plots the optimal radius \( r \) of the antenna ports around the central port as a function of

\[
\sigma_z^2 = \sum_{j=1}^{6} \sum_{j=1}^{N} \frac{\gamma_j}{C(r(p_{ij}, u))} + \sigma_n^2,
\]

where we assumed that all antenna ports in all neighboring cells are using the same transmit power of \( S \). Therefore, by replacing \( \sigma_z^2 \) with the variance of the noise \( \sigma_n^2 \) in (16) or (17), we can consider the effect of interference on the expected ergodic capacity. The same stochastic update algorithm of Section IV can be applied here for placement optimization of antenna ports, simply by replacing the capacity formula in (19). Note that we assumed similar relative positions for the ports in all neighboring ports. This forces the final optimal layout of the antennas (after convergence of the algorithm) to be identical in all the cells \( j = 0, 1, \ldots, 6 \). As we will show in the next section, the effect of interference in the optimal location of the antenna ports is to move the ports towards the center of the cell in order to minimize the power of the antenna ports that leaks outside the cell boundaries, thereby causing interference.

V. Numerical Results

In this section we illustrate the performance of the placement optimization algorithm introduced in Section IV. Figure 2 illustrates the optimal radius \( r \) to deploy three ports, for different values of path loss \( \alpha \). We used the stochastic update method for two different scenarios described in Section III and plotted the optimal deployment radius as a function of path loss. As we can see, depending on using (11) or (10) for capacity, the optimal placement strategy is different, especially for small path loss exponent \( \alpha \).

![Fig. 2. Optimal placement radius \( r \) of DAS(3,1) with stochastic update method in hexagonal cell of radius \( R = 1000 \)](image)
interference coefficient $\gamma$ for different values of path-loss $\alpha$. As we see in the figure, the optimal layout shrinks towards the center of the cell as the interference coefficient $\gamma$ increases.

In order to illustrate the performance of optimally placed DAS as a function of cell size, we use the area spectral efficiency (ASE) metric. This efficiency metric [6] is defined as the average ergodic rate of the user per unit bandwidth per unit area. Figure 5 illustrates this metric for $N = 3$ single antenna ports, optimally placed in a cell with a varying radius. The same figure also plots the area spectral efficiency of a system with $N = 3$ ports, randomly placed inside the cell. For both cases we scale the power proportional to the area of the cell in order to have the same power consumption per unit area. Also for both cases we assumed that the CSI is only available to the receiver. As we see in the figure, in comparison with random placement, the optimal placement of the ports in DAS has a significant effect on area spectral efficiency of the system when we have CSIR. Considering this result and the fact that even randomly placed DAS outperforms colocated antenna systems [8], we can conclude that optimally placed distributed antenna systems can improve the spectral efficiency of cellular system significantly.

Finally we should note that, in our simulations, we observed that the optimal location of the antenna ports do not change when we have shadowing or multipath Rayleigh fading in our model. In other words, the optimal layout is not affected by either multipath or shadow fading.

VI. CONCLUSION

We have presented a general framework for placement optimization of distributed antennas in cellular networks under general performance metrics. Our numerical results indicate that a 15dB power reduction or a doubling of area spectral efficiency results from this optimization. In addition we show that, as interference increases, the distributed antennas should be placed closer to the center of the cell. This optimization framework can also be extended to optimize relay placement in cellular networks.

REFERENCES