Second-order optical differentiator based on a mechanically-induced LPFG with a single \(\pi\)-shift

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Abstract—An all-optical second-order temporal differentiator based on a mechanically-induced long-period fiber grating (MI-LPFG) with a single \(\pi\)-shift is demonstrated. The MI-LPFG is created by pressing a fiber between two periodically grooved plates with a \(\pi\)-shift located at the 3/4 length from the input end of LPFG. The coupling coefficient \(\kappa\) can be adjusted through changing the pressure applied on the fiber. The experimental results show that the transfer function of the proposed MI-LPFG can be adjusted to be close to the ideal transfer function of a second-order differentiator. The differential performance of the designed differentiator to a Gaussian pulse is also analyzed.

Keywords-optical differentiator; long-period fiber-grating; \(\pi\)-shift; optical signal processing

I. INTRODUCTION

Optical differentiators are the basic devices for all-optical signal processing in the next generation ultra-high speed fiber-optic communication networks [1], which can achieve the differential operation of the optical signal intensity or the electric field envelope. Many optical devices can implement an optical differential operation, such as applying cross-polarization modulation or cross-gain modulation in a semiconductor optical amplifier (SOA) [2-3], cross-phase modulation in an optical fiber [4], critical coupling of silicon micro-ring resonator [5], fiber Bragg grating (FBG) [6] and long-period fiber grating (LPFG) [7]. A Program employing micro-ring resonator [5], fiber Bragg grating (FBG) [6] and long-period fiber grating (LPFG) [7]. A Program employing fiber gratings is compatible with optical communication networks, and has a low insertion loss, so it is a relatively simple and practical solution.

Either a FBG or a LPFG can complete the differential operation, while the latter has a wider bandwidth and is easier to fabricate than the former. Specially-designed uniform LPFG or phase-shifted LPFG can realize first- and higher-order all-optical differentiator [8]. There are many methods to fabricate a LPFG, such as exposing an optical fiber to an ultraviolet laser through an amplitude mask [9], point-by-point writing on a fiber using a CO2-laser [10] or electric-arc technique [11], mechanically-induced LPFGs [12] and so on. However, an arbitrary-order all-optical differentiator with an extremely high precision requires the LPFG inscription to be accurately controlled, which is difficult in practical fabrication. Recently, a band-pass filter based on a mechanically-induced multi-\(\pi\)-shifted LPFG and a first-order differentiator based on a MI-LPFG are reported [13-14]. The mechanically-induced LPFG technology is flexible and simple for specifically customized phase-shifted LPFGs, and can be used in the fabrication of the arbitrary-order optical differentiators.

In this paper, an all-optical second-order temporal differentiator based on a mechanically-induced LPFG (MI-LPFG) with a single \(\pi\)-shift is demonstrated. The MI-LPFG is created by pressing a fiber between two periodically grooved plates with a \(\pi\)-shift located at the 3/4 length from the input end of LPFG. The experimental results show that the performance of the differentiator is influenced by the notch depth of the transmission spectrum, the central frequency consistency of the notch in the transmission spectrum and the input pulse.

II. OPERATION PRINCIPLE

LPFGs with a single \(\pi\)-shift can achieve second-order transient differential of the optical signal transient electric field. Assume that the optical field envelope is \(E_0(t)\), and its Fourier transform is \(E_\omega(\omega-\omega_b)\), where \(\omega\) and \(\omega_b\) are the optical frequency and the central optical frequency of the signal, is the second-order temporal differential of \(E_\omega(t) = \partial^2 E_0/\partial t^2\), then its Fourier transform is

\[
\tilde{E}_\omega(\omega-\omega_b) \propto \int \frac{\partial^2 E_0(t)}{\partial t^2} \exp[-j(\omega-\omega_b)t] dt
\]

\[
\propto (\omega-\omega_b)^2 \tilde{E}_0(\omega-\omega_b)
\]

Thus, the second-order differential operation can be obtained through a filter that has the transfer function of \((\omega-\omega_b)^2\).

According to the coupled-mode theory of the uniform LPFG, a uniform LPFG (corresponding to the \(i\)th segment of our LPFG) is described by the transfer matrix

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$$F_i = \begin{bmatrix} \cos \gamma L_i + j \frac{\sigma}{\gamma} \sin \gamma L_i \\ \frac{j \kappa}{\gamma} \sin \gamma L_i - \frac{j \sigma}{\gamma} \sin \gamma L_i \end{bmatrix}$$  \quad (2)$$

Where $\gamma = \sqrt{\kappa^2 + \sigma^2}$; $\kappa$ is the coupling coefficient between the core mode and a chosen cladding mode (determined by the period $\Lambda$ of the LPFG for a given wavelength); $\sigma$ is the detuning factor given by

$$\sigma = (\beta_{\text{co}} - \beta_{\text{cl}})/2 - \pi/\Lambda$$  \quad (3)

where $\beta_{\text{co}}$ and $\beta_{\text{cl}}$ are the propagation constants of the core mode and the cladding mode, respectively; $L_i$ is the length of the $i$th LPFG segments. The $\pi$ phase shift between consecutive LPFG segments is introduced via the matrix

$$\Phi = \begin{bmatrix} e^{-j \pi/2} & 0 \\ 0 & e^{j \pi/2} \end{bmatrix} = \begin{bmatrix} j & 0 \\ 0 & -j \end{bmatrix}$$  \quad (4)

Thus, the overall transfer characteristics of the system is computed as

$$F = F_n \times \Phi \times F_{n-1} \times \ldots \times F_2 \times \Phi \times F_1$$  \quad (5)

If the input and output signals are propagating in the fiber core mode, the field transfer function of the filter is characterized by element. For the second-order differentiator, the transfer matrix is

$$F = F_2 \times \Phi \times F_1$$  \quad (6)

Hence, we can get transfer function as

$$F(1,l) = -\frac{j \sigma^2}{\kappa^2 + \sigma^2} \cos \sqrt{\kappa^2 + \sigma^2} (L_1 + L_2)$$

$$-\frac{j \kappa}{\kappa^2 + \sigma^2} \cos \sqrt{\kappa^2 + \sigma^2} (L_1 - L_2)$$

$$-\frac{\sigma}{(\kappa^2 + \sigma^2)^{3/2}} \sin \sqrt{\kappa^2 + \sigma^2} (L_1 + L_2)$$  \quad (7)

which can be expanded into a Taylor series at the central frequency $\omega_b$ as

$$F(1,l) = F(\sigma) + \frac{1}{2} F'(\sigma) \cdot \sigma + \frac{1}{6} F''(\sigma) \cdot \sigma^2 + \ldots \ldots$$  \quad (8)

Meanwhile, $\sigma$ can also be expanded into a Taylor series with the terms including and beyond the second order neglected as

$$\sigma = (\beta_{\text{co}} - \beta_{\text{cl}} - 2\pi / \Lambda) - \frac{1}{2} \left[ \frac{\partial^2 \beta_{\text{co}}}{\partial \omega^2} \right]_{\omega = \omega_b} - \frac{\partial \beta_{\text{cl}}}{\partial \omega} \right]_{\omega = \omega_b}$$  \quad (9)

which shows a linear relation of $\sigma \approx (\omega - \omega_b)$. The influence of the terms including and beyond the third order is generally much smaller than that of the first three ones in Eq. (8). Hence, a second-order differentiator can be realized through setting $F(\sigma) = 0$ and $F'(\sigma) = 0$, which yield

$$\cos \kappa (L_1 - L_2) = 0$$  \quad (10)

$$\sin \kappa (L_1 + L_2) = 0$$  \quad (11)

i.e.,

$$\kappa L_1 = \frac{3}{4} \pi$$  \quad (13)

$$\kappa L_2 = \frac{1}{4} \pi$$  \quad (14)

which means that the position of the $\pi$ phase shift should split the grating asymmetrically in the proportion of 3:1.

### III. EXPERIMENTAL SETUP AND RESULTS

M. Kulishov and other researchers have designed optical differentiators based on LPFGs, while the LPFGs were fabricated using the point-by-point technique with laser injected into a Boron-codoped fiber. In this paper, an all-optical second-order temporal differentiator based on a MI-LPFG with a single $\pi$-shift is demonstrated. The coupling coefficient ($\kappa$) can be adjusted through changing the pressure applied on the fiber. Figure 1 presents the periodical pressure device and the experimental setup. The light launched from a wideband source (Koheras: Superth Compact) is polarized using a polarizer, and the polarization direction is then controlled by a polarization controller. The transmission spectra of the MI-LPFGs are measured by an optical spectrum analyzer (Yokogawa: AQ6370B) with a resolution of 0.5 nm. The grooved plate in the setup consists of 200 periods having grating pitch $\Lambda=600 \mu m$, groove width $b=400 \mu m$ and bulge width $a=200 \mu m$. The $\pi$-shift is located at the 3/4 length from the input end of LPFG.

![Figure 1: Experimental system (P: polarizer; PC: polarization controller; OSA: optical spectrum analyzer).](image_url)
end of LPFG is illustrated in Fig. 2 (a) with the blue (solid) line. A fitting to the experimental data is also given Fig. 2 (a) with the black (dot) line, and the ideal transfer function of a second-order differentiator is shown in the same figure with the red (dash dot) line. Assuming a Gaussian pulse (its spectrum is shown in Fig. 2(b)) with a half bandwidth of 2 THz (at 1/e intensity) is used as the input, and transmits through the MI-LPFG; the output spectrum is obtained as shown in Fig. 2 (c). Through inverse Fourier transform, the output pulse in the time domain can be obtained as shown in Fig. 2 (d). It can be seen in Fig. (2) that the transfer function of the designed filter is very close to an ideal second-order differentiator with a bandwidth of 6 THz, which indicates that a second-order differentiator can be achieved by the designed MI-LPFG. The height of the two lateral side-lobes in Fig. 2 (d) will be affected by the fact that the resonance loss of the transmission spectrum of the MI-LPFG (as shown in Fig. 2 (a) by the blue (solid) line) cannot reach zero.

In order to study the influence of the central wavelength mismatch between the optical pulse and the differentiator on the differential performance, the central wavelength of the second-order differentiator is artificially shifted away from that of the input pulse (192.7277 THz) as shown in Fig. 2 (b). The resonant wavelength of black (dot) line is 192.7277 THz, frequency shift $\Delta f_1$ between the black (dot) line and blue (solid) line is 0.3 THz; likewise, $\Delta f_2$ is 0.2THz. Figure 3 presents the simulation results. It can be seen in Fig. 3 that a small central wavelength mismatch can make the notches between the main peak and the two lateral side-peaks rise rapidly, greatly weakening the second-order differential performance. Therefore, the central wavelength match is crucial for the second-order differential operation.
IV. CONCLUSION

An all-optical second-order temporal differentiator based on a mechanically-induced LPFG (MI-LPFG) with a single π-shift is demonstrated. The pressure applied on the fiber is adjusted to get the required coupling intensity of the grating. The experimental results show that the MI-LPFGs with a single π-shift splitting the grating asymmetrically in the proportion of 3:1 can achieve the second-order differential operation. The proposed scheme is flexible, controllable and simple for the application in optical temporal differentiator.

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Figure 3. (a) Transfer functions of simulated transmission spectra of LPFG, expressions of $y_1=-0.2397(x-192.4277)^2$, $y_2=-0.2397(x-192.5277)^2$, $y_3=-0.2397(x-192.7277)^2$, with a blue (solid), red (dash dot), and black (dot) line respectively. (b) shows that product of lines in Figure 3 (a) and Figure 2 (b) respectively, (c) shows the inverse Fourier transform of lines in (b).