An I/Q Imbalance Estimation and Compensation Strategy for 3GPP LTE Systems

Jiang Chang and I-Tai Lu

Department of ECE, Polytechnic Institute of New York University, Brooklyn, NY, 11201, USA

Abstract — A novel receiver I/Q imbalance estimation and compensation strategy is proposed for the 3GPP Long Term Evolution (LTE) system using the proposed synchronization signals available in each frame. System architecture and algorithm design for the receiver I/Q imbalance estimation and compensation are described in detail. Algorithm optimization is also discussed for further enhancement of the design. Simulation results indicate that the proposed approach can effectively compensate the receiver I/Q imbalance.

Index Terms — In-phase and quadrature-phase (I/Q) imbalance, orthogonal frequency division multiplexing (OFDM), Long Term Evolution (LTE).

I. INTRODUCTION

OFDM modulation scheme has been adopted by many modern communication systems, such as WLAN [1], WiMAX [2] and 3GPP LTE [3] systems, etc.

In recent years, the direct-conversion architecture has been widely used in low-power, fully integrated receiver design. However, in direct-conversion receivers, the gain and phase imbalances between the I and Q branches will be introduced due to the analog component imperfection. Many algorithms have been proposed for I/Q imbalance estimation and compensation in OFDM systems [4]-[7]. Among these work, some are based on the IEEE 802.11a/n systems, in which the preamble can be used for I/Q imbalance estimation. Others estimate I/Q imbalance with the aid of specially-designed pilot structure. However, these requirements on frame structure can not be met in LTE systems. In [8] and [9], standard independent I/Q imbalance estimation algorithms are proposed based on blind signal processing.

In this paper, we proposed a novel receiver I/Q imbalance estimation and compensation strategy using synchronization signals in LTE systems. The LTE downlink (DL) and uplink (UL) are composed of physical channels and physical signals. Physical channels are used to carry user data and control information. Physical signals are used for cell search and channel estimation purpose [3]. The main DL physical signals are the primary and secondary synchronization signals (P-SCH and S-SCH). P-SCH and S-SCH carry cell-specific sequence for cell search and synchronization purpose. The proposed algorithm will make use of the S-SCH and P-SCH for receiver I/Q imbalance estimation and compensation.

The paper is organized as follows. Section II describes the receiver signal model in the presence of receiver I/Q imbalance in OFDM systems. In section III, the time and frequency structures of LTE frame and synchronization signals are first briefly introduced and the proposed I/Q imbalance estimation and compensation algorithm are then illuminated in detail. The simulation results and evaluation of the propose algorithm are given in the section IV. Conclusions are drawn in the Section V.

II. SIGNAL MODELING

A. Receiver I/Q Imbalance Modeling

In a communication receiver, denote the RF signal before down conversion as $r(t)$ and for convenience ignore the noise term for the moment

$$r(t) = y(t)e^{j2\pi f_c t} + y^*(t)e^{-j2\pi f_c t}$$

(1)

where $y(t)$ is the equivalent low pass complex baseband signal of $r(t)$ and $f_c$ is the carrier frequency.

In direct-conversion receiver, $r(t)$ is down-converted by a mixer with I/Q imbalance. This imperfection can be modeled by a complex local oscillator (LO) with time function [8]

$$\tilde{x}_{LO}(t) = \cos(2\pi f_c t) - jg \sin(2\pi f_c t + \phi)$$

(2)

where parameter $g$ is the receiver I/Q amplitude imbalance and $\phi$ is the phase imbalance.

The amplitude imbalance expressed in decibels is defined as

$$e = 10 \log[1 + (1 - g)/(1 + g)]$$

(3)

Define two parameters $K_1$ and $K_2$ which are functions of the I/Q imbalance parameters

$$K_1 = \frac{1 + ge^{-j\phi}}{2}, K_2 = \frac{1 + ge^{j\phi}}{2}, K_2 = \frac{1 - ge^{-j\phi}}{2}$$

(4)

where $\gamma = ge^{-j\phi}$ and $[\cdot]^*$ denotes complex conjugate.

Then $\tilde{x}_{LO}(t)$ can be reformulated as

$$\tilde{x}_{LO}(t) = K_1 e^{j2\pi f_c t} + K_2 e^{-j2\pi f_c t}$$

(5)

In the presence of receiver I/Q imbalance, the received signal $r(t)$ after down-conversion and low pass filtering becomes

$$z(t) = LPF\{r(t)\tilde{x}_{LO}(t)\} = K_1 y(t) + K_2 y^*(t)$$

(6)
Equivalently, the RF imperfection can be also described in frequency domain as

\[
Z(f) = K_y f Y(f) + K_y^* f (-f)
\]

where \(Z(f)\) and \(Y(f)\) are the Fourier transform of \(z(t)\) and \(y(t)\) respectively.

### B. I/Q Imbalance in OFDM Systems

In an OFDM system, suppose the transmitted symbol on the \(k\)th subcarrier of the \(l\)th OFDM symbol is \(X_k(l)\). After being transmitted through a frequency selective fading channel with the equivalent lowpass channel impulse response \(h(\tau, l)\), the received signal is sampled and demodulated with FFT. For the moment, if we assume the channel to be time invariant during the transmission of one OFDM symbol. The demodulated data symbol of the \(l\)th OFDM symbol can be expressed by

\[
Y_k(l) = H_k(l) X_k(l) + n_k(l), \quad k = -K/2, ..., (K/2 - 1)
\]

where \(n_k(l)\) is the complex additive white Gaussian noise, 
\(H_k(l)\) is the channel transfer function at subcarrier \(f_k\), \(K\) is the number of data subcarriers in each OFDM symbol.

In the presence of I/Q imbalance, according to (7) I/Q imbalance will introduce image interference from mirrored subcarriers. From (7) and (8), the demodulated signals on the \(k\)th and \((-k)\)th subcarriers of the \(l\)th OFDM symbol with I/Q impairment will be

\[
\begin{aligned}
Z_k(l) &= K_y^* Y_k(l) + K_y Y_k^*(l) + W_k(l) \\
Z_k^*(l) &= K_y^* Y_k^*(l) + K_y Y_k(l) + W_k^*(l)
\end{aligned}
\]

where \(W_k(l)\) and \(W_k^*(l)\) are additive noise term on the corresponding subcarriers.

Equation (9) can be rewritten in a matrix form as

\[
Z = KY + W
\]

with

\[
Z = \begin{bmatrix} Z_k(l) \cr Z_k^*(l) \end{bmatrix}, \quad Y = \begin{bmatrix} Y_k(l) \cr Y_k^*(l) \end{bmatrix}, \quad W = \begin{bmatrix} W_k(l) \cr W_k^*(l) \end{bmatrix}, \quad K = \begin{bmatrix} K_1 & K_2 \\
K_1^* & K_2^* \end{bmatrix}
\]

In (11), \([\cdot]^T\) denotes the matrix transposition operation.

### III. I/Q IMBALANCE ESTIMATION AND COMPENSATION ALGORITHM DESCRIPTION

Since the proposed I/Q imbalance estimation algorithm is designed to exploit the synchronization signal in each LTE frame, the time and frequency structure of LTE frame and synchronization signals will be introduced first.

#### A. Structure of LTE Frame and Synchronization Signals

The frame structure type 1 is applicable to FDD in LTE system and is shown in Fig.1.

![Fig. 1. Structure of the LTE frame and synchronization signals](image)

Each frame is 10 ms long and consists of 10 subframes numbered from 0 to 9. Each subframe is 1 ms long and consists of 2 slots. Thus each frame contains 20 slots numbered from 0 to 19. The primary and secondary synchronization signals (P-SCH and S-SCH) are transmitted twice for each frame in slot 0 and slot 10. P-SCH and S-SCH are carried by two consecutive OFDM symbols. The P-SCH and S-SCH symbols are transmitted, respectively in the 6th and 7th OFDM symbols of the slots 0 and 10. Both signals occupy 63 (including DC) subcarriers centered at the DC subcarrier.

The P-SCH is generated from a frequency-domain Zadoff-Chu sequence \(d_u(n)\) according to

\[
d_u(n) = \begin{cases} 
e^{-\frac{\pi un(n+1)}{63}}, & n = 0, 1, ..., 30 \\
e^{-\frac{\pi un(n+1)(n+2)}{63}}, & n = 31, 32, ..., 61 \end{cases}
\]

where the Zadoff-Chu root sequence index \(u = \{25, 29, 34\}\).

The sequences \(d(0), ..., d(61)\) used for the S-SCH are interleaved concatenation of two length-31 binary sequences. The concatenated sequence is scrambled with a scrambling sequence given by the primary synchronization signal [3].

#### B. I/Q Imbalance Estimation and Compensation Using Synchronization Signals

After cell search the S-SCH and P-SCH will be known and can be exploited as the training data for I/Q imbalance estimation. As shown in Fig.2, S-SCH is conveyed by the \(l\)th OFDM symbol and the P-SCH is conveyed on the \((l+1)\)th OFDM symbol. The data on the symmetric and adjacent subcarriers can be used to set up a set of equations. The unknown parameters (I/Q imbalance and
channel response) in the set of equations can be estimated with least square (LS) algorithms [5].

In Fig. 2, the data on the \( k \)th subcarrier and \( l \)th OFDM symbol can be represented by \( X_{k}(l) \). Thus equation (10) can be rewritten in matrix form as

\[
Z = P \cdot C + W \tag{13}
\]

Where

\[
Z = \begin{bmatrix} Z_{k}(l) & Z'_{k}(l) \end{bmatrix}^T
\]

\[
P = \begin{bmatrix} X_{k}(l) & 0 & X'_{k}(l) & 0 \\
0 & X_{k}(l) & 0 & X'_{k}(l) \end{bmatrix}
\]

\[
C = [c_1, c_2, c_3, c_4]^T
\]

\[
= [K_i H_k(l) \ K'_i H'_k(l) \ K_i H'_k(l) \ K'_i H'_k(l)]^T
\]

\( W \) is the noise vector defined in (11).

To use the LS estimation algorithm effectively, the number of independent equations must be greater or equal to the number of unknown parameters. In equation (13), there are four unknown parameters needed to be estimated. Thus we should have at least four equations for the LS based estimation algorithm. Furthermore, we assume that the frequency-domain transfer functions for the adjacent symbols are invariant. For instance, if two adjacent symbols of two pair of symmetric subcarriers ( \( X_{k}(l) \), \( X'_{k}(l) \), \( X_{k}(l+1) \) and \( X'_{k}(l+1) \) ) are used for estimation, we assume

\[
H_{k}(l) = H'_{k}(l+1) \quad \text{and} \quad H'_{k}(l) = H'_{k}(l+1). \tag{14}
\]

Using the four training data, equation (13) can be extended and reformulated as

\[
\tilde{Z} = \tilde{P} \cdot C + \tilde{W} \tag{15}
\]

where \( \tilde{Z} = [Z_{k}(l) \ Z'_{k}(l) \ Z_{k}(l+1) \ Z'_{k}(l+1)]^T \), \( \tilde{W} \) is the noise vector corresponding to the four subcarriers and

\[
\tilde{P} = \begin{bmatrix} X_{k}(l) & 0 & X'_{k}(l) & 0 \\
0 & X_{k}(l) & 0 & X'_{k}(l) \\
X_{k}(l+1) & 0 & X'_{k}(l+1) & 0 \\
0 & X_{k}(l+1) & 0 & X'_{k}(l+1) \end{bmatrix} \tag{16}
\]

Then the parameter vector \( C \) can be estimated with LS method

\[
\hat{C} = [\hat{c}_1, \hat{c}_2, \hat{c}_3, \hat{c}_4]^T = (\tilde{P}^H \tilde{P})^{-1} \tilde{P}^H \tilde{Z} \tag{17}
\]

where \([\cdot]^T\) is the Hermitian transposition operation.

After \( C \) is estimated, we can obtain the estimation of \( \alpha = K'_2 / K_1 \) by

\[
\hat{\alpha} = \frac{1}{2} \left( \frac{\hat{c}_1}{\hat{c}_4} + \frac{\hat{c}_3}{\hat{c}_2} \right) \tag{18}
\]

From equation (4) we know

\[
\alpha = \frac{1 - \gamma}{1 + \gamma} \tag{19}
\]

Thus \( \gamma \) can be estimated by

\[
\hat{\gamma} = \frac{1 - \hat{\alpha}}{1 + \hat{\alpha}} \tag{20}
\]

Consequently, we can get the estimate of \( K_1 \) and \( K_2 \) to be \( \hat{K}_1 \) and \( \hat{K}_2 \) with equation (4).

To improve the estimation performance, \( \alpha \) can be estimated by averaging over a number of subcarriers and OFDM symbols. From equation (10), the demodulated signal without I/Q imbalance can be recovered by

\[
\hat{Y} = \hat{K}^{-1} Z \tag{21}
\]

\( \hat{Y} \) is the estimate of \( Y \) defined in (10) and \([\cdot]^T\) denotes the matrix inversion operation. After I/Q imbalance compensation with, regular algorithms can be used for subsequent channel estimation and detection.

C. Optimal Training Data Selection for Estimation

An important issue in I/Q imbalance estimation is to guarantee the matrix \( \tilde{P} \) defined in (16) to be non-singular and well-conditioned. Thus in our algorithm, an efficient algorithm is used to check this condition before estimation. Only optimal selected training data will be used in I/Q imbalance estimation.

IV. SIMULATION RESULTS

The simulation is set up based on the LTE system specifications [3]. FFT size for OFDM is \( N = 1024 \), the number of data subcarriers in each OFDM symbol is \( N_d = 600 \). Modulation scheme used on each subcarrier is 64-
QAM. The reference channel model used in simulation is 6-ray TU (Typical Urban) with vehicle speed \( v = 100 \) Km/h. The amplitude I/Q imbalance \( \epsilon \) in (3) is 0.2 dB and the phase I/Q imbalance \( \phi \) is 2 degree. The symbol error rate (SER) performances with and without I/Q imbalance estimation and compensation are shown in Fig.3. From the simulation results, we can see that the proposed I/Q estimation and compensation algorithm can effectively compensate the I/Q imbalance even if one LTE frame is used for I/Q imbalance estimation.

In Fig.4, we demonstrate the performance improvement obtained by using optimally selected data of I/Q imbalance estimation. Before optimization, all valid data (\( \hat{P} \) is non-singular) is used in estimation. In the optimized solution, we discard data lead to ill-conditioned system matrix and only optimally selected data is used in estimation. Optimal selection can be based on the condition number of the data matrices. The condition number threshold can be determined through simulation. From the simulation results shown in Fig.4, we can see the significant performance improvement after optimization.

V. CONCLUSIONS

In this paper, a novel receiver I/Q imbalance estimation and compensation strategy is proposed for LTE system. The synchronization signals embedded in each LTE frame are employed for receiver I/Q imbalance estimation. The system architecture and algorithm are described in detail. The computer simulation results indicate that the proposed algorithm can effectively compensate the receiver I/Q imbalance. The performance can be improved further by optimally selecting the training data used for estimation.

![Fig. 3. Performance of the I/Q imbalance estimation and compensation algorithm, I/Q imbalance: \( \epsilon = 0.2 \) dB, \( \phi = 2 \) degree.](image)

![Fig. 4. Performance gain through optimal training data selection, I/Q imbalance: \( \epsilon = 0.2 \) dB, \( \phi = 2 \) degree.](image)

ACKNOWLEDGEMENT

The authors would like to thank InterDigital for their support and help in this work.

REFERENCES