An Approach to Reliability Growth Planning based on Failure Mode Discovery and Correction using AMSAA Projection Methodology

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SUMMARY & CONCLUSIONS

Exact expressions for the expected number of surfaced failure modes and system failure intensity as functions of test time are presented under the assumption that the surfaced modes are mitigated through corrective actions. These exact expressions depend on a large number of parameters. Functional forms are derived to approximate these quantities that depend on only a few parameters. Such parsimonious approximations are suitable for developing reliability growth plans and portraying the associated planned growth path. Simulation results indicate that the functional form of the derived parsimonious approximations can adequately represent the expected reliability growth associated with a variety of patterns for the failure mode initial rates of occurrence. A sequence of increasing MTBF target values can be constructed from the parsimonious MTBF projection approximation based on the following: (1) planning parameters that determine the parsimonious approximation; (2) corrective action mean lag time with respect to implementation and; (3) test schedule that gives the number of planned Reliability, Availability, and Maintainability (RAM) test hours per month and specifies corrective action implementation periods.

1. INTRODUCTION

To mature the reliability of a complex system under development it is important to formulate a detailed reliability growth plan. One aspect of this plan is a depiction of how the system’s reliability is expected to increase over the developmental test period. The depicted growth path serves as a baseline against which reliability assessments can be compared. Such baseline planning curves for Department of Defense (DoD) systems have frequently been developed in the past utilizing the assumed reliability growth pattern specified in Military Handbook 189 (MIL-HDBK-189) [1]. This growth relationship is between the reliability, expressed as the mean test duration between system failures and a continuous measure of test duration such as time or mileage. The equation governing this growth pattern was motivated by the empirically derived linear relationship observed for a number of data sets by Duane (1964), [2], between the developmental system cumulative failure rate and the cumulative test time when plotted on a log-log scale.

In this paper we obtain a non-empirical relationship between the mean test duration between system failures and cumulative test duration that can be utilized for reliability growth planning. This relationship is derived from a fundamental relationship between the expected number of failure modes surfaced and the cumulative test duration. For convenience, we shall refer to the test duration as test time and measure the reliability as the mean time between system failures (MTBF). The functional form of this fundamental relationship is well known and is easily established without recourse to empiricism (see, for example [3]). We obtain an approximation to this relationship that is suitable for reliability growth planning. One significant advantage to our approach is that it does not rely on an empirically derived relationship such as the Duane based approach. We shall show how the cumulative relationship between the expected number of discovered failure modes and the test time naturally gives rise to a reliability growth relationship between the expected system failure intensity and the cumulative test time. The presented approximation for the resulting growth pattern avoids a number of deficiencies associated with the Duane/MIL-HDBK-189 approach to reliability growth planning.

Section 2 contains a list of notation and model assumptions. In Section 3 we highlight a number of issues associated with the Duane/MIL-HDBK-189 approach to reliability growth planning. Section 4 develops the exact expected system failure intensity and parsimonious approximations suitable for reliability growth planning. These functions of test time are derived from the exact and planning approximation relationships between the expected number of surfaced failure modes and the cumulative test time. The exact relationship is expressed in terms of the number of potential failure modes, $k$, and the individual initial failure mode rates of occurrence. Parsimonious approximations to this relationship are obtained. The first approximation utilizes $k$ and several additional parameters. The second approximation discussed is the limiting form of the first approximation as $k$ increases. This approximation is suitable for complex systems or subsystems. The approximations are derived through consideration of an MTBF projection equation. This equation arises from considering the problem of estimating the system MTBF at the start of a new test phase after implementing corrective actions to failure modes surfaced in a proceeding test phase. This MTBF projection
has been documented in [4] and is described in Section 4. Section 5 contains simulation results. The simulations are conducted to obtain actual patterns for the system MTBF versus test time for random draws of initial mode failure rates from several parent populations and numbers of failure modes. The stochastic MTBF realizations are compared to the reciprocal of the theoretical expected system failure intensity and to the parsimonious approximations. Random draws for mode fix effectiveness factors (FEFs) (fraction reductions in initial failure mode rates of occurrence due to mitigation) are used to simulate corrective actions to surfaced failure modes. Using the simulated corrective actions, the relationship between the expected system failure intensity and cumulative test time is simulated for various sets of mode initial failure rates. This relationship is obtained under the assumption that the system failure intensity associated with a cumulative test time \( t \) reflects implementation of corrective actions to the modes surfaced by \( t \) with the associated randomly drawn FEFs. The resulting system MTBF versus test time relationship is compared to the corresponding relationship established for planning purposes.

Section 6 indicates how to construct a sequence of MTBF target values that start at an expected or measured initial MTBF and end at the goal MTBF. It is shown that the parsimonious approximation to the reciprocal of the expected system failure intensity can be used for this purpose in conjunction with a test schedule that specifies the expected monthly RAM hours to be accumulated on the units under test and the planned corrective action periods.

2. NOTATION AND ASSUMPTIONS.

2.1 Notation.

- \( k \) – total number of potential failure modes.
- \( m \) – number of surfaced failure modes.
- \( T \) – total duration of a developmental test.
- \( t_1 \) – length of the initial test phase.
- \( M_1 \) – average initial MTBF over \( t_1 \).
- \( M_G \) – goal MTBF.
- \( \alpha_M \) – growth rate.
- \( MS \) – management strategy.
- \( \mu_M \) – average fix effectiveness.
- \( M(t) \) – number of modes surfaced by time \( t \).
- \( \mu(t) \) – expected value of \( M(t) \).
- \( \mu_i(t) \) – approximation of \( \mu(t) \).
- \( \Lambda_i(t) \) – system failure intensity for unsurfaced modes.
- \( \Lambda(t) \) – system failure intensity, after mode mitigation.
- \( \Lambda_i \) – system failure intensity for unsurfaced modes.
- \( \hat{\lambda}_i \) – standard estimate of \( \lambda_i \).
- \( \tilde{\lambda}_i \) – the Stein estimate of \( \lambda_i \).
- \( \theta_M \) – true but unknown Stein shrinkage factor.
- \( h(t) \) – expected rate of occurrence of new modes at time \( t \).
- \( d_i \) – true but unknown fix effectiveness for mode \( i \).
- \( \rho(t) \) – expected failure intensity at time \( t \).
- \( \rho_i(t) \) – approximation of \( \rho(t) \).
- \( MTBF(t) \) – MTBF at time \( t \).
- \( MTBF_i(t) \) – approximation of \( MTBF(t) \).

2.2 Assumptions.

The system has a large number of potential failure modes with initial rates of occurrence \( \lambda_1, ..., \lambda_k \). The modes are candidates for corrective action if they are surfaced during test. All failure modes independently generate failures according to the exponential distribution and the system fails whenever a failure mode occurs. It is also assumed that corrective actions do not create new failure modes.

3. MIL-HDBK-189 IDEALIZED GROWTH CURVE.

The frequently referenced United States Department of Defense MIL-HDBK-189 [1] utilizes an idealized reliability growth pattern with instantaneous MTBF function given by,

\[
MTBF(t) = \begin{cases} 
M_i & 0 \leq t \leq t_i \\
M_i \left( \frac{t}{t_i} \right)^{\alpha_M} & t > t_i
\end{cases}
\]

(1)

This relationship is motivated by Duane’s empirical relationship [2]. Duane observed, for a number of data sets, that the logarithm of the cumulative failure rate versus the logarithm of the cumulative test time tended to display a linear relationship. Equation (1) is consistent with this relationship. Note that (1) utilizes the power law expression to represent MTBF growth over a sequence of test phases following the initial test phase of length \( t_i \). One can see that the power law expression would not be a realistic growth pattern over the initial test phase, since it would imply the initial MTBF is zero. In the above expression \( t_i \) should be chosen large enough such that corrective actions are scheduled for implementation by the start of the subsequent test phase.

The growth rate \( \alpha_M \) is used as a measure of programmatic risk with respect to being able to grow from \( M_i \) to \( M_G = MTBF(T) \) in test time \( T \). The higher the \( \alpha_M \) relative to past experience the greater the risk of attaining \( M_G \). From Equation (1) we can see that \( MTBF(T) \) is a strictly increasing function of the ratio \( T / t_i \) and can be made as large as desired by making \( t_i \) sufficiently small. Thus for any given \( T, M_i \), and growth rate \( \alpha_M \), one can always find a small enough \( t_i \) such that \( MTBF(T) \) will equal the desired value. This implies that \( \alpha_M \) as a measure of programmatic risk is only as meaningful as the choice of \( t_i \). In particular, one should guard against artificially lowering \( \alpha_M \) by selecting \( t_i \) so small that no significant amount of fix implementation is
expected to occur until during a corrective action period that is beyond \( t_j \). The strong dependence of the global parameter \( \alpha_M \) on the length of the initial test phase is not a desirable attribute for planning purposes.

Finally, we note that Equation (1) implies that, even with a reasonable choice for \( t_1 \), any value of \( M_G \) can eventually be obtained since there is no upper limit implied by Equation (1). However, one must keep in mind that if the planning curve extends over many thousands of hours, the planned growth rate may not be sustainable due to resource constraints besides test time and due to technological constraints.

4. DERIVED RELIABILITY GROWTH PATTERNS.

The first step in obtaining a functional form for the expected failure intensity as a function of test time and planning parameters that is based on non-empirical considerations involves the relationship between the expected number of failure modes surfaced and test duration. This relationship was considered by Crow [3] for the case where test duration is continuous. In this paper we are measuring a generic measure of test duration for this continuous case. In particular, let \( I_i(t) \) denote the indicator function for mode \( i \). The indicator function takes on the value one if mode \( i \) occurs by \( t \) and equals zero otherwise. The number of modes surfaced by \( t \) is given by,

\[
M(t) = \sum_{i=1}^{k} I_i(t)
\]

(2)

The expected value of \( M(t) \) is equal to,

\[
\mu(t) = \sum_{i=1}^{k} E(I_i(t)) = \sum_{i=1}^{k} (1 - e^{-\lambda t}) = k - \sum_{i=1}^{k} e^{-\lambda t}
\]

(3)

This expected value function implies a functional form for the expected failure intensity and corresponding MTBF as a function of test time \( t \) given that corrective actions have been incorporated to all the failure modes surfaced by \( t \). One component of the expected failure intensity is due to the failure modes not yet surfaced by \( t \). This component is simply given by the derivative of \( \mu(t) \). Note,

\[
\frac{d\mu(t)}{dt} = \sum_{i=1}^{k} \lambda_i e^{-\lambda_i t}
\]

(4)

In [5] it is shown that the expression in (4) is the expected failure intensity due to all the modes not surfaced by \( t \). To show this observe that the failure intensity due to these modes can be expressed as the random variable \( \Lambda_U(t) \) where

\[
\Lambda_U(t) = \sum_{i=1}^{k} \Lambda_i [1 - I_i(t)]
\]

(5)

The expected value of \( \Lambda_U(t) \) is given by,

\[
E(\Lambda_U(t)) = \sum_{i=1}^{k} \Lambda_i [1 - E(I_i(t))] = \sum_{i=1}^{k} \lambda_i e^{-\lambda_i t} = \frac{d\mu(t)}{dt}
\]

(6)

Before considering the other components of the system failure intensity, we shall address obtaining parsimonious approximations to the expected number of failure modes surfaced by \( t \) and the corresponding failure intensity due to the unsurfaced failure modes. The exact expressions for these quantities are given by (3) and (6). Note that these expressions depend on \( k + 1 \) parameters, namely the number of potential failure modes \( k \) and the initial failure mode rates of occurrence \( \lambda_i \) for \( i = 1, \ldots, k \).

4.1 Parsimonious Approximations for the Expected Number of Modes, and its Derivative.

To obtain parsimonious approximations to the expected number of modes surfaced by \( t \) and its derivative, we shall consider an optimization problem under the assumption that all corrective actions are delayed until \( t \). Let \( N_i \) denote the number of failures that occur by \( t \) due to mode \( i \). Then \( \bar{\lambda} = \frac{N_i}{t} \) denotes the standard maximum likelihood estimate of \( \lambda_i \). Consider the estimator for \( \lambda_i \) given by,

\[
\bar{\lambda}_i = \theta \cdot \bar{\lambda} + (1 - \theta) \cdot \text{avg}(\lambda_i)
\]

(7)

where \( \text{avg}(\lambda_i) \) denotes the arithmetic average of the \( k \) \( \bar{\lambda} \), and \( \theta \in [0,1] \) is chosen to minimize the expected sum of squared errors between \( \bar{\lambda}_i \) and \( \lambda_i \), i.e. \( E \left[ \sum_{i=1}^{k} (\bar{\lambda}_i - \lambda_i)^2 \right] \). The value of \( \theta \) that solves this optimization problem can be shown to be \( \theta_S \) [4]

where

\[
\theta_S = \frac{\text{Var}[\bar{\lambda}_i]}{\lambda \cdot \text{Var}[\bar{\lambda}_i] + \frac{k}{1 - \theta} - \frac{1}{k} + \theta \cdot \frac{k}{1 - \theta}}
\]

(8)

for \( \lambda = \sum_{i=1}^{k} \lambda_i, \bar{\lambda} = \lambda / k, \) and \( \text{Var}[\bar{\lambda}_i] = \sum_{i=1}^{k} (\bar{\lambda}_i - \bar{\lambda})^2 / k \). The estimate of \( \lambda_i \) given by (7) with \( \theta \) equal to \( \theta_S \) has been called the Stein estimate of \( \lambda_i \) [4]. Note this is a theoretical estimate in the sense that it cannot be computed from the data since it involves the unknown values of \( k, \lambda, \) and \( \lambda \).

From the definition of \( \bar{\lambda}_i \) and the fact that \( N_i \) equals zero for a failure mode unobserved by \( t \) we have that the Stein assessment of the failure intensity due to all the failure modes not surfaced by \( t \) equals

\[
\sum_{i=\text{obs}} \bar{\lambda}_i = (k - m) \cdot (1 - \theta_S) \cdot \left[ \frac{N_i}{k \cdot t} \right] = \left[ \frac{k - m}{k} \right] \cdot (1 - \theta_S) \cdot \left[ \frac{N_i}{t} \right]
\]

(9)

where \( m \) denotes the number of surfaced modes by \( t \) and \( \text{obs} \) denotes the index set for the failure modes not surfaced by \( t \). From (8) one can show,
\[ \sum_{i \in \text{obs}} \tilde{\lambda}_i = \left( 1 - \frac{1}{k} \right) \left( 1 - \frac{m}{k} \right) \left( \frac{N}{t} \right) \left( 1 - \frac{1}{k} \right) + \frac{\sum_{i \in \text{obs}} \tilde{\lambda}_i}{\tilde{\lambda} - \frac{\lambda}{k}} t \]  

(10)

Note that \( m \) can be regarded as an estimate of the expected number of modes surfaced by \( t \), i.e. \( \mu(t) \). Additionally, in light of Equation (6), the left hand side of (10) can be viewed as an estimate of \( \frac{d\mu(t)}{dt} \). From (4), it follows that the derivative of \( \mu(t) \) at \( t = 0 \) equals \( \lambda \). Finally, observe that \( \frac{N}{t} \) in (10) is the maximum likelihood estimate of the initial failure rate \( \lambda \) under the assumption that all corrective actions are delayed to \( t \). Let \( h(t) \) denote \( \frac{d\mu(t)}{dt} \). Simulation results for a number of cases (where \( k \) and the \( \lambda_i \) are known) conducted in support of [4] have indicated that the Stein assessment given in (10) yields good estimates of \( h(t) \) when all the corrective actions are delayed. The value \( h(t) \) that is being estimated does not depend on the corrective action process. Only the estimate of \( \lambda \) given by \( \frac{N}{t} \) depends on the assumption that all corrective actions are delayed until \( t \). Thus the right hand side of (10) with \( m \) and \( \frac{N}{t} \) replaced by good approximations of \( \mu(t) \) and \( \lambda = \frac{d\mu(t)}{dt} \bigg|_{t=0} \) respectively should yield a good approximation for \( h(t) \) regardless of the corrective action process for the cases where the Stein estimate of \( h(t) \) given by (10) are accurate. This suggests that we choose our parsimonious approximation for \( \mu(t) \), denoted by \( \mu_k(t) \), as the unique solution to the differential equation obtained from (10) by replacing \( m \) by \( \mu_k(t) \), \( \frac{N}{t} \) by \( \lambda \), and

\[ \sum_{i \in \text{obs}} \tilde{\lambda}_i \text{ by } \frac{d\mu_k(t)}{dt} \text{ with initial conditions } \mu_k(0) = 0 \text{ and } \frac{d\mu_k(t)}{dt} \bigg|_{t=0} = \tilde{\lambda} \]  

For the case where all the \( \lambda_i \) are equal, one can show that \( \mu_k(t) = \mu(t) \) for all \( t \geq 0 \). Thus, in what follows we shall only consider the case where not all the \( \lambda_i \) are equal. The solution to the resulting differential equation, for this case, with the specified initial conditions is,

\[ \mu_k(t) = k \left[ 1 - (1 + \beta_k \cdot t)^{-\rho_k} \right] \]  

(11)

where

\[ \beta_k = \left( 1 - \frac{1}{k} \right)^{-1} \left( \frac{1}{k} \sum_{i \in \text{obs}} \tilde{\lambda}_i - \frac{\lambda}{k} \right) \]  

(12)

and

\[ p_k = \frac{\lambda}{k \cdot \beta_k} \]  

(13)

The solution was obtained by the method of integrating factors [6]. The solution can be verified by directly substituting (11) and its derivative into the differential equation for \( \mu_k(t) \) and noting that \( \mu_k(t) \) satisfies the specified initial conditions. Observe that \( \mu_k(t) \) can be expressed in terms of \( t \) and three constants, namely \( k \), \( \lambda \) and \( \beta_k \). The corresponding parsimonious approximation for \( h(t) \) is \( \frac{d\mu_k(t)}{dt} \), which we shall denote by \( h_k(t) \).

4.2 Parsimonious Approximations for the Expected System Failure Intensity and MTBF.

Next we shall consider the expected system failure intensity after \( t \) test hours and a corresponding parsimonious approximation, given that corrective actions are implemented to all the surfaced failure modes. We shall let \( d_i \) denote the fraction reduction in the rate of occurrence of mode \( i \) due to the corrective action (termed a fix). The reduction factor is termed the fix effectiveness factor (FEF) for failure mode \( i \). Let \( \Lambda(t) \) denote the failure intensity of the system given that fixes have been incorporated to all the failure modes surfaced by \( t \). Then,

\[ \Lambda(t) = \sum_{i=1}^{k} (1 - d_i I_i(t)) \lambda_i \]  

(14)

The corresponding expected failure intensity is \( \rho(t) \) where

\[ \rho(t) = E(\Lambda(t)) = \sum_{i=1}^{k} (1 - d_i E(I_i(t))) \lambda_i = \sum_{i=1}^{k} (1 - d_i) \lambda_i + \sum_{i=1}^{k} d_i \lambda_i e^{-\lambda_i t} \]  

(15)

This expression for the expected failure intensity was presented in [3].

For reliability growth planning purposes, assessments of individual failure mode FEFs will not be available. Thus, in place of \( \rho(t) \) we shall use a parsimonious approximation, denoted by \( \rho_k(t) \), that utilizes an average fix effectiveness factor. It follows from (6) that \( \lambda - h(t) \) is the expected failure intensity due to the failure modes surfaced by \( t \) prior to mitigation. Assume these modes are mitigated with an average FEF of \( \mu_d \). Then the expected failure intensity due to the surfaced failure modes after mitigation can be approximated by \( (1 - \mu_d) \cdot (\lambda - h(t)) \). Thus the parsimonious approximation for \( \rho(t) \) will be defined as follows:

\[ \rho_k(t) = (1 - \mu_d) \cdot (\lambda - h_k(t)) + h_k(t) \]  

(16)

We also define the parsimonious MTBF approximation of \( MTBF(t) \) defined as \( (\rho(t))^{-1} \) for reliability growth planning by,

\[ MTBF_k(t) = (\rho_k(t))^{-1} \]  

(17)
For planning, it can be useful to add a term, \( \lambda_s \), to the expressions for \( \rho(t) \) and \( \rho_s(t) \) given by (15) and (16) respectively. This term represents the failure rate due to all the failure modes that will not be corrected, even if surfaced (referred to as A-modes in [3]). This term for planning purposes would be given by the quantity \((1-MS)\lambda_s\). However, since this term does not contribute to the difference between \( \rho(t) \) and \( \rho_s(t) \) we shall not consider it further.

It may be difficult to select a value of \( k \) for planning purposes. For complex systems or subsystems it is reasonable to use the limiting forms of \( \mu_1(t) \), \( h_1(t) \), and \( \rho_1(t) \) as \( k \to \infty \). Consider the limit as \( k \to \infty \) of these functions. In taking the limit we shall hold \( \lambda \) fixed and assume the limit of \( \beta_k \) is positive as \( k \) increases, say \( \beta_\infty \in (0,\infty) \). Under these conditions one can show the three functions converge to limiting functions which we shall denote by \( \mu_\infty(t) \), \( h_\infty(t) \), and \( \rho_\infty(t) \), respectively. One can show,

\[
\mu_\infty(t) = \left( \frac{\lambda}{\beta_\infty} \right) \ln(1+\beta_\infty \cdot t) \tag{18}
\]

and

\[
h_\infty(t) = \frac{d\mu_\infty(t)}{dt} = \lambda(1+\beta_\infty \cdot t)^{-1} \tag{19}
\]

Also, \( \rho_\infty(t) \) is given by (16) with \( h_1(t) \) replaced by \( h_\infty(t) \).

5. SIMULATION.

5.1 Simulation Overview.

We wish to compare the parsimonious approximations to realized and expected reliability growth patterns with respect to a number of quantities. To do so we shall generate a number of realized reliability growth patterns via simulation in Mathematica. We shall consider cases where the failure mode initial rates of occurrence are realizations of a specified parent population for several choices of the parent distribution. We shall also generate reliability growth patterns for a deterministically specified sequence of failure mode initial rates of occurrence that have been found to be useful in representing initial bug rates of occurrence in software programs under development [7].

The simulation consists of the following steps: (1) Specify inputs such as test duration, the number of failure modes, and the sequence or distribution governing the parent population of the initial mode failure rates. (2) Produce mode initial failure rates. Failure rates are either stochastically generated, or deterministically calculated. (3) Generate mode failure times via randomly generated uniform numbers, and the mode initial failure rates. (4) Finally, generate mode fix effectiveness factors by drawing realizations of a random sample from a beta distribution with mean 0.80, and coefficient of variation 0.10.

5.2 Simulation Results.

Results below display plots of the reciprocals of the expected and realized system failure intensities (i.e. MTBF) for loglogistic \( \lambda_1 \) (Figure 1), and geometric \( \lambda_i \) (Figure 2). The geometric initial mode failure rates are given by

\[
\lambda_i = a \cdot b^i \tag{20}
\]

for \( i = 1,\ldots,k \) where \( 0 < a \) and \( 0 < b < 1 \). All the displayed quantities have been averaged over ten replications of simulation steps (2) through (4) above.

The intent of the plots is to see whether the functional form of the parsimonious approximations are reasonably compatible with respect to the reciprocal of the expected system failure intensity as a function of test time. Corrective actions are assumed to be implemented to all the failure modes surfaced by \( t \) with the simulated mode fix effectiveness factors. The value of \( \mu_d \) in Equation (16) is set equal to

\[
\frac{1}{k} \sum_{i=1}^{k} d_i
\]

to generate the parsimonious approximations to the exact expected failure intensity and corresponding MTBF. Additionally, for the results displayed below, \( k = 1,500 \) and \( \lambda = 10^{-1} \).

The value of the scale parameter obtained from Equation (12) does not provide adequate parsimonious approximations except when the parent population is gamma or the scale parameter is sufficiently small. Thus for the specified \( k \), \( \mu_1 \), and \( \lambda \), the scale parameters \( \beta_k \) and \( \beta_\infty \) of the parsimonious approximations were fitted to the exact expected number of surfaced failure modes function by using maximum likelihood estimates. These estimates were obtained from the simulated mode first occurrence times. This procedure provided a “best statistical fit” of the parsimonious functional approximations for \( \mu(t) \) and \( \rho(t) \), with respect to the scale parameter, over the entire planning period of interest, i.e. 10,000 hours.

The parsimonious approximations for \( \mu(t) \) and \( \rho(t) \) based on the limiting forms for \( \mu_1(t) \) and \( \rho_1(t) \) as \( k \) increases will tend to be too large for values of \( t \) when \( \mu(t) \) is too close to \( k \). We have observed that the limiting approximations are adequate for \( \mu(t) \) and \( \rho(t) \) over the range of \( t \) for which \( \mu(t) \leq k^\frac{1}{2} \). Thus for complex systems, or subsystems, the limiting approximation functional forms should be adequate representations of \( \mu(t) \) and \( \rho(t) \) over most test periods of interest.

The red curves in the figures represent the reciprocal of the exact expected system failure intensity (Figures 1 and 2). The dots in each figure represent a corresponding stochastic realization. The green curves display the finite \( k \) approximations while the blue curves display the corresponding limiting approximations. Similar results were obtained for the cases where the \( \lambda_i \) were generated from gamma, lognormal, and Weibull parent populations.

For comparison purposes, the MIL-HDBK-189 system MTBF based on Equation (1) was fitted to the reciprocal of the expected system failure intensity (the red curves). The MIL-HDBK-189 curves are displayed in yellow and were fitted utilizing all the observed simulated cumulative times of
failure. The use of all cumulative failure times requires that fixes be implemented when failure modes are observed. The simulation was carried out in this manner to allow the parameters of the MIL-HDBK-189 curves to be statistically fitted via the maximum likelihood estimation procedure in [1]. As for the other displayed quantities, the averages of 10 replicated MIL-HDBK-189 MTBF curves are shown.

Figure 1. Inverse of the Failure Intensity (Loglogistic).

Figure 2. Inverse of the Failure Intensity (Geometric).

6. GENERATING A PLANNED RELIABILITY GROWTH PATH.

Once the planning parameters are chosen, the parsimonious approximation for the expected failure intensity can be used to generate a detailed reliability growth planning curve. For example, suppose a test schedule is laid out that gives the planned number of RAM miles accumulated on the units under test per month. Also suppose the test schedule specifies blocks of calendar time for implementing corrective actions. Finally, for planning purposes let us assume that in order for a failure mode to be addressed in an upcoming corrective action period, it must occur four months prior to the start of the CAP. Since \( MTBF(t_i) \) depends on a large number of parameters it would be approximated by the parsimonious approximation \( \{\rho_+(t_i)\}^{-1} \) or \( \{\rho_-(t_i)\}^{-1} \). In such a manner a sequence of target MTBF steps would be generated that grow from the initial MTBF value to a goal MTBF value.

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BIOGRAPHIES

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