Common Errors in Application of MIL-STD-781

Terrance W. Elliott; Pacific Missile Test Center; Point Mugu

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Abstract

This paper discusses some common errors in application of MIL-STD-781 that lead to inflated estimates of true reliability. These errors have occurred because of the availability and acceptability of this standard for electronic equipment. As electronic equipment has become more complex and flexible, testing schemes have deviated from Poisson requirements and functional test equipment often will not detect all the possible failure modes. This paper will discuss the impact of these errors along with recent work in mathematical deviation of equations for correct test design.

Introduction

MIL-STD-781, in its various versions, has become a commonly applied test specification for reliability demonstration tests in government acquisition contracts of electronic systems. This has probably occurred for the following three reasons: (1) MIL-STD-781 is easy for managers to apply at the contract level, (2) MIL-STD-781 is applicable to systems with Poisson probability distributions which is believed to be true for complex electronic systems which covers the bulk of major government acquisition today, and (3) sequential tests are efficient. This has led to MIL-STD-781 being increasingly misapplied in government acquisition contracts without due consideration given to the applicability of this Standard and, correspondingly, inflated estimates of true reliability are advertised from the results of these tests.

The three most common application errors are (1) using this Standard, for equipment with a Poisson failure probability distribution, when the test is run as a Bernoulli trial, (2) functional testing of the electronic system with test equipment that does not measure 100 percent of the failure rate or possible failure modes, and (3) combining two different kinds of functional tests, such as a self-test and a full system test, neither of which check 100 percent of the failure rate.

Poisson Vice Binomial

The original development effort on sequential tests centered around one-shot devices, or number defective out of a given sample size. The failure probability for these situations is Binomial and so the original sequential test equations were Binomial based. Later, it was recognized that electronic systems fail with a Poisson failure probability distribution (random in time) so sequential test designs for Poisson failure probability became increasingly important. Eventually, MIL-STD-781 was developed to cover this important case, to standardize test design, and to increase the applications of sequential testing.

Since then, Binomial failure probability test designs have been largely ignored for testing of electronic systems (there is no Binomial MIL-STD comparable to MIL-STD-781). This has contributed to a lack of "thinking the problem through" to assure that a Poisson failure probability is appropriate. (MIL-STD-105, "Sampling Procedures and Tables for Inspection by Attributes" is based on a Binomial failure distribution when the AQL is less than 10.0. However, it is not intended as a sequential test specification with repeated tests of the same unit under specified risks. It is noted, that with a little effort, the analogous Binomial sequential test can be derived for the sampling plans of MIL-STD-105.)

To help in thinking it through, first, review the necessary conditions for when each distribution is applicable. The Poisson derived test plan may be used when the underlying probability density function is given by the exponential density function

\[ f(t,e) = \frac{1}{\theta} \exp(-t/\theta) \]

and the functional status is known at all times (failures are detected when they occur). The Binomial distribution is applicable:

(a) to evaluation of the probability of successes in \( n \) trials,
(b) to predicting the reliability of go-no-go devices,
(c) to hardware which operates in a cyclic fashion.

The third area of applicability for the Binomial is the source of one of the most common errors in application of MIL-STD-781. MIL-STD-781 specifies that only "on" time shall be counted to determine test length and accept/reject criteria. However, many classes of electronic equipment are periodically dormant during in-service usage and the reliability parameter of interest is the MTBF over both the "on" and "off" time summation. (A typical example is captive carry of air-launched guided missiles where the missile is normally dormant except when functionally tested to determine status.) For these classes of equipment, a MIL-STD-781 test plan is often applied, ignoring the requirement to count only "on" time, however, when electronic equipment, with a Poisson failure distribution, is operated cyclically (status is determined at specific time intervals) the Binomial distribution describes the detectable failure probability for test design purposes over succeeding cycles.

Consider an item of electronic equipment operated cyclically (some period of "on" time followed by a period of "off" time). This equipment may be subjected to some physical environments during the "off" period that may cause failures or weaken some part such that failures are precipitated during the "on" period. During a test to simulate in-service usage, time is counted as the total test time not merely the "on" time (in-service field data is also counted this way). However, failures only become

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apparent when the system is "off" so the knowledge of the failure is "event-based," when the system is "on," rather than "time-based." Consequently the knowledge of the failure distribution, i.e., the data collected, is Binomial and not Poisson. (The probability of a failure within a time interval is Poisson, but the probability distribution of detected data over multiple cycles is Binomial.) Thus, any test design for this case should be based on a Binomial distribution since the data to be analyzed is Binomial.

In real world use of periodic functional testing, where the test interval is a large fraction of the MTBF, multiple failures often occur. When this data is analyzed, the analyst has the option of considering it as one system failure (the unit is "dead") or counting each failure independently. In the first case it is obviously a Binomial analysis. In the second case, however, it can be shown that when multiple failures are counted the Poisson distribution is applicable. One caveat, however, is the assumption that one failure does not affect the probability of occurrence of another failure (as might happen if a power supply failed removing interval heat generation which could contribute to a temperature related failure). Thus, as test interval decreases (or off time) and the probability of failure becomes small the Poisson is a good assumption, however with longer intervals, as is usually encountered, the Binomial is more accurate.

Consider another situation that frequently occurs. A complex electronic system is being tested for reliability. Functional checkout is accomplished by another complex electronic test set. The test sets are expensive and limited in number such that they must be shared between different activities and one test set cannot be totally dedicated to the reliability test. Typically, the test is performed in a chamber for some period of time, the test item removed from the test and checked out then, if unfailed, returned to test. Again, the knowledge of the failure distribution is "event based," or Binomial.

The important point to remember is that the test design should be based upon the knowledge of the failure distribution obtained from the functional test scheme and not the assumed true failure distribution.

Figure 1 is an example of the impact on test design when the test is based upon Binomial vice Poisson. In Figure 1 the risks are 10 percent on each side, the discrimination ratio is 2.0, and the functional test is performed at different fractions of the lower mean-time-between-failures (MTBF). The divergence of the Poisson and Binomial lines are apparent and it can also be seen that at some point the Poisson "accept" line will cross the Binomial "reject" line. Figure 1 also shows that as the test interval becomes small the Binomial lines approach the Poisson lines. It also shows that with no failures both distributions are equivalent.

Test Set Effectiveness

As electronic systems have grown more complex, the test equipment to functionally check the system has grown even more complex and more costly. The effort and cost to design and build a test set that can test all possible failure modes, in some cases, is essentially unaffordable. This has resulted in test sets which measure less than 100 percent of the failure rate. The percent, or proportion, of failure rate measured by a test set has become a common design specification. This parameter is called the effectiveness, e, of the test sets and is specified as the actual proportion (usually 0.90 - 0.95) of the total failure rate measured.

When a MIL-STD-781 reliability demonstration test is specified, recognition is not usually made of the fact that the test equipment is less than 100 percent effective. Consider that a test set with an effectiveness, e, is used to determine an MTBF, \( \theta \), of a system. The true MTBF will be less than the test measured MTBF by the factor e.

\[ \theta_{true} = e \theta_{test} \]  
(2)

Thus, the calculated probability of apparently passing a test of time, \( t \), will be greater since the test observed MTBF is greater than the true MTBF. Thus, the probability of passing is given by

\[ P(t) = \exp\left(\frac{-t}{\theta_{true}}\right) \]  
(3)

and the probability of observing \( x \) failures, in the Poisson case, is given by

\[ P(x) = \binom{t}{x} \exp(\theta) \frac{\exp(-\theta)}{x!} \]  
(4)

Then, new accept-reject lines for a sequential test, adjusting for test set effectiveness, can be constructed from the following (reference (1)):

\[ \frac{e t}{\theta_{1}} \frac{1}{\theta_{0}} + \ln \frac{1 - B}{\alpha} \leq x \leq \frac{1}{e \theta_{1}} - \frac{1}{\theta_{0}} + \ln \frac{B}{1 - \alpha} \]  
(5)

where the \( \theta \) values are the upper and lower MTBFs and \( \alpha \) and \( B \) are the risks as used in MIL-STD-781.

For the Binomial case the probability of a failure corresponding to each MTBF is...
$P_0 = 1 - \exp(-\delta t/\theta_0)$ ($P_0 =$ Probability of 1 or more failures when $\theta_0$ is true) \hspace{1cm} (6)

$P_1 = 1 - \exp(-\delta t/\theta_1)$ ($P_1 =$ Probability of 1 or more failures when $\theta_1$ is true)

where $\delta$ is the time between functional tests on the test set. The sequential test accept-reject lines can then be constructed from the following:

\[
\ln \left(\frac{P_1}{P_0}\right) = \ln \left(\frac{1-P_1}{1-P_0}\right)
\]

\[
\ln \left(\frac{P_1}{P_0}\right) = \frac{1}{\delta} \ln \left(\frac{1}{1-P_0}\right) \leq x \leq \frac{1}{\delta} \ln \left(\frac{1}{1-P_0}\right)
\]

\[
\ln \left(\frac{P_1}{P_0}\right) = \frac{1}{\delta} \ln \left(\frac{1}{1-P_0}\right)
\]

where

- $n =$ number of test intervals of $\delta$
- $x =$ number of failures

The impact on the sequential test lines is easy to visualize by recalling eq. (2) which says that for $i$ failures the test time must be $1/e$ times longer for the same conclusion when test set effectiveness is considered. This implies that a simple correction for sequential test plans would be to make the abscissa $1/e$ vice $0$, implying test time will be $1/e$ longer for $\theta$ to be true.

**Multiples Functional Tests**

As electronics have grown more complex, with more possible failure modes, and higher MTBFs are required, a self-test mode, or built-in-test (BIT), is often incorporated to provide on-call status of the equipment. The BIT serves the same purpose as the full systems test set but typically has a lower effectiveness. The BIT may be a useful addition to the full systems test set during a reliability test, particularly if the time between full systems tests is long. It does, however, introduce some additional complexity into the design of the test.

Recall that the sequential test is a probability ratio test. The previous sequential test equations were derived from solutions to eq. (8).

\[
\frac{P_1}{P_0} < B < \frac{P_1}{P_0}
\]

where $A$ and $B$ are accept and reject criteria, $P_1$ is the probability of getting a particular set of results given that MTBF is equal to the lower value, and $P_0$ is the same probability given that MTBF is equal to the upper value.

When the functional testing is periodic, $P_1$ and $P_0$ are Binomial probabilities. When these probabilities are substituted into eq. (8) the Binomial coefficients, which define the number of possible paths to a particular set of results, cancel. Therefore, the previously discussed Binomial sequential tests may be considered path independent.

When two kinds of functional tests, BIT and full systems test, are combined this path independency is not generally true. Figure 2 shows some possible test result state points achievable with the combined test scheme. The dots are the test result state points reachable through full system test failures. In general, the full system test result state points are a subset of the BIT test result state points. The number of paths from one state point to another state point with one more failure is much higher for BIT tests than for full systems tests.

When equation (8) is solved for the combined systems test, each $P_i$ must include all possible combinations of full system test detectable failures and BIT detectable failures that add up to the correct total number of failures. Each term, in turn, will have its own path coefficient which is different than path coefficients for the other terms. This path factor will consist of two terms, the number of paths for the full systems test detectable failures and the number of paths for the BIT detectable failures.

\[
P_1 = \frac{\sum_{i=0}^{N} K_{IF} P_i (N-I) \times K_{IB} P_0(I) BIT}{P_0 = \sum_{i=0}^{N} K_{IF} P_0 (N-I) \times K_{IB} P_0(I) BIT}
\]

where

- $K_{IF} =$ path factor, $I$, for full system failures,
- $K_{IB} =$ path factor, $I$, for BIT failures,
- $P_1 =$ probability given lower MTBF is true
- $P_0 =$ probability given upper MTBF is true
- $N =$ total number of failures
- $I =$ number of BIT failures

In general, the path factor coefficients do not cancel as before, thus, there is not path independency and the equation is not easily solved.
One solution to this problem is to assume that the failures will be distributed between BIT detection and full systems test detection in the same ratio as the BIT effectiveness (i.e., if $e = 0.33$ then $1/3$ of the failures are detected by BIT and $2/3$ by the full systems test). This is consistent with the definition of test set effectiveness. Then the terms to either side of this case may be neglected such that

$$\frac{P_1}{P_0} = \frac{K_{JB} P_0(N(1-e))}{K_{JB} P_0(N(1-e))} \times \frac{K_{JB} P_1(Ne)}{K_{JB} P_1(Ne)}$$

and the path factors now cancel. Some numerical examples have shown this approximation to be sufficiently close to the ratio obtained when all terms are present that its use appears reasonable.

Then, a test plan may be derived for the combined testing scheme, although, not in a simple format. It is probably easier to iteratively solve for various points on each accept or reject line to specify the test design. Figure 3 is an example of a combined test showing Poisson lines and full system test only lines for comparison.

**Conclusions**

1. Test design should be based on the knowledge of the failure distribution obtained from the test and not the assumed failure distribution.

2. Test set effectiveness less than unity ($e < 1$) require $1/e$ more testing for the same conclusion since, effectively, only $e$ of the system is being tested.

3. Combined functional testing schemes are more efficient than single testing schemes but it is more difficult to determine the correct test design.

**References**


**Biography**

Terrance W. Elliott, B.S.M.E., M.B.A.
Pacific Missile Test Center
Point Mugu, California 93042 USA

Mr. Elliott is a senior reliability test engineer for the U.S. Navy at the Pacific Missile Test Center. Since 1963 he has been involved in environmental and reliability of air-carried guided missiles. He currently works in the Production Acceptance Test and Evaluation (PATE) Division which performs all Navy prosecuted acceptance programs of new production air-carried guided missiles. He was deeply involved in all environmental and reliability testing of the PHOENIX missile during production, performed the HARPOON Production Reliability Test, which is a long term reliability test using temperature cycling and acoustically induced vibration, and is currently involved in the reliability development and testing of the AMRAAM missile. He holds a B.S.M.E. from the University of California, Berkeley, and an M.B.A. from California Lutheran College.