Interference Suppression Using Knowledge-Aided Subarray Pattern Synthesis

1st Lt David New
Air Force Institute of Technology
Wright-Patterson AFB, OH 45433
Email: david.new@afit.edu

Lt Col Phillip Corbell, Ph.D
Air Force Institute of Technology
Wright-Patterson AFB, OH 45433
Email: phillip.corbell@afit.edu

Abstract—Phased array systems often subarray many antenna elements into far fewer digitized channels. While having more degrees of freedom (DOF) yields better performance, adding channels to create more digital DOF increases system cost and data throughput requirements. A subarray itself constitutes a phased array with as many DOF as it has antenna element weights. Typically, only one degree of freedom is specified to steer the maximum gain direction of the subarray pattern. For typical antenna geometries a single subarray will provide many more spatial DOF than there are digitized channels. The inherent DOF of the subarrays could be used to mitigate selected interference signals with the subarray pattern if the antenna manifold, the angle of arrival (AOA), and the power of interference sources at the array face are known (i.e., Knowledge-Aided (KA)). The AOA and power estimates can then be used to synthesize an adapted subarray pattern to null interference, preserving digital DOF for other purposes. Simulations are used to illustrate the implementation for a simple antenna scenario.

I. INTRODUCTION

For a bevy of good reasons, many modern phased array antennas consist of a large array size with hundreds or thousands of antenna elements, but only a handful of digitized channels (i.e., the number of analog-to-digital converters (ADCs)). The array size is determined by the desired beamwidth, gain, monopulse accuracy, etc. while the number of channels is typically dictated by the application (Moving Target Indication (MTI), Synthetic Aperture Radar (SAR), etc.) or a set of Electronic Protection (EP) requirements. Connecting elements to channels is the antenna manifold: the analog network of combiners, phase shifters or time delays, amplifiers and attenuators that constitute mini-phased arrays (or subarrays) for each channel. Subarrays must be dynamically steered in the direction of interest in real time, effectively forming a real subarray beampattern through which each channel senses the electromagnetic world. Having sampled the world through the lens of the subarray beampattern and stored it in digital bits, the channel data enters the adaptive signal processing chain responsible for canceling interference and/or clutter, finding moving targets, or measuring a signals direction of arrival (DOA), etc. A notional system processing chain of a subarrayed antenna is shown in Figure 1.

![Notional system processing chain of a subarrayed antenna.](image)

The number of channels, \( C \), sets the number of digital DOF of a system, \( C - 1 \), the amount of which has a strong correlation to system capability, performance, and system cost. The ideal scenario of having a dedicated ADC behind every antenna element is only warranted (or possible) in a small handful of phased array systems. The costs of increasing channel count is not limited to the material and manufacturing costs of more receive chains and ADCs, but also the burden of processing more digital data and the testing and calibration of more channels.

There are cases when such a system would greatly benefit from additional spatial DOF. There can be hundreds, or in some cases thousands of DOF afforded by the antenna elements in each subarray. By the necessity of being steerable, a subarray contains the essential hardware (i.e., hardware weights and a combiner) to create real adaptive beampatterns, but without sampled data from their individual elements the adaptive weight set can not be directly estimated. If one could somehow discern how to adapt what those weights to null interference, the non-adaptive steerable subarrays could begin to leverage their spatial DOF.

While digital nulling is preferable to analog, there are certainly situations where both may provide superior performance over digital alone. Adaptive spatial nulling done in the subarrays could mitigate interference in the sidelobes, thereby freeing up digital DOF for Space-Time Adaptive Processing (STAP). All of this is possible through the use of KA techniques. Through careful measurement of the antenna manifold it would be possible to both discern the AOA of strong interference sources and use that AOA to synthesize adaptive subarray beampatterns (i.e., subarray pattern synthesis) with nulls in the directions of those sources. The body of KA adaptive radar processing brought about by the DARPA KASSPER program supports the validity of this approach [1].

The objective of this paper is to illustrate and explore a concept for increasing the spatial interference suppression (i.e.,
nulling) capabilities of a phased array radar by leveraging KA techniques to employ the inherent but dormant DOF of its subarrays. It will postulate a calibrated (i.e., known) phased array manifold design that when combined with cognitive KA algorithms, and utilizing subarray adaptive beamforming prior to the ADC, may be capable of effectively creating many more spatial nulls than what C channels would conventionally allow. Further detail and results are available in [2].

II. RADAR MODEL

Various radar models have been developed and improved over the years, principally for the purposes of Space-Time Adaptive Processing (STAP) research [3]–[5]. The model, in [6], which encompasses planar arrays and subarrays, is the basis for the model used in this paper. Integral to this research is the assumption of a known and calibrated phased array manifold, made possible via detailed in-situ calibration or calibration techniques. Inclusion of uncalibrated array manifold errors would result in disparities between the ideal and realized adapted patterns, leading to inaccurate null placement [7].

A. Geometry

The radar model for this research considers an active electronically scanned phased planar array. Figure 2 shows the geometry of the array, which exists in the x-z plane. Radar boresight points in the positive y-axis direction. Distance d_x separates the N columns uniformly in the horizontal dimension, and distance d_z separates the P rows uniformly in the vertical dimension. The radar elevation angle, θ, is measured from the positive y-axis to the positive z-axis, indicating that negative elevations point toward the ground. The radar azimuth angle, φ, is measured from the positive y-axis towards the positive x-axis. The face of the array is organized into subarrays which are fed separately through a single channel [8]. This model defines N_ch columns of channels and P_ch rows of channels across the array face. Each channel is connected to N_sub x P_sub elements and is steerable through the antenna manifold.

The effects of subarraying have been simulated in the past as shown in Chapter 6 of [9]. This research follows the subarray model in [6] in which a subarray pattern is modeled as an element pattern in an array in which the array factor is calculated from the phase centers of the subarrays. Both approaches are equivalent, but the approach in [6] is more efficient for modeling large arrays.

B. Beam Patterns

With the array geometry established, the array beam pattern, G(θ, φ), can be expressed as

\[
G(\theta, \phi) = |W_{ch}(\theta, \phi)|^2 G_{sub}(\theta, \phi),
\]

in which \(W_{ch}(\theta, \phi)\) is the array factor as synthesized from the phase centers of the subarray channels, and \(G_{sub}(\theta, \phi)\) is the subarray gain pattern.

When each subarray consists of a single element (\(N_{sub} = P_{sub} = 1\)), the subarray pattern is simply the element pattern. This research models a microstrip radiating element pattern which provides a relatively flat response across look angles [10], to approximate element patterns found in modern radars. Manipulation of the spacing between the slots within the microstrip allows the element designer to widen or narrow the element beamwidth as necessary. Throughout this research the spacing is set so that 60° off boresight corresponds to a 2 dB drop in gain.

C. Noise and Interference Model

A noise and interference model is used to illustrate the benefits of subarray pattern synthesis. This model development is valid for non-overlapping, like-sized subarrays. The development of the noise and interference models follow the development in [11]. Summarized here, greater detail can also be referenced in Chapter 2 of [4] or Chapter 3 of [5].

1) Noise and Interference Model: The noise in this research is simulated as independent and identically distributed complex white Gaussian noise with a power \(\sigma^2\) present in the receive chain behind each element. The assumption of white noise remains valid as long as the Pulse Repetition Frequency (PRF) of the radar is significantly less than the bandwidth of the radar waveform. The statistics of the noise as received by a single subarray are captured in the \(N_{sub}P_{sub} \times N_{sub}P_{sub}\) noise covariance matrix, \(R_{n,sub}\). The statistics of the noise as received through the digitized channels are captured in the \(N_{ch}P_{ch} \times N_{ch}P_{ch}\) noise covariance matrix, \(R_{n,chn}\). The noise covariance matrices, \(R_{n,sub}\) and \(R_{n,chn}\), are given by

\[
R_{n,sub} = \mathbb{E} \{X_{n,sub}X_{n,sub}^H\} = \sigma^2 I_{MN_{sub}P_{sub}},
\]

\[
R_{n,chn} = \mathbb{E} \{X_{n,chn}X_{n,chn}^H\} = \sigma^2 N_{sub}P_{sub}I_{MN_{chn}P_{chn}},
\]

where \(\mathbb{E}\) is the expectation operator, and \(H\) denotes a Hermitian transpose. The spatial noise snapshots, \(X_{n,sub}\) and \(X_{n,chn}\), are vectors denoting the noise measured at each element or within each channel. Note that in the case of \(R_{n,chn}\) the noise is summed across the elements of the subarray, and so the noise power increases linearly.
The matrices $\mathbf{R}_{n,sub}$ and $\mathbf{R}_{n,ch}$ refer to the estimated covariance matrices. Given the i.i.d. noise assumption, the maximum likelihood estimate of the noise covariance matrix is achieved through Sample Matrix Inversion (SMI), expressed in matrix form as

$$\hat{\mathbf{R}}_{n,sub} = \frac{1}{t} \left[ \mathbf{X}_{n,sub} \mathbf{X}_{n,sub}^H \right], \quad (4)$$

$$\hat{\mathbf{R}}_{n,ch} = \frac{1}{t} \left[ \mathbf{X}_{n,ch} \mathbf{X}_{n,ch}^H \right], \quad (5)$$

in which $t$ is the number of training data samples used to estimate the covariance matrix, and $\mathbf{X}_{n,sub}$ or $\mathbf{X}_{n,ch}$ are matrices of spatial noise snapshots. The quality of this estimate as a function of $t$ is established by the Reed, Mallett and Brennan Rule [12]. The matrices of snapshots, $\mathbf{X}_{n,sub}$ or $\mathbf{X}_{n,ch}$, are created by concatenating $t$ realizations of $\mathbf{x}_{n,sub}$ or $\mathbf{x}_{n,ch}$, respectively.

The interference is simulated as complex white Gaussian noise in the far field. The number, power, and location of the interference sources are determined by the scenario. The second order interference statistics as received by a single subarray are represented in the spatial covariance matrix, $\mathbf{R}_{j,sub}$. The second order statistics of the interference as received through the digitized channels are captured in the spatial covariance matrix, $\mathbf{R}_{j,ch}$. The interference covariance matrices, $\mathbf{R}_{j,sub}$ and $\mathbf{R}_{j,ch}$, are given by

$$\mathbf{R}_{j,sub} = \mathbb{E} \left\{ \mathbf{X}_{j,sub} \mathbf{X}_{j,sub}^H \right\}, \quad (6)$$

$$\mathbf{R}_{j,ch} = \mathbb{E} \left\{ \mathbf{X}_{j,ch} \mathbf{X}_{j,ch}^H \right\}, \quad (7)$$

where the spatial interference snapshots, $\mathbf{X}_{j,sub}$ and $\mathbf{X}_{j,ch}$, are vectors denoting the received interference signals behind each element or within each channel. The power of the $b^{th}$ interference source is either received through the element pattern, $\mathbf{g}$, or the subarray pattern, $\mathbf{G}_{sub}$, in the cases of $P_{j,sub,b}$ and $P_{j,ch,b}$ respectively. The powers are given by

$$P_{j,sub,b} = \frac{S_{j,b} g(\theta_b, \phi_b) \lambda_0^2}{(4\pi)^2 R_{j,b}^2 L_r}, \quad (8)$$

$$P_{j,ch,b} = \frac{S_{j,b} G_{sub}(\theta_b, \phi_b) \lambda_0^2}{(4\pi)^2 R_{j,b}^2 L_r}, \quad (9)$$

where $R_{j,b}$ is the range to the $b^{th}$ interference source, $L_r$ is the loss of the receiver, $S_{j,b}$ is the $b^{th}$ interference source's effective radiated power spectral density, and $(\theta_b, \phi_b)$ is the AOA of the $b^{th}$ interference source.

The interference covariance matrices can be estimated from the matrix of $t$ concatenated spatial interference snapshots, $\mathbf{X}_{j,sub}$ and $\mathbf{X}_{j,ch}$, as

$$\hat{\mathbf{R}}_{j,sub} = \frac{1}{t} \left[ \mathbf{X}_{j,sub} \mathbf{X}_{j,sub}^H \right], \quad (10)$$

$$\hat{\mathbf{R}}_{j,ch} = \frac{1}{t} \left[ \mathbf{X}_{j,ch} \mathbf{X}_{j,ch}^H \right], \quad (11)$$

The covariance matrices representing the subarray or channel for noise and interference, $\hat{\mathbf{R}}_{sub}$ or $\hat{\mathbf{R}}_{ch}$ respectively, are found by summing the noise and interference covariance matrices. This also holds for the estimated covariance matrices, $\hat{\mathbf{R}}_{sub}$ and $\hat{\mathbf{R}}_{ch}$.

Both $\mathbf{R}_{ch}$ and $\mathbf{R}_{sub}$ are non-singular and full rank (since the noise covariance matrix is full rank). The number of adaptive DOF of the digital array factor is $N_{ch} P_{ch} - 1$. By following the same logic it can be concluded that $N_{sub} P_{sub} - 1$ is the number of adaptive DOF of the subarray factor. Therefore, if the subarray pattern is made adaptive, one could potentially utilize $(N_{ch} P_{ch} - 1) + (N_{sub} P_{sub} - 1)$ spatial adaptive DOF.

### III. METHODOLOGY

The investigative method of this research is software simulation. The subarray antenna patterns and digital array factors which constitute different spatial filters are formed using three different methods within this research. In the case of a radar which is not subarrayed, Adaptive Digital Beamforming (ADBF) is used in adapting the digital array factor. For a subarrayed antenna, conventional operation dictates that the subarray pattern will be steered towards the location of interest, but not made adaptive. The digital array factor is still adapted. This method is referred to as Subarray Beam Steering (SBS)-ADBF. The Knowledge-Aided Subarray Pattern Synthesis (KASPS)-ADBF method is the novel contribution of this research, by which both the subarray pattern and digital array factor are adapted.

When every element is sampled individually and weights are adapted to null interference sources, as described in [13], it is referred to as ADBF. The adaptive spatial filter, $\mathbf{w}_{ch}(\theta_0, \phi_0)$, is the adaptive matched filter for the estimated interference covariance matrix [12], $\hat{\mathbf{R}}_{ch}$, given the antenna manifold and desired look angle specified by the steering vector, $\mathbf{v}_{ch}(\theta_0, \phi_0)$. This technique is shown pictorially in Figure 3(a). The filter is given by

$$\mathbf{w}_{ch}(\theta_0, \phi_0) = \hat{\mathbf{R}}_{ch}^{-1} \mathbf{v}_{ch}(\theta_0, \phi_0). \quad (12)$$

Conventional subarrayed architectures steer the subarray pattern toward the look direction of interest. To steer a subarray in the direction of a look angle $(\theta_0, \phi_0)$, the subarray weights are set equal to a steering vector derived from the antenna manifold geometry. The subarray weights are given as

$$\mathbf{w}_{sub}(\theta_0, \phi_0) = \mathbf{v}_{sub}(\theta_0, \phi_0). \quad (13)$$

Even though the subarrays are non-adaptive, the digital weights are still adapted. The digital weights are calculated from the covariance matrix (as sensed through the subarray pattern) and a steering vector derived from the phase centers of each subarray, equivalent to the formulation in Equation (12). This nulling configuration, using steered but non-adaptive subarray weights and adaptive digital weights is referred to as SBS-ADBF. This technique is shown pictorially in Figure 3(b).

In order to improve the interference suppression capabilities of subarrayed systems, it could be advantageous to use the subarray weights to null interference as well. However, calculating adaptive subarray weights in practice presents a challenge not faced in calculating adaptive digital weights. In calculating
the adaptive digital weight set the covariance matrix, $\hat{\mathbf{R}}_{ch}$, can be estimated directly from the channel measurements. By contrast, there is no way to directly observe the subarray covariance matrix, $\mathbf{R}_{sub}$, within the antenna manifold. Thus, using knowledge of the manifold and interference, a synthetic covariance matrix, $\hat{\mathbf{R}}_{sub}$, must be synthesized to calculate the adaptive subarray weights, shown as

$$\mathbf{w}_{sub}(\theta_0, \phi_0) = \hat{\mathbf{R}}_{sub}^{-1} \mathbf{v}_{sub}(\theta_0, \phi_0).$$

The matrix, $\hat{\mathbf{R}}_{sub}$, can be synthesized using a model [2] with estimates of the manifold, the interference source locations in azimuth and elevation, and the interference source power at the array face. There are many possible methods for determining interference source parameters, including monopulse beams, external apriori information, and other onboard systems. This paper assumes the availability of this information. With the set of adaptive subarray weights and adaptive digital weights generated, the beam pattern may be formed as depicted in Figure 3(c). This nulling configuration is referred to as KASPS-ADB.

It will not always be advantageous to null all interference sources in the scenario with the subarray pattern [2]. In choosing the best subarray pattern, combinations of subarray pattern nulls are iteratively tested for the pattern that best minimizes Jamming and Noise-to-Signal Ratio (JNSR). The JNSR is the inverse of Signal-to-Interference and Noise Ratio (SNR), and is given by

$$JNSR(\theta, \phi) = \frac{\mathbf{w}_{ch}(\theta_0, \phi_0) \mathbf{H} \mathbf{R}_{ch}^{-1} \mathbf{w}_{ch}(\theta_0, \phi_0)}{\sigma^2 N_{sub} \sigma^2 N_{ch} \xi_m(\theta, \phi) | \mathbf{w}_{ch}(\theta_0, \phi_0) \mathbf{H} \mathbf{R}_{ch}^{-1} \mathbf{w}_{ch}(\theta_0, \phi_0) |^2},$$

in which $\xi_m$ represents the per-sample Signal-to-Noise Ratio (SNR) for a single subarray.

A system’s coverage area is defined as the scannable area in azimuth and elevation below a maximum JNSR threshold. Coverage statistics (i.e., Figure 8) for a range of JNSR thresholds are used in determining an EP systems effectiveness in suppressing interference [14].

### IV. RESULTS

The JNSR plot shown in Figure 4 and the following JNSR plots throughout this section are calculated using unique beam patterns for each look angle. For instance, at a given look angle, $(\theta_0, \phi_0)$, the beam pattern is adapted according to the method in use and the JNSR is calculated. At the next look angle the beam pattern is uniquely adapted again, and again the JNSR is calculated, until the entire field of view has been analyzed.

Figure 4 shows a noise-only JNSR baseline for an $8 \times 8$ array with no subarraying. The contour of the JNSR output in Figure 4 correlates to the dropoff in element pattern gain (through both transmit and receive). Thus, minimum JNSR, which has been set at $-10$ dB, occurs at boresight, $(0^\circ, 0^\circ)$, and increases to $-6$ dB as either azimuth $(\phi^o)$ or elevation $(\theta^o)$ reach $60^\circ$ (the extent of the field of view examined). Figure 4 represents the clairvoyant noise-only bound on performance. The blue (+) line of Figure 8 shows the coverage statistics of the noise-only response by indicating the percentage of the field of view (from $-60^\circ$ to $60^\circ$ in elevation and azimuth) from Figure 4 for which JNSR is less than the value specified.

![Antenna Manifold](image)

Fig. 3. Application of subarray and digital weights under three different interference suppression schemes.

<table>
<thead>
<tr>
<th>Param.</th>
<th>$N_{ch}$</th>
<th>$P_{ch}$</th>
<th>$N_{sub}$</th>
<th>$P_{sub}$</th>
<th>$P_t$ (kW)</th>
<th>$\sigma^2$ (kW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>2</td>
<td>2</td>
<td>4</td>
<td>4</td>
<td>10</td>
<td>$1.25 \times 10^{-18}$</td>
</tr>
</tbody>
</table>

Table I. Antenna Properties for the scenarios shown in Figures 6 and 7.

The blue (+) line of Figure 8 shows the JNSR coverage statistics for these
conditions, in which the digital DOF outnumber the jamming signals, 64 to 4.

Figure 6 shows the JNSR when digital adaption is used on the subarrayed antenna in Table I and interference sources 1-4 from Table II. Under these conditions, the black (x) line in Figure 8 shows the coverage statistics. The three digital DOF are not sufficient to null all four interference sources, resulting in a large inoperable area for the radar, as expected. Examination of Figure 6 reveals regions of performance comparable to that of the non-subarrayed scenario (i.e. the regions surrounding (5°, −10°), (−5°, 15°), etc). These regions occur where one or more of the jamming signals has fallen into a quiescent null of the subarray pattern, $G_{sub}$. When this occurs, the channel data contains three or fewer interference signals, and the digital DOF become sufficient to digitally null those interference sources. However, this phenomenon does not provide consistent coverage across the desired scan area for reliable radar operation. The effects of having insufficient DOF can be observed for large areas of the field of view.

Figure 7 shows the JNSR output from the environment consisting of interference sources 1-4 from Table II and the antenna from Table I when using the KASPS algorithm to selectively place subarray pattern nulls. The purple (o) line of Figure 8 shows the JNSR coverage statistics.

Figure 8 overlays the coverage statistic curves as calculated from Figures 4, 5, 6, and Figure 7. To summarize, the “Ideal, No Jamming” curve shows the coverage statistics for the case in which there is an ADC behind every element in a noise only environment. As such, this curve represents the best possible coverage statistics, with signal degradation being only a function of the dropoff of the element pattern. The “Ideal ADBF” curve shows the coverage statistic for the case in which there is an ADC behind every element, interference sources 1-4 from Table II present, and in which ADBF is being used to suppress the interference. This curve can be seen to represent the ideal clairvoyant nulling solution for this particular interference source laydown. The “SBS-ADBF” curve shows the coverage statistics for the subarrayed antenna from Table I, interference sources 1-4 from Table II, and using ADBF to suppress the interference. The curve shows large amounts of degradation due to the insufficient DOF. Finally, the “KASPS-ADBF” curve shows the coverage statistics for the subarrayed antenna from Table I, interference sources 1-
A set of subarray weights could be synthesized and applied to performance in the processor. This research has illustrated how manifold can augment and enhance adaptive digital nulling signals. Mitigation of selected interference sources in the array antenna to mitigate spatially localised interference signals is an untapped resource that could be employed via modest upgrades to existing radars. KASPS offers some of the EP benefits of more digitized channels without their cost. The existing DOF inherent in phased array subarrays is an untapped resource that could be used synergistically with digital DOF for clutter suppression. Finally, a proof-of-concept test on real hardware utilizing a known manifold is envisioned to prove real-world viability.

In an era of ever increasing electronic attack, it is increasingly important to provide means of EP at an affordable cost. KASPS will provide the radar designer with an affordable and powerful tool in maximizing the operable field of view when confronted by a highly populated hostile interference environment.

4 from Table II, and using KASPS in concert with ADBF to suppress the interference. This curve approaches the ideal nulling solution, showing vast improvement over using ADBF alone.

Up through this point, all results have been calculated for a single interference source laydown as described in Table II. In order to investigate the sensitivity of the technique to various interference source laydowns, Figure 9 shows the coverage statistics of the four different nulling methodologies as averaged over 100 different four interference source laydowns. Each interference source laydown consists of four interference sources whose azimuth and elevation are independently and uniformly distributed across the field of view. The error-bars in Figure 9 show the standard deviation of the data. The KASPS-ADB technique outperforms the conventional SBS-ADB technique by many standard deviations for most JNSR thresholds.

**V. CONCLUSION AND CONTINUING WORK**

This research demonstrates a proof of concept for utilizing untapped spatial DOF inherent to the subarrays of a phased array antenna to mitigate spatially localised interference signals. Mitigation of selected interference sources in the array manifold can augment and enhance adaptive digital nulling performance in the processor. This research has illustrated how a set of subarray weights could be synthesized and applied to place subarray pattern nulls on multiple interference sources in the scene.

To further examine the application of this technique for improving the performance of existing radar systems, a number of topics are being pursued for further study. Among them are to repeat this analysis while accounting for mutual coupling, hardware effects, aperture errors, and other sources of error. The analysis will also be expanded to investigate how subarray DOF may be used synergistically with digital DOF for clutter suppression. Finally, a proof-of-concept test on real hardware utilizing a known manifold is envisioned to prove real-world viability.

In an era of ever increasing electronic attack, it is increasingly important to provide means of EP at an affordable cost. KASPS offers some of the EP benefits of more digitized channels without their cost. The existing DOF inherent in phased array subarrays is an untapped resource that could be employed via modest upgrades to existing radars. KASPS will provide the radar designer with an affordable and powerful tool in maximizing the operable field of view when confronted by a highly populated hostile interference environment.

**References**


