Range Sidelobe Control for LFMCW Waveforms in OTHR with Direct Digital Receivers

Yuri I. Abramovich  
WR Systems  
Fairfax, VA 22030 USA

Geoffrey San Antonio  
US Naval Research Laboratory  
Washington, DC 20375 USA

Index Terms—HF OTHR, range-sidelobes, mismatch filtering, periodic waveforms

I. INTRODUCTION

Current generation Over-the-Horizon Radar (OTHR) operating in skywave or surface wave propagation modes exploit predominantly periodic linear frequency modulated continuous wave (LFMCW) waveforms in conjunction with so-called “stretch” or “deramping” receiver technology [3]. In these receivers, a shifted in time-delay replica of the transmitted LFMCW waveform is used as a mixing signal in a heterodyne receiver. In addition narrowband intermediate frequency (IF) selectivity filters with bandwidth $\Delta f$ at the output of the mixer stage specify the range “depth” available for surveillance.

In most cases

$$\Delta f \ll B_w$$ (1)

where $B_w$ is the bandwidth of the transmitted waveform. The number of range resolution cells available for surveillance by the above described “deramping” receivers is equal to $\eta_\Delta$

$$\eta_\Delta = \Delta f / \text{WRF}$$ (2)

which is much smaller than the total number range cells $\eta_\text{t}$ within the waveform repetition interval:

$$\eta_\text{t} = B_w / \text{WRF} \gg \eta_\Delta$$ (3)

where WRF is the waveform repetition frequency in Hz. In the context of specific radar operating conditions such as limited range intervals of useful oblique backscattered signals or surface wave returns, this receiver/system output limitation did not necessarily create problems and at the same time provided an important invariance property. Indeed, the use of a narrowband IF filter and heavy range taper applied to the output time-domain data, created conditions closely resembling conventional (non-circular) convolution of a single LFM waveform for the limited number of ranges $\eta_\Delta$ within the passband filter $\Delta f$. It is well known that the LFM waveform is tolerant to a Doppler frequency shift that may lead to a peak time-shift at the output of the tapered matched filter, but retains the low range sidelobe level irrespective of this shift. In this paper we introduce a mismatched filtering technique that can be used in modern digital receivers that exploits the cyclic waveform structure to achieve low range sidelobe and low white noise mismatch losses.

II. OPTIMAL CYCLIC MISMA TCHED FILTERS

Modern OTHR development with the introduction of wide-band direct digital receivers (DDRx) provide a possibility to dramatically increase the number of simultaneous observed ranges, ultimately up to the entire range span $\eta_\text{t}$ (3). Yet, the conventional (tapered) matched filtering in this case does not mimic single LFM waveform matched filtering, since the “range processing” is now cyclic and not the conventional linear convolution of a single waveform. For cyclic matched processing, the well-known properties of a single LFM pulse are not retained anymore. In figure (1) we show the spectrum of a “hanning” window amplitude taper. In figure (2) we show a comparison between linear convolution range processing and circular convolution range processing using the “hanning” taper. One can see two effects. First, in both cases, the far range sidelobes do not fall off as might be expected from the “hanning” taper spectrum as shown in figure (1). Second, the cyclic processing causes an increase in the far range sidelobes compared to the linear convolution case. This is an indication that the proper taper should account for the cyclic processing. One can see that in the immediate vicinity of the main peak, the range sidelobes are very close to the sidelobes of a single LFM pulse with the tapered matched processing.

In the case of significant time-delay shifts, the range sidelobes get very large. If for example these distant range cells are to be used as training cells for adaptive interference mitigation beamforming, the large range sidelobes of distant targets may create significant problems. Therefore, for such applications, the traditional range processing is not appropriate and has to be reconsidered with respect to the cyclic convolution that takes place for LFMCW waveforms.

First, for cyclic convolution, a mismatched filter that controls cyclic range sidelobes within a certain range-Doppler frequency area could be proposed. For example an $N = B_w / \text{WRF}$ element mismatched filter that drives all (N-1) sidelobes to zero could be designed. However without appropriate care, this filter usually has unacceptable signal-to-white-noise ratio (SWNR) losses. The actual area of the main peak that is not suppressed, as well as diagonal loading allow for efficient cyclic sidelobe control within the entire range depth with insignificant SWNR losses. Note, in [1] a sidelobe suppression technique for periodic sequences is presented, but it does not consider the issues mentioned with respect to radar waveforms.
Consider a cyclic type waveform such as a periodic continuous wave linear frequency modulated having a fundamental waveform consisting of \( N \) samples \( x = [x_1, x_2, \ldots, x_N]^T \). Then the mismatched filter \( w \) of length \( N \) designed to reject all \( N - 1 \) range sidelobes can be computed as

\[
w = \frac{R^{-1}x}{x^HR^{-1}x}
\]

where \( R \) is the \( N \times N \) covariance matrix computed as

\[
R = \sum_{i=2}^{N} x(i)x^H(i) + \sigma^2I.
\]

Here the notation \( x(i) \) denotes the waveform vector circularly shifted by \( i \) samples, \( x(i) = [x_i, x_{i+1}, \ldots, x_N, x_1, \ldots, x_{i-1}]^T \). The scalar \( \sigma^2 \) multiplying the identity matrix can be used to control the tradeoff between the mismatch filter white noise mismatch loss and the peak-to-sidelobe level. It is evident that without any loading the covariance matrix will be rank deficient. In addition, it is possible to oversample the fundamental waveform and use some waveform delays to control desired width of the mainbeam.

The mismatched filter solution as presented applies exactly only for the zero-Doppler target case. An alternate formulation could compensate for a single non-zero Doppler frequency or more generally, some interval of Doppler frequencies. The covariance calculation may be modified as follows to accommodate a set of Doppler frequencies to optimize the range sidelobe behavior over.

\[
R = \sum_{j=1}^{N_1} \sum_{i=2}^{N} (x(i) \otimes v(f_j))(x(i) \otimes v(f_j))^H + \sigma^2I.
\]

Here \( v(f_j) \) is defined as \( v(f_j) = \exp(j2\pi f_j \times [0, 1, \ldots, N - 1]^T/f_s) \), where \( f_j \) is the Doppler frequency in Hz and \( f_s \) is the waveform sample rate in Hz.

III. Simulations

In figure (5) we present the results of such cyclic sidelobe control by the optimized mismatched cyclic compression filter. One can see that unlike the conventional taper that allows for range sidelobe control in close vicinity of the main peak only, the cyclic mismatched filter provides very efficient control of all range sidelobes with SWNR losses that even smaller than for a conventional amplitude taper. In figures (3,4) we show the magnitude and phase respectively of the three filtering schemes. Notice the difference in amplitude between the optimized cyclic mismatched filter and the conventional hanning. Also, notice the phase behavior of the optimized cyclic mismatched filter. For reference we have included figure (6) which shows the range-filter output for various single pulse LFM range filters. In this case linear convolution has been applied. One can see that neither the optimal \( N \)-length filter or \( 3N - 2 \)-length filter can suppress the range-sidelobes to the extent that is possible with the optimal cyclic mismatched filter.

Unlike the single LFM pulse processed by the tapered matched filter, the cyclic mismatch filters are not Doppler-tolerant and for significant Doppler mismatches relative to the WRF, the sidelobe level rises back to that of the original matched cyclic filter level of -40dB. Figure (7) show the case of a 4Hz Doppler offset. Even with this slight Doppler offset the far-range sidelobes have begun to rise. The “brute force” approach is to use a “matched to every Doppler frequency” (i.e. on the grid \( \Delta f = 1/T_{CIT} \)) mismatched circular filter. In this case, strong targets at any Doppler Frequency, being processed by individually matched (in frequency) mismatched filter, will demonstrate the same sidelobes as per figure (5).

The solution proposed above should be treated as an over-kill since the Doppler Frequency tolerance of cyclic range processing compression filter with memory equal to the repetition interval should not depend on the number of repetition intervals within \( T_{CIT} \) (coherent integration time).

Since a mismatched cyclic filter may be designed for a broader Doppler frequency band along the range sidelobe ridge (with a certain SWNR degradation) the number of range compression filters may be reduced from \( \text{WRF} \times T_{CIT} \) (for the “matched as every Doppler frequency approach) to a more manageable number. Figure (9) illustrates the performance of the mismatched cyclic compression filter calculated for range sidelobe suppression within the Doppler frequency band of \( \pm 7 \)Hz. One can see that the achieved sidelobe level (-65dB), accompanied by the SWNR losses of -0.64dB may be treated as acceptable with 3 different cyclic compression filters required to cover the entire WRF. For reference, figure (8) shows the range-sidelobes for a target at 0Hz Doppler using the modified cyclic optimized mismatched filter.

IV.Conclusion

This paper has introduced a mismatched filtering technique for controlling the range sidelobe behavior in cyclic type waveforms such as LFM CW. We have derived and simulated the performance for mismatched filters with and without Doppler tolerance. Finally note that the approach introduced thus far is far from what may be considered optimal processing for small target detection since the SWNR losses are equally applicable to strong and weak targets. The optimum processing in this regard is delivered by the well-known CLEAN technique [5], [6], that induces minimal SWNR losses for weak targets. In a full paper to follow, we consider and compare all options discussed above applicable to strong target range sidelobe control in OTHR with periodic LFM CW (or any periodic CW) waveform in direct digital receiver systems.

Acknowledgment

This work was sponsored by the Office of Naval Research under an NRL 6.1 Base Program

References

Fig. 1. Hanning window Spectrum

Fig. 2. Linear vs. circular weighted matched filtering.

Fig. 3. Magnitude response of conventional, weighted, and optimized filters.

Fig. 4. Phase response of conventional, weighted, and optimized filters.

Fig. 5. Conventional, weighted, and optimized filter outputs for 0 range and 0Hz target. 0Hz Doppler filter optimization.

Fig. 6. Conventional, weighted, and optimized filter outputs for 0 range and 0Hz target. 0Hz Doppler filter optimization using single LFM pulse and linear convolution.
Periodic LFM Cross-Ambiguity Function - Single Doppler Cut = 4.2021Hz
Fs = 20kHz, N = 501, T = 25.05ms, BW = 5kHz, WRF = 39.9202Hz
Opt Filter Param: N/r/Nk = 2/8, Diag. Load = 40dB
Dopp Window = 0 - 0Hz, f = 0.1Hz
Conv. Matched Filter: WNL = -0.0025168dB
Conv. Weighted Matched Filter: WNL = -1.7523dB
Opt. Mismatched Filter: WNL = -1.2702dB

Fig. 7. Conventional, weighted, and optimized filter outputs for 0 range and 4.2Hz target. 0Hz Doppler filter optimization.

Periodic LFM Cross-Ambiguity Function - Single Doppler Cut = 0Hz
Fs = 20kHz, N = 501, T = 25.05ms, BW = 5kHz, WRF = 39.9202Hz
Opt Filter Param: N/r/Nk = 2/8, Diag. Load = 40dB
Dopp Window = 0 - 0Hz, f = 0.1Hz
Conv. Matched Filter: WNL = 0dB
Conv. Weighted Matched Filter: WNL = -1.7523dB
Opt. Mismatched Filter: WNL = -1.2702dB

Fig. 8. Conventional, weighted, and optimized filter outputs for 0 range and 0Hz target. ±7Hz Doppler filter optimization.

Periodic LFM Cross-Ambiguity Function - Matched Filter
Fs = 20kHz, N = 501, T = 25.05ms, BW = 5kHz, WRF = 39.9202Hz
Matched Filter: WNL = -0.0025168dB
Weighting Param: Hanning

Fig. 9. Conventional, weighted, and optimized filter outputs for 0 range and 4Hz target. ±7Hz Doppler filter optimization.

Fig. 10. Matched filter periodic LFM cross-ambiguity function (auto in this case).

Periodic LFM Cross-Ambiguity Function - Weighted Matched Filter
Fs = 20kHz, N = 501, T = 25.05ms, BW = 5kHz, WRF = 39.9202Hz
Opt Filter Param: N/r/Nk = 2/8, Diag. Load = 40dB
Dopp Window = 0 - 0Hz, f = 0.1Hz
Conv. Matched Filter: WNL = -0.0025168dB
Conv. Weighted Matched Filter: WNL = -1.7523dB
Opt. Mismatched Filter: WNL = -1.2702dB

Fig. 11. Weighted matched filter periodic LFM cross-ambiguity function.

Fig. 12. Optimized (single Doppler) filter periodic LFM cross-ambiguity function.

Fig. 13. Optimized (multi-Doppler) filter periodic LFM cross-ambiguity function.

Fig. 14. Various filter schemes white noise mismatch loss as a function of Doppler frequency.