Planar Frequency Diverse Array Receiver Architecture

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Abstract—In this paper, we extend the beamforming theory for frequency diverse arrays (FDA) to planar array geometries and give spatial pattern snapshots from the transmit and receive aspects. This is a natural extension of linear FDA work. Prior to this paper, FDA has only been discussed as either a linear array on transmit or with a receive processing chain that excludes some transmitted signals. We propose a technique that allows all transmitted signals to be seen and processed by each element on receive in a monostatic configuration.

I. INTRODUCTION

Frequency diverse array radar systems have future use in multi-mission, multi-mode structures, as suggested in [1]. Specifically, when the energy distribution requirements of a radar would benefit from a controllable range dependency, the FDA architecture could satiate this need. Much has been investigated regarding the linear frequency diverse (FD) array but many radar applications require the ability to beamsteer in higher dimensionalities than a linear array affords, prompting the planar array development suggested in this paper.

Typically, planar array systems allow beamsteering in two dimensions (azimuth and elevation) but is range independent. This requires additional signal processing to determine the range of a target of interest. FDA systems allow for three-dimensional beamsteering with a planar array geometry and could possibly lead to elimination of additional range processing, although that is not discussed here.

Here-to-date, the literature has mentioned either transmit arrays or receiver architectures that reject or exclude the return signals from frequencies other than what was transmitted from that element. Specifically, in [2], the concept of a uniform linear array with FD on transmit was first presented and subsequently developed in [3]. Farooq and Baier, in [4] and [5], both look at FDA for synthetic aperture radar purposes but reject signals on receive that are not frequency matched to the transmit element. Sammartino has investigated novel beamforming techniques for both transmit and receive in [6], but again, only investigates the linear array. These practices neglect practical considerations, such as the desire to beamsteer in multiple dimensions, in receiver implementation and therefore motivates research to develop an architecture that does not exclude signals and extends to planar geometries.

In this paper, we propose a planar array with frequency offsets along both axes such that a linear increase is witnessed in both dimensions. We also propose a receiver architecture that allows beamforming on receive to include all frequencies from a planar geometry. This will allow the receiver to maximize the signal-to-noise ratio for the available power. The signal structure herein is flexible enough to include other frequency progressions or waveform types although not investigated here. The 4-D pattern snapshot visualization technique allows the complex pattern to be directly compared to the familiar constant frequency (CF) planar array pattern, providing insight into the value of planar FDA.

The remainder of this paper is outlined as follows. Section II discusses the geometry and scenario, including parameters used for simulation. Section III develops the transmit signal structure, gives a closed-form expression after making a narrowband approximation, and charts the transmit spatial pattern when compared to a standard CF array. Section IV develops the receiver architecture and spatial pattern. Lastly, section V contains conclusions and future work suggestions.

II. GEOMETRY AND SCENARIO

The elements of the planar FDA are assumed to be ideal isotropic radiators and without noise interference. For this analysis, let the array span the X-axis with inter-element spacing $d_x = \frac{\lambda_{R\min}}{2}$, and Y-axis with inter-element spacing $d_y = \frac{\lambda_{R\min}}{2}$, such that the reference element of the array is located at $(0, 0, 0)$, and $\lambda_{R\min}$ corresponds to the wavelength of the maximum frequency transmitted by the array, see Figure 1 from [7]. In this paper, the FDA will transmit a set of linearly increasing frequencies, on the X and Y-axes with step sizes $(\Delta f_x = \Delta f, \Delta f_y = (N + 1)\Delta f)$ respectively, where $|\Delta f|$ is limited in order for the system to be considered narrowband. Doing so, the set is succinctly represented as $f_{nm} = f_c + n\Delta f_x + m\Delta f_y$ for $n = 0...N - 1$ and $m = 0...M - 1$, where $f_c$ is the carrier and $N$ and $M$ are the number of elements in the array along the X and Y-axes, respectively. See Figure 2 for a graphical depiction of this configuration.

We assume the radar is operating in continuous wave (CW) mode and that graphics of the patterns are snapshots for fixed time $t$ where $t \gg \frac{2R}{c}$, $2R$ is the two-way range,
and \( c \) is the RF speed of propagation. This is an important aspect of our analysis as it helps frame the approach used to describe and depict time-dependent patterns. However, it must be remembered that FDA spatial patterns are periodic in time.

In the following receiver architecture discussion, we note that linear frequency progression on either axis is not necessary, but it does require complete spectral diversity, meaning no repeated frequencies on transmit. Without this spectral diversity to distinguish each signal on receive, an additional method, such as coding, would be necessary to separate the signals to apply the appropriate beamforming weights. In the following two sections, the transmit and receive signals are developed and simulated using the parameters found in Table I. It provides values for quantifiable parameters such as, the number of elements in the array, the grid limits, grid spacing for the points in space where the signal was calculated and measured, as well as the target location.

### Table I

<table>
<thead>
<tr>
<th>Simulation Parameters</th>
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<tbody>
<tr>
<td>number X-axis elements: ( N )</td>
</tr>
<tr>
<td>number Y-axis elements: ( M )</td>
</tr>
<tr>
<td>element spacing: ( d_x ) and ( d_y )</td>
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<tr>
<td>carrier frequency: ( f_c )</td>
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<tr>
<td>X-axis frequency offset: ( \Delta f_x )</td>
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<tr>
<td>Y-axis frequency offset: ( \Delta f_y )</td>
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<td>X grid limits</td>
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<td>X grid spacing</td>
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<td>Z grid limits</td>
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<td>Z grid spacing</td>
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<td>target Y location: ( y_o )</td>
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<td>target Z location: ( z_o )</td>
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### III. Transmit Spatial Pattern

In this section, we discuss the complete transmit signal and present a closed-form expression of the planar transmit spatial pattern and graphically depict the shape of the main beam spatial pattern. The signal transmitted by the \((n,m)\) emitter is a complex sinusoid given as

\[
s_{nm}(t) = a_{nm}(t) \exp\{j2\pi f_{nm}t\}.
\]  

(1)

The signal when delayed to a target location is

\[
s_{nm}(t) = a_{nm}\left(t - \frac{R_{nm}}{c}\right) \exp\left\{j2\pi f_{nm}\left(t - \frac{R_{nm}}{c}\right)\right\},
\]

(2)

where \( a_{nm}(t) \) is a complex weighting factor that represents transmission and propagation effects that may be neglected (i.e., \( a_{nm}(t) = 1 \) \( \forall (n,m) \)) for our purposes. The signal is measured for a target at \((x_o, y_o, z_o)\) for element locations \((x_n, y_m, 0)\) by setting \( R_{nm} = \sqrt{(x_o - x_n)^2 + (y_o - y_m)^2 + z_o^2} \). Letting the reference point be \((0,0,0)\) and making a far-field approximation, we express range as

\[
R_{nm} \cong R_o - nd_x \sin \theta_o \cos \phi_o - md_y \sin \theta_o \sin \phi_o,
\]

(3)

where \( R_o = \sqrt{x_o^2 + y_o^2 + z_o^2} \) \( \cos \theta_o = \frac{z_o}{R_o} \) and \( \tan \phi_o = \frac{y_o}{x_o} \), and boresight is measured perpendicular to the reference element along the \( Z \)-axis (see Figure 1). This allows (2) to be rewritten as

\[
s_{nm}(t) \cong \exp\left\{j2\pi f_{nm}\left(t - \frac{R_o}{c} - \frac{nd_x \sin \theta_o \cos \phi_o}{c}
+ \frac{md_y \sin \theta_o \sin \phi_o}{c}\right)\right\}.
\]

(4)

In order to beamform on transmit, an additional phase term is necessary that comprises two components (angle and range).
We steer the beam in angle \((\hat{\theta}_o, \hat{\phi}_o)\) and range \(\hat{R}_o\) to yield a composite beam-weighting factor
\[
\alpha(\hat{R}_o, \hat{\theta}_o, \hat{\phi}_o) = \exp\left\{j 2\pi f_{nm} \left(\frac{\hat{R}_o}{c} - \frac{nd_x \sin \hat{\theta}_o \cos \hat{\phi}_o}{c}\right)
- \frac{md_y \sin \hat{\theta}_o \sin \hat{\phi}_o}{c} \right\},
\]
where \((\hat{\theta}_o, \hat{\phi}_o)\) and \(\hat{R}_o\) are relative to the reference element. The transmit signal from a single element as seen by a point target in space is
\[
s_{nm}(t; \hat{R}_o, \hat{\theta}_o, \hat{\phi}_o) = \alpha(\hat{R}_o, \hat{\theta}_o, \hat{\phi}_o)s_{nm}\left(t - \frac{R_{nm}}{c}\right),
\]
\[
= \exp\left\{j 2\pi f_{nm} \left(t - \frac{R_o - \hat{R}_o}{c}\right)
+ \frac{nd_x \sin \hat{\theta}_o \cos \hat{\phi}_o - \sin \hat{\theta}_o \cos \hat{\phi}_o}{c}
+ \frac{md_y \sin \hat{\theta}_o \sin \hat{\phi}_o - \sin \hat{\theta}_o \sin \hat{\phi}_o}{c} \right\},
\]
Continuing, consider the transmit signal in (6) from each element and sum over all \(X\) and \(Y\)-axes contributions to give the total observed signal at \((x_o, y_o, z_o)\):
\[
s(t; \hat{R}_o, \hat{\theta}_o, \hat{\phi}_o) = \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} \alpha(\hat{R}_o, \hat{\theta}_o, \hat{\phi}_o)s_{nm}\left(t - \frac{R_{nm}}{c}\right),
\]
\[
= \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} \exp\left\{j 2\pi f_{nm} \left(t - \frac{R_o - \hat{R}_o}{c}\right)
+ \frac{nd_x \sin \hat{\theta}_o \cos \hat{\phi}_o - \sin \hat{\theta}_o \cos \hat{\phi}_o}{c}
+ \frac{md_y \sin \hat{\theta}_o \sin \hat{\phi}_o - \sin \hat{\theta}_o \sin \hat{\phi}_o}{c} \right\}.
\]
Making a plane wave approximation (target range, \(R \gg \frac{D^{2\pi}}{4\lambda_{\text{co}}^2}\), \(D\) is largest dimension of aperture) and narrowband assumption (bandwidth \(\ll f_c\)) we derive a closed-form expression where \(\omega_x = \pi \Delta f_x\), \(\omega_y = \pi \Delta f_y\), \(\omega_o = \pi \Delta f_o\), and \(\lambda_c = \frac{\lambda}{c}\). The term \(\exp\left\{j \Phi\right\}\) contains additional phase factors associated with the geometry of the set-up, but do not necessarily contribute to the structure of the pattern, which is of most importance. It is important to note that by making the narrowband assumption and manipulating the signal into a sinc-like structure, we are excluding the quadratic phase terms that manifest when multiplying out the frequency components.

In Figure 3, we display the 10-dB mainbeam width. While sidelobes are present, they are greater than 10-dB down from the mainbeam and therefore are not visible in this portrayal. To get a better feel for the iso-surface presentation, observe the cross section of the CF transmit spatial pattern and notice the concentric rings of varying color that represent receding gain values. As expected, the CF pattern does not vary in range but the FDA pattern appears periodic in angle and range in three dimensions. Additionally, we display the patterns of four different offset configurations in Figure 4. Notice that even though the offsets can have the same magnitude, the pattern is also dictated by the “direction” of the offset (+, −) and along which axis \((X, Y)\) the offset progresses. We are visualizing a single pattern ambiguity, if we computed the pattern for a larger volume the periodicity would be evident. This phenomenon could be a nuisance for the radar designer, but with range fall-off and selective frequency offset choices such that the ambiguities are below the minimum discernible signal of the system, it could be an uncontroversial point.

IV. RECEIVE SPATIAL PATTERN

We now develop the receive architecture and signal structure and provide a 4-D visualization of the main beam receive spatial pattern. Begin with the complete transmit signal in (7) and account for the two-way delay to arrive at the received signal at each node
\[
y_{pq}(t) = \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} \alpha(\hat{R}_o, \hat{\theta}_o, \hat{\phi}_o)s_{nm}\left(t - \frac{R_{nm} + R_{pq}}{c}\right),
\]
\[
\approx \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} \exp\left\{j 2\pi f_{nm} \left(t - \frac{2R_o - \hat{R}_o}{c}\right)
+ \frac{nd_x \sin \hat{\theta}_o \cos \hat{\phi}_o - \sin \hat{\theta}_o \cos \hat{\phi}_o}{c}
+ \frac{md_y \sin \hat{\theta}_o \sin \hat{\phi}_o - \sin \hat{\theta}_o \sin \hat{\phi}_o}{c}
+ \frac{qd_z \sin \hat{\theta}_o \cos \phi_o}{c} + \frac{qd_y \sin \hat{\theta}_o \sin \phi_o}{c} \right\}.
\]
In order to apply the appropriate phase to each frequency and reconstruct to beamform, it is necessary to filter at each receive element to parse the received signal according to its transmitting node. Each filter bank contains a set of ideal bandpass filters, i.e., flat passband and complete attenuation of all other signals. We then apply the beamform weighting and combine to form the final output. Figure 5 shows the architecture necessary to complete the reconstruction.
\[ s(t; \hat{R}_o, \hat{\theta}_o, \hat{\phi}_o) \cong \exp \left\{ j \Phi \right\} \times \frac{\sin \left( \omega_x N \left( t - \frac{R_c - \hat{R}_x}{c} \right) + \omega_o N (\sin \theta_o \cos \phi_o - \sin \hat{\theta}_o \cos \hat{\phi}_o) \right)}{\sin \left( \omega_x N \left( t - \frac{R_c - \hat{R}_x}{c} \right) + \omega_o N (\sin \theta_o \cos \phi_o - \sin \hat{\theta}_o \cos \hat{\phi}_o) \right)} \times \frac{\sin \left( \omega_y M \left( t - \frac{R_c - \hat{R}_y}{c} \right) + \omega_o M (\sin \theta_o \sin \phi_o - \sin \hat{\theta}_o \sin \hat{\phi}_o) \right)}{\sin \left( \omega_y M \left( t - \frac{R_c - \hat{R}_y}{c} \right) + \omega_o M (\sin \theta_o \sin \phi_o - \sin \hat{\theta}_o \sin \hat{\phi}_o) \right)}, \]

(8)

Additional signal labels are for intermediate steps that, for convenience and succinctness, are not discussed. However, for a more complete derivation of a uniform linear array see [8]. The beamforming weight is

\[ \beta_{pm} \left( \hat{R}_o, \hat{\theta}_o, \hat{\phi}_o \right) = \exp \left\{ j 2 \pi f_{nm} \left( \frac{\hat{R}_x}{c} \right) - \frac{p d_x \sin \hat{\theta}_o \cos \hat{\phi}_o}{c} - \frac{q d_y \sin \hat{\theta}_o \sin \hat{\phi}_o}{c} \right\}, \]

(10)

such that the beam formed received signal is expressed as

\[ y(t; \hat{R}_o, \hat{\theta}_o, \hat{\phi}_o) = \sum_{p=0}^{P-1} \sum_{q=0}^{Q-1} \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} \exp \left\{ j 2 \pi f_{nm} \left( t - \frac{2(\hat{R}_o - \hat{R}_x)}{c} + \frac{(n + p) d_x (\sin \theta_o \cos \phi_o - \sin \hat{\theta}_o \cos \hat{\phi}_o)}{c} + \frac{(m + q) d_y (\sin \theta_o \sin \phi_o - \sin \hat{\theta}_o \sin \hat{\phi}_o)}{c} \right\}, \]

(11)

Maintaining the assumptions of a plane wave and narrowband signal on receive, we arrive at a closed-form expression given

Fig. 4. Various transmit spatial pattern snapshots from a planar FDA radar for different frequency offsets. (upper left) \( \Delta f_x = 1 \) kHz, \( \Delta f_y = 10 \) kHz, (upper right) \( \Delta f_x = -1 \) kHz, \( \Delta f_y = -10 \) kHz, (lower left) \( \Delta f_x = 10 \) kHz, \( \Delta f_y = 1 \) kHz, (lower right) \( \Delta f_x = -10 \) kHz, \( \Delta f_y = -1 \) kHz.

Fig. 5. Receive beamforming chain of the FDA architecture. Including filter banks at each receive node that apply appropriate phase shifts to steer the beam in range and angle for a planar array.
as where $\omega_o = \frac{\pi d}{\lambda}$, $\omega_n = \frac{\pi d}{\lambda}$ and $\exp\{j \Psi\}$ contains additional phase factors associated with the geometry of the set-up.

In Figure 6, the 4-D composite transmit and receive spatial pattern snapshot is shown for 10-dB off the peak value. Notice that the main beam does not have as simple a pattern as the transmitter. Of importance to note is that the composite receive signal does not match the structure of the transmit pattern. This is due to the fact that the transmit and receive pattern are not the same. The objective of this receiver architecture is to cohere energy at the target location, not preserve any other attribute of the pattern. While other configurations are available, they are not investigated here. Again, we are visualizing a single pattern ambiguity, if we computed the pattern for a larger volume the periodicity would be evident.

The FDA pattern in Figure 6 shows that, in this configuration, the main beam no longer traces out a spiral in space but is simply surrounding the target area. The volume around the target is the 10-dB beamwidth of the main beam on receive. Three views are shown (top row) and compared against a CF array (bottom row) to fully understand the pattern. While it appears that energy is only around the target, sidelobe energy is present but not visible. Figure 7 further investigates this by allowing the beamwidth to increase in consecutive plots by 5-dB at a time. As we increase the beamwidth that is visualized, more of the sidelobe structure of the pattern becomes apparent.

V. CONCLUSIONS

These patterns are extremely unique and are even more complex than what is visualized due to additional sidelobe structure greater than 10-dB off the peak. However, with this additional degree of freedom it has been suggested that 3-D null steering, or null steering in range could be possible. Additionally, it could warrant multiple target tracking at different azimuths, elevations and ranges within a single beam, although this may require a different receiver architecture. While these ideas seem probable, there is also the issue of periodicity and time dependence for this type of pattern. It is currently being investigated if the time dependency could be eliminated while still maintaining the range-angle dependency. Moving forward we also suggest investigation of more complex waveform schemes, such as exponential or other non-linear frequency offsets in an effort to improve mainbeam peak-to-sidelobe ratio.

ACKNOWLEDGMENT

The authors would like to thank the Air Force Office of Scientific Research for support. This work completed by funding under project #2311. We would also like to thank Mr. Douglas Glass for assisting in the MATLAB visualization technique development.

REFERENCES


Fig. 6. The 10-dB receive pattern structure snapshot is shown for a $9 \times 9$ planar array with (top) frequency diversity and (bottom) constant frequency transmit waveforms, for three views.

Fig. 7. Composite receive pattern snapshot for a $9 \times 9$ FDA for various beamwidths. (upper left) 10-dB beamwidth (upper right) 15-dB beamwidth (lower left) 20-dB beamwidth (lower right) 25-dB beamwidth