Abstract

URSA Minor is a 21 cavity linear transformer driver (LTD) at Sandia National Laboratories to evaluate LTD technology for driving radiographic diodes. Operating at ~2.0 MV, there is substantial electron flow in the central magnetically insulated transmission line (MITL). An important concern is electron loss to the vacuum insulators, which are recessed only ~2 cm from the MITL. Electron flow has been analyzed with both 2-D, r-z particle-in-cell simulations of the full 21 cavity system, and full $2\pi$ r-$\phi$-z 3-D simulations of the last seven cavities to address azimuthal asymmetry. The simulation results show that electron loss to the insulators is negligible, even with large perturbations in the cavity trigger timing. However, they predict peak load currents ~10 – 15% higher than experiment. This is probably because a comparable fraction of the switches are triggering very late – well after peak power.

I. INTRODUCTION

The linear transformer driver (LTD) is a promising technology for building a compact, high-voltage driver for radiographic applications [1-3]. The 21 cavity URSA Minor LTD [3] at Sandia National Laboratories, shown in Fig. 1, provides the first opportunity to evaluate an LTD for radiographic diodes with substantial electron flow in the central magnetically insulated transmission line (MITL). Fig. 2 shows a 2-D r-z cross-section of URSA Minor. It consists of three groups of seven cavities. The inner surfaces of the cavities form the MITL anode at a radius of $r_a = 14.5$ cm. The cathode stalk inside the cavities consists of three uniform impedance sections – 16.3 $\Omega$, 33.7 $\Omega$, and 49.5 $\Omega$ -- with relatively abrupt conical transitions between them. The first is between cavities 6 and 7, and the second between cavities 13 and 14. Each cavity is driven with ten “bricks” uniformly spaced in azimuth, each consisting of two capacitors and a low inductance switch. Initial testing of this system was done with the capacitors charged to $\pm 75$ kV, driving a large area electron beam diode (LAD) [3]. Following these initial tests, most shots have been done at full charge voltage, $\pm 90$ kV, driving both LAD loads and self-magnetic pinch (SMP) diodes. We have fired ~60 SMP shots with a peak voltage of 1.5 – 2 MV. Experiments at 2.0 MV have produced total X-ray dose of 12 – 20 rad in a 50 ns radiation pulse.

Power flow on URSA Minor is diagnosed with anode and cathode B-dots at locations B – E shown in Figs. 2 – 4. Prior to July 2012, the experiments showed clear evidence of current loss between locations C and D. However, with improved cleaning and preparation of the surfaces, this loss is no longer observed. However, measured load currents are still smaller than the simulations by ~10 – 15%. Voltage monitors for each cavity indicate that a comparable fraction of the switches are not being triggered until very late in the pulse – well after peak power. We believe that if a switch is triggered too late, neighboring bricks that have fired drop the voltage across the switch, and it doesn’t fire until after the neighboring bricks are significantly discharged.
We compare the simulation current waveforms with experiment, and also monitor the current loss to the anode and the insulators, an important concern since these are recessed only ~2 cm from the MITL, as shown in Fig. 3.

II. 2-D SIMULATIONS

The 2-D simulation r-z geometry is shown in Figs. 2-4. We use a cell size of $\Delta z = 1$ mm across the 2.2 cm A-K gap of each cavity feed. We use a non-uniform radial grid with the highest resolution, $\Delta r = 0.5$ mm, at the radial cathode emission surfaces, and $\Delta r = 1$ mm at the insulators. We use a slanted dielectric surface model to simulate the 45° insulator surfaces without having any “stairsteps”. Each insulator surface cell has a slanted dielectric/vacuum boundary connecting opposite corners of the cell. Electrons incident on an insulator are killed at the surface and removed. We use a triangular weighting scheme to avoid leaving behind any charge at the vacuum corner of the cell when a particle is killed [5]. However, the simulation electric field is consistent with the killed particle charge remaining on the surface, i.e. $\text{div}(E) = (\rho_{\text{part}} + \rho_{\text{killed}})/\varepsilon_0$. Were this charging of the surface to be an issue, we would need to model electron transport in the dielectric. In practice, there is almost no electron loss to the insulators, so this is not an issue.

![Figure 2](image2.png)

**Figure 2.** The full simulation geometry with 21 feed lines (note extreme aspect ratio), and the experimental anode/cathode B-dot locations.

![Figure 3](image3.png)

**Figure 3.** Simulation geometry showing cavities 18 – 21 with the insulators, and the simulation MITL diagnostic locations and z-bins.

![Figure 4](image4.png)

**Figure 4.** Simulation geometry showing the last cavity, downstream MITL, load, and anode/cathode B-dot locations.

Each cavity is modeled with the circuit shown in Fig. 5. The main capacitor, $C_0 = 100$ nF, is charged to 180 kV for these simulations. When triggered, the switch resistance $R_s$ falls from $10^5$ to 0.19 Ω exponentially with a decay time of 2 ns. The switch inductance is $L_s = 21.5$ nH. $R_{\text{loss}}$ and $L_{\text{loss}}$ model core losses; $L_{\text{loss}} = 26.6$ nH is fixed, while $R_{\text{loss}}$ decays exponentially from 10 to 1 Ω in 37 ns. Simulations using a single cavity discharging into a fixed resistive load are in very good agreement with corresponding circuit code simulations. These agree well with single cavity experiments up to peak power, and very slightly underestimate the decay late in the pulse.

![Figure 5](image5.png)

**Figure 5.** External circuit used to drive the simulation feed lines. The terminals on the right are connected to the two conductors of the 2-D simulation feed.

Electrons are emitted from the cathode using a conventional space-charge-limited emission model. When the normal electric field exceeds a threshold value, 200 kV/cm here, electrons are emitted with enough space charge to satisfy Gauss’ Law with $E_{\text{surf}} = 0$. To diagnose power flow and electron loss in the MITL, we define a set of diagnostic locations $\{z_{d,i}, 1 \leq i \leq 21\}$ approximately 10 cm downstream of each cavity, illustrated in Fig. 3. We save time histories of the voltage, anode and cathode currents, and electron flow current at each location. We also save time histories of electron loss current and power to the anode, feed conductors and insulators divided into “z-bins”, where the $i$’th bin covers $z_{d,i-1} < z < z_{d,i}$. To compare directly with experiment, we also save the voltage and currents at the experimental locations B – E, and at the cathode tip, as shown in Fig. 4.
We have done many 2-D simulations, since they take only a few hours to run on 16 CPUs of one of our compute servers. We present results that compare with an experimental series at 90 kV varying the LAD anode-cathode gap over six values – 7.62, 6.35, 5.1, 3.8, 3.3, and 2.54 cm. The baseline runs use ideal cavity trigger timings, with each cavity having a delay time relative to the first one of $\Delta t_i = (z_i - z_1)/c$. We have also done simulations adding random jitter to the timing, selected from a normal distribution with $\sigma = 5$ ns. This has very little effect on the load currents or electron loss to the anode, as seen previously in simulations at 75 kV [6].

The electron distribution from a baseline run with a 7.62 cm gap at peak power is shown in Fig. 6. Electrons emitted just upstream of the first impedance transition form turbulent vortices that flow downstream, carrying a substantial fraction of the total flow current at the diode. Vortices are also launched from the second transition. From now on, all results presented here are for simulations with this largest gap, unless specifically mentioned otherwise. This geometry has been studied most extensively, since it has the largest electron flow and losses to the anode.

![Figure 6](image)

**Figure 6.** Electron distribution at peak power from a run for a gap of 7.62 cm, color-coded by creation location.

### A. Voltage and Current Measurements

Fig. 7 shows the MITL voltage and currents at three locations from the same baseline run. The peak voltage at the load is 2.24 MV. The anode and cathode currents show the increase in electron flow moving down the MITL. The peak anode and cathode currents at the load are 69.4 and 35.6 kA respectively.

![Figure 7](image)

**Figure 7.** Time history of (a) voltage, and (b) anode and cathode current (solid and dashed lines respectively), at MITL locations B and C, and the load.

Fig. 8, compares the anode and cathode currents at the load (i.e. location E) between simulation and experiment. The most obvious difference is that the simulation $I_a$ reaches a higher peak value faster, and then drops off faster. This is most likely due to the very late triggering of $\sim 10 - 15\%$ of the capacitors mentioned in the introduction. This drops the experimental peak current, and causes it to decay more slowly. Both simulation and experiment exhibit the qualitative feature that late in the pulse, $I_a$ is smaller than $I_c$. The reason for this is at late times most feed electrons are being lost to the outer anode boundary at $r = 14.5$ cm upstream of the anode B-dot at location E. In contrast, at peak power, feed electrons are lost to the $z_{\text{max}}$ boundary, “inside” the anode B-dot.

![Figure 8](image)

**Figure 8.** Anode and cathode currents at the load for experiment and simulation.

Fig. 9 compares the peak anode and cathode currents between experiment and simulation for the six diode gaps. The cathode current is in fairly good agreement, but the simulations are predicting higher anode current at larger diode gaps. It is not yet clear why the agreement is better at smaller gaps. We have not studied the small gaps in great detail since power flow issues are not as important for lower impedance loads. We plan to do more simulations to understand this trend.
B. Electron Loss to the Anode and Insulators

Fig. 10 provides a concise summary of where the electrons are being lost to the anode and insulators along the MITL for three runs with a diode gap of 7.62 cm. The dotted lines show the z-boundaries of the central section of the MITL cathode.

The energy deposition shown in Fig. 10(b) is very modest, resulting in a peak anode temperature increase of only ~6 °C. We thus conclude that electron deposition heating of the anode is completely negligible. Unlike runs at 75 kV [6], there is some electron loss to the insulators, but the energy deposition is very small. In these three simulations, only three cavities had deposition greater than 0.001 J – in the Ran1 run, 0.016 J into insulator 21, and in the Ran2 run, 0.034 J and 0.019 J into insulators 21 and 20 respectively.

III. 3-D SIMULATIONS

Since it is clear that some of the switches are not triggering until after peak power, there is azimuthal asymmetry in the power flow of some of the cavities. This was proposed as one possible explanation for the current losses originally observed between locations C and D in the experiments. As mentioned in the introduction, this loss was later mitigated by improved cleaning of the conductors and insulators. However, prior to this, we set up 3-D simulations to study azimuthal asymmetry.

The 3-D geometry is fully $2\pi$ in azimuth, with 280 cells in $\phi$. We model only the last seven cavities, using the restricted r-z geometry shown in Fig. 11. To reduce the total cell count, we also use coarser gridding than the 2-D simulations, with $\Delta r = 1.25$ mm at the cathode, and 16 cells across each feed gap instead of 22. 1-D transmission lines are attached to the $z_{\text{min}}$ and $z_{\text{max}}$ ends of the system. The line at $z_{\text{min}}$ is driven with a forward-going wave representing the first 14 cavities. The drive waveform is obtained from a vacuum (i.e. without particles) 2-D simulation of the first 14 cavities into a matched load. The line at $z_{\text{max}}$ models the inductance downstream of the 3-D
region, and is terminated with a time-dependent resistor, $R_{\text{load}}(t)$, obtained from the load voltage and currents of 2-D simulations. For a 7.62 cm diode gap, this resistance starts out very high, and drops to $45.6 \, \Omega$ over the first 10 ns after particle emission begins.

Figure 12. The r-z geometry for the 3-D simulations shown in green, overlaying the full 21 cavity geometry. The upstream and downstream coaxial sections are several anode-cathode gaps long.

Each cavity is driven with 10 independent external circuits – scaled versions of the one shown in Fig. 5. To be independently driven, each circuit must be connected to a closed, 2-D anode-cathode geometry at the 3-D system boundary. We use ten radial rectangular coaxial lines equally spaced in azimuth, with the circuits attached at $r = r_{\text{max}} = 35 \, \text{cm}$. All ten feeds are connected to a disk transmission line at $r = 26 \, \text{cm}$.

Figure 13 compares the voltage, anode current and electron flow current 10 cm downstream of cavity 21 for a 3-D simulation with azimuthally-symmetric drive and a 2-D simulation. We only ran the 3-D simulation to $t = 120 \, \text{ns}$, since the largest electron losses are expected early in the pulse, and we were initially unsure how long these runs would take. They are surprisingly affordable – with ~14.7 M cells and an average of 28.2 M particles, they run in ~10 hours on 64 CPUs of a Sandia compute server. The agreement between the two simulations is fairly good, but the differences increase after ~80 ns. The main problems with the 3-D setup is that (a) they lack the electron flow from the first 14 cavities, and (b) the power feed at $z = z_{\text{min}}$ is obtained from a vacuum simulation, artificially enhancing electron emission from the single upstream impedance transition. Nevertheless, the agreement in Fig. 13 is close enough to believe that the 3-D electron flow is fairly similar to a full 21 cavity simulation.

We have done 3-D simulations with combinations of the following perturbations:

1. 5 ns jitter in the trigger timing for each brick.
2. Disabling the triggering of ~20% of the 70 bricks in the seven cavities, randomly chosen. The azimuthally-symmetric power feed at $z_{\text{min}}$ is unchanged.
3. Misalignment of the anode and cathode. We offset the cathode by $3\Delta r = 3.75 \, \text{mm}$ ($\pm 4.6\%$ in the final section).

No combination of these perturbations, including all three together, has significantly enhanced electron loss to the anode or the insulators. Fig. 14 illustrates highly asymmetric power flow for perturbation #2. Four bricks in this cavity are not triggered – 2, 6, 7, and 9. The radius $r = 15.9 \, \text{cm}$ is just inside the insulators, while $r = 14.5 \, \text{cm}$ is the outer MITL radius. One can see the reduction in azimuthal asymmetry as power flows radially inward, but the perturbations at the MITL are still very substantial, and clearly correlated with the pattern of the untriggered bricks. Nevertheless, despite this asymmetry, electron loss to the insulator in this cavity is not enhanced.

Figure 14. Lineouts of $2\pi B_{\phi}(\phi)/\mu_0$ at two radii: $r = 15.9$ and $14.5 \, \text{cm}$, in cavity 21 for a 3-D simulation at peak power with bricks 2, 6, 7 and 9 not being triggered.
Finally, we note that perturbation #2 does not significantly reduce peak power downstream of cavity 21 – peak current and voltage are reduced by ~1% and ~3% respectively. This is a consequence of not changing the effective source impedance very much and having an overmatched load, $Z_L = 45.6 \, \Omega$. The source impedance of a single cavity with all 10 bricks firing is ~1 $\Omega$. If we approximate the full system using a simple circuit model with 21 of these circuit elements in series, the effective source impedance is ~21 $\Omega$. With 20% of the bricks in the last seven cavities not triggered, this is only increased to $\sim (14 + 7/0.8) = 22.8 \, \Omega$. Although many details are missing from this simple circuit model, the overall conclusion about power coupled to the load is fairly insensitive to such details. However, these results suggest that late-firing bricks may not be the sole cause of the discrepancy between the simulated and experimental load currents.

Increasing the 3-D simulation domain to model all 21 cavities in 3-D would allow us to simulate the actual behavior on URSA Minor, with ~10 – 15% of bricks in all cavities triggering very late. Given our experience with the resources required for the 3-D simulations described here, it is clearly feasible to model all 21 cavities and the load in 3-D. The only limitation is that we cannot extend the load geometry down to $r = 0$ in 3-D. However, for a LAD load, there are ways to deal with this issue.

**IV. SUMMARY AND FUTURE PLANS**

We have a detailed 2-D r-z, PIC simulation model of URSA Minor, the 21 cavity Sandia LTD for radiography. Each feed is driven with its own external circuit, and the 45$^\circ$ vacuum/insulator boundary is accurately modeled with a slanted surface model that avoids having to use any stairsteps that could perturb electron trajectories. Simulations can be run in just a few hours on 16 processors of a high-end parallel system, enabled many simulations to be performed quickly for system studies. We also have a 3-D PIC model of the last seven cavities, each driven with 10 external circuits equally, spaced in azimuth, to address azimuthal asymmetry issues.

We have done many simulations of shots on URSA Minor at 90 kV charge voltage. They all show only modest electron losses to the conductors near the insulator rings, and negligible electron losses to the insulators themselves. The same result is seen even for major, 5 ns, jitter in trigger timing, failing to trigger entire cavities in 2-D, or randomly selected bricks in 3-D. The 3-D simulations also allow us to misalign the cathode and the anode. For an offset of ±4.6%, no enhancement of electron losses in the MITL is observed either.

The one major discrepancy is that the simulations predict peak load currents ~10 – 15 % higher than experiment. We believe that this is an issue with the triggering of the cavities in the experiment. Voltage monitors for each cavity indicate that some fraction (of order 10 – 15 %) of the bricks fail to trigger until very late in the power pulse – after peak power. It thus appears that if a switch triggers too late, neighboring bricks that have fired can drop the voltage across the switch and inhibit the late brick from triggering until its neighbors have substantially discharged.

Our future plans are to set up and run 3-D simulations of all 21 cavities and the load. This will enable us to model late-firing bricks in all cavities in a manner comparable to what is observed on URSA Minor.

**V. REFERENCES**


