Outage Probability of Multihop MIMO Networks with Transmit Antenna Selection (TAS) in a Poisson Field of Interferers

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Abstract—In this paper, we analyze the performance of a multiple-input multiple-output (MIMO) multi-hop relay network using transmit antenna selection and maximal ratio combining (TAS/MRC) employing amplify-and-forward relay mode over a Rayleigh fading channels in the presence of Poisson field of interferers, that is applied practically in current cellular networks like LTE (Long Term Evolution). The aggregated interference per hop is modeled as shot noise with α-stable distribution, which is assumed to be non-identical per receive antenna. In particular, we derive compact analytical expressions for exact, and upper bound outage probability of the system. To gain further insights on the proposed system behaviors, we derive asymptotic outage probability at high signal-to-ratio (SNR) which explicitly characterizes the diversity order and coding gain of the network. All derived analytical results are corroborated by simulation results to show the correctness of our derived results.

I. INTRODUCTION

The need for higher data rate along the wide range of applications has become the main challenge for most of the players in the industry. Several studies were done to extend coverage, and increase data rate at lower infrastructure. This can be done by using the relaying technology, where idle nodes that are closer to the transmitter than the destination may be used to relay the signals from the source to the destination [1]. Multi-hop wireless networking has been studied in the context of ad-hoc and mesh networks. The application of multi-hop networking in wide-area cellular system was studied in [2]. Multi-hop techniques promise to enhance spectral efficiency, and network coverage for the future wireless communication systems [3, 4]. The relays in multi-hop network have number of protocols for operation. One of the most common protocols is amplify-and-forward (AF), where received signal at each node is amplified then forwarded to the successive node [5]. The performance for multi-hop systems have been studied with the limitation of Gaussian thermal noise [6], or generic noise [7].

Relaying technology has added the extension of MIMO (Multiple Input Multiple Output) technology to benefit from the diversity offered by MIMO to lower the effect of fading [8, 9]. The performance of multi-hop wireless networks over fading channels has been extensively evaluated in terms of outage probability, error probability, and channel capacity [10].

The transmit antenna selection with maximal-ratio combining (TAS/MRC) is used to provide low feedback overhead. In point-to-point MIMO communications and multi-hop MIMO relaying, TAS/MRC is proved to achieve full transmit and receive diversity [11, 12], where single transmit antenna that maximizes the post-processed signal-to-noise ratio (SNR) at the receiver is selected, index of the selected antenna is fed back to the transmitter. All the received signals are combined using MRC. Most of the work for relaying systems was considering thermal noise and channel fading only, while in fact, due to the limitation of spectrum resources and exponential growth of data traffic, the need to reuse the frequency of transmission is heavily increasing, that leads to co-channel interference. Some researchers have focused on channel configuration without considering the effect of co-channel interference (CCI) during the transmission process, where interference is assumed either at the relay, or at the destination [13, 14], while others have explored the interference effect for the multi-hop relaying system, however, they have put some limitations to relax the problem by assuming fixed number of interferers at fixed known locations, or single dominant interferer. Interference is modeled as fixed number of interfering signals coming from fixed locations experiencing Rayleigh fading channels[15, 16]. AF relaying with interference-limited relaying with a Rician fading interferer, and Nakagami-m fading interferers are studied in [17, 18]. These models can give a good approximation for the wireless network at specific regular planned node location realization but they are no longer applicable for irregular planned node locations for emerging wireless networks e.g., cognitive radios, heterogeneous cellular networks, and femtocells, or for any other node location realization. Stochastic geometry has provided statistical physical models based on the physical noise generation process to model the interference for these emerging networks, these models include the α-stable model initially proposed by Furutsu, and Ishida [19], and later advanced by Giordano [20], and Sousa [21], where Interferers are distributed using random spatial model called Poisson Point Process (PPP), which accurately models the aggregated interference produced by random number of interferers located at random location from the receiving nodes. Some recent works discussed the impact of random interference on the performance of relaying systems. In par-

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ticular, references in [22–25] have treated the multi-hop system in interference-limited environments, and presented exact, and upper bound expressions for the outage probability, symbol error rate (SER), and the average capacity.

From the discussion above, we aim to study the impact of random number of interferers distributed in the 2D plane and additive white Gaussian noise on the performance a multi-hop MIMO system employing TAS/MRC per hop over Rayleigh fading channels. In particular, we derive a compact expression for the outage probability based on the moment generating function (MGF)-based approach. A simple tight upper bound expression for the outage probability is also derived. In addition, asymptotic high signal-to-noise ratio (SNR) approximations are presented, which enables the characterization of the key performance indicators such as the diversity order and coding gain, thereby revealing the impact of the key system parameters such as number of interferers and antenna selection on the performance of the system. Our results reveal that the diversity order is affected by the path loss parameter $v$ of the interferers, which is not the case for the fixed interfere locations.

II. SYSTEM MODEL AND NETWORK INTERFERENCE MODEL

This section composed of two parts. In the first part, we briefly discuss the proposed system model. In the second part, we briefly present the interference model based on random locations.

A. System Model

We consider a multi-hop MIMO communication system in 2D plane consisting of a source $S$, a destination $D$, and arbitrary number of $K−1$ relays operating in AF mode as shown in Fig. 1. All nodes are equipped with multiple antennas, where $S$ is communicating with $D$ through the intermediate MIMO relays. Without loss of generality, we assume that the number of transmit and receive antennas at each hop are equal. The channel gain matrix at each hop is denoted by $H_k$ with elements of complex channel gain between $i^{th}$ transmit antenna and $r^{th}$ receive antenna at hop $k$ denoted by $h_{k,i,r}$, which captures the channel impairments (path loss, fading, and shadowing). All nodes operate using TAS/MRC. The source $S$ sends pilot and data signals to multi-antenna destination $D$. At the node $D$, the channel gain matrix $H_k$ elements are combined over all receive antennas then transmit antenna with highest channel gain is selected by sending back the index $i$ of the transmit antenna back to $S$. This requires perfect channel state information (CSI) at the receiver with error free feedback, and perfect time synchronization between source, relays, and destination is assumed while asynchronous transmission for the interferers. The source is located at the origin, and the relays are located at fixed locations in the 2D plane. The interferers are distributed randomly according to spatial point process in the 2D plane, which will be explained extensively in section B.

The received signal at $k^{th}$ hop $(k \in \{1, 2, \ldots, K\})$ hop is given by

$$Z_k(t) = H_k X_k(t) + I_k(t) + W_k(t)$$  \hspace{1cm} (1)

where the term $H_k X_k(t)$ denotes the filtering of the transmitted signal $X_k(t)$ by the channel gain matrix $H_k$ per hop $k$. At each hop $k$, the aggregated interference received from all interferers located at random locations is denoted by $I_k(t)$. The term $W_k(t)$ accounts for the additive white gaussian noise that is circularly normal random variable with zero mean and variance $\sigma^2$. We denote the sum of channel gains by $L_{k,i} = \sum_{n=1}^{N_{r}} |h_{k,i,r}|^2$, and the maximum per each hop is denoted as $L_k = \max_{n=1, \ldots, N_{r}} \{L_{k,i}\}$. We have assumed AF mode(i.e. $X_k = A_k Z_k$). The relay amplifying gain as in [26] is to be $A_k = \sqrt{\frac{P_{R_k}}{P_{R_{k-1}} \sigma^2_{e-1}}}$ based on the concept of inverting the channel gain of the $k^{th}$ link, where $P_{R_k}$ and $P_{R_{k-1}}$ are the received power at nodes $k$ and $k−1$, respectively. The fading power is denoted by $\sigma^2_{e-1}$.

We can write the end-to-end signal model per each receive antenna at the destination $K$ as follows

$$Z_{K,r}(t) = h_{K,r} X_0 \left( \prod_{k=1}^{K-1} A_k L_k \right) + I_{K,r}(t) + w_{K,r}(t)$$

$$+ \sum_{n=1}^{K-1} h_{K,r} \left( \prod_{k=1}^{K-1} A_k \right) \left( \prod_{r=1}^{N_r} \sum_{n=1}^{N_{r}} h_{n,r}^* w_{n,r} \right) \left( \prod_{k=n+1}^{K-1} L_k \right)$$

$$+ \sum_{n=1}^{K-1} h_{K,r} \left( \prod_{k=1}^{K-1} A_k \right) \left( \prod_{r=1}^{N_r} \sum_{n=1}^{N_{r}} h_{n,r}^* I_{n,r} \right) \left( \prod_{k=n+1}^{K-1} L_k \right)$$  \hspace{1cm} (2)

where $X_0$ is the signal transmitted from the source, $w_{n,r}$ and $I_{n,r}$ are the additive white gaussian noise per each receive antenna, and aggregated interference per each receive antenna, respectively. The operator $*$ denotes complex conjugate. Now the instantaneous end-to-end received SINR is defined as the sum of of the inverse of the individual per hop SINR, while assuming ideal gain, and ignoring the noise, we can write it as follows

$$\gamma_{eq} = (\sum_{k=1}^{K} \frac{1}{\gamma_k})^{-1}$$  \hspace{1cm} (3)
Where the instantaneous SINR per hop $\gamma_{Tk}$ can be written as

$$\gamma_{Tk} = \frac{L_k}{I_k + \sigma_k^2} = \frac{\gamma_k}{\gamma_{Tk} + 1}$$

where $L_k$ is the signal power for the selected transmit antenna, $I_k$ is the aggregated interference power, and $\sigma_k^2$ is the noise power, $\gamma_k = \frac{L_k}{I_k}$ is the signal-to-noise ratio (SNR) at node $k$, and $\gamma_{Tk} = \frac{I_k}{\sigma_k^2}$ is the interference-to-noise ratio (INR) at node $k$.

### B. Network Interference Model

Interference is modeled statistically using spatial point process to distribute randomly the interferers at random locations. We use the PPP to model the interferers locations. The aggregated interference depends mainly on the random nature of interferers location and propagation parameters. Interference rises from incoherent sources is modeled as a shot noise that has symmetric $\alpha$-stable distribution.

We assume that interferers are distributed at an intensity of $\lambda$. The number of interferers in an area $A$ is a poisson random variable with mean $\lambda A$. We consider the case where each receive antenna sees interference from interferers belonging to independent PPP processes $\phi$. In our case, we have $N_r$ independent poisson process $\phi_r$ with same $\lambda$. We use the notation $\{\} _{k,j,r}$ to denote the parameter value at $k$th hop for the $j$th interferer at $r$th receive antenna. We can write the aggregated interference $I_k(t)$ as below, where each instantaneous propagation parameter depends on both the interferer index $j$ and antenna index $r$

$$I_k(t) = \sum_{r=1}^{N_r} \sqrt{(P_r^t)_{k,j}} \sum_{j \in \phi_r} \exp(\sigma_{k,j,r}G_{k,j,r}) ||d_{k,j,r}||^{-\upsilon_{k,j,r}}$$

$$\times \alpha_{k,j,r}X_{k,j,r}$$

where $(P_r^t)_{k,j}$ is the transmitted power, $\sigma_{k,j,r}G_{k,j,r}$ is the lognormal shadowing factor, $d_{k,j,r}$ is the distance between the interferer and tagged node, $\upsilon_{k,j,r}$ is the path loss component between the interferer and the tagged node, and $\alpha_{k,j,r}$ is the fading coefficient between the interferer and the tagged node. We use the MGF of the aggregated interference, which follows $\alpha$-stable distribution $I_k(t) \sim \text{S\textsc{oS}}(\alpha_I, \gamma_I)$. We will use the expressions derived in [27] for the characteristic function of the aggregated interference. The characteristic function will have the form

$$\psi_I(\theta_I) = \exp(-\gamma_I|\theta_I|^{-\alpha_I})$$

where we define the parameters as below

$$\alpha_I = \frac{2}{\upsilon_I} ; \quad \gamma_I = \frac{\gamma_{k_I} \gamma_{I_0}}{\gamma_I} ; \quad \Omega_I = E_I(\sigma_{I(k,r)})$$

Where $\upsilon_I$ is the path loss exponent between the tagged node and the interferer, which is assumed equal for all hops, $\alpha_{I(k,r)}^2$ is the fading coefficient between the interferer and tagged node at hop $k$. Now the characteristic function for interference model can be written as in (4), where $F$ is a factor capturing the signal waveform, modulation and delay, $\alpha_{k,r}^2$ is the fading coefficient at the $k$th hop, $k_I$ is the path loss coefficient constant for the interferer. The term $C_{2/\upsilon_I}$ can be formulated as follows

$$C_x \triangleq \begin{cases} \Gamma[1-x] \cos(\pi x/2) & , x \neq 1 \\ \frac{2}{\pi} & , x = 1 \end{cases}$$

Depending on the fading channel type, the characteristic function of the interference model is exact or upper bound, however, this will also gives an insight of the system performance.

### III. End-to-End SINR Performance Analysis

In this section, we investigate the end-to-end outage probability of multi-hop MIMO networks with TAS/MRC over Rayleigh fading channels in a Poisson field of interferes. In particular, we derive an exact, and an upper bound of the outage probability of the proposed system model. The asymptotic outage probability is also provided, which yields the key insights on the achievable diversity order and coding gain.

#### A. Exact Outage Probability

The outage probability of the end-to-end SINR is defined as the probability that SINR falls below a predetermined threshold $\gamma_0$. The end-to-end outage probability can be expressed as follows [23]

$$P_{out}(\gamma_0) = Pr(\gamma_{eq} \leq \gamma_0) = Pr(\frac{1}{\gamma_{eq}} > \frac{1}{\gamma_0})$$

$$= 1 - \mathcal{L}^{-1}\left(\frac{\mathcal{M}_{\frac{1}{\gamma_0}}(s)}{s}\right)|_{1/\gamma_0}$$

where $\mathcal{M}(.)$ is moment generation function, and $\mathcal{L}^{-1}(.)$ is the inverse Laplace transform [30]. To obtain the outage probability in (6), the end-to-end MGF can written as below [23]

$$\mathcal{M}_{1/\gamma_0}(t) = \prod_{k=1}^{K} \mathcal{M}_{1/\gamma_{Tk}}(t)$$

where $\mathcal{M}_{1/\gamma_{Tk}}$ is the inverse SINR MGF of $k$th hop that can be obtained directly from the cumulative distribution function (CDF) [22] as

$$\mathcal{M}_{1/\gamma_{Tk}}(t) = 1 - t \int_{0}^{\infty} \gamma_0^{-2} e^{-t/\gamma_0} \mathcal{F}_{\gamma_{Tk}}(\gamma_0) d\gamma_0$$

where $\gamma_a$ to be equal $\gamma_0(\gamma_{I_0} + 1)$. The term $\mathcal{F}_{\gamma_{Tk}}(\gamma_0)$ is found equal to $\mathcal{F}_{\gamma_{k}}(\gamma_a)$

$$\mathcal{F}_{\gamma_{Tk}}(\gamma_0) = Pr\left(\gamma_k \leq \frac{\gamma_{I_0} + 1}{\gamma_{I_k} + 1} \leq \gamma_0\right)$$

$$= \int_{0}^{\gamma_k} \mathcal{F}_{\gamma_k}\left(\left(\gamma_{I_k} + 1\right) \gamma_0 I_k = \gamma_{I_k} \right) f_{I_k}(\gamma_{I_k}) d\gamma_{I_k}$$

(8)

Now we write the expression for CDF of $\gamma_k$, which is defined as the SNR at node $k$. The CDF of $\gamma_k$ follows an independent identically distributed (i.i.d) chi-squared distribution is
\[ \gamma_x = \left( \sum_{r=1}^{N_r} \alpha_{k,r}^2 \right)^{1/v_x} ; \quad \hat{\gamma}_x = 4\lambda \pi C_2^{-1} k_x^2 \exp(2\sigma_y^2/v_x \Omega_x) F \]

\[ \psi_x(\theta_x) = \exp \left( -4\lambda \pi C_2^{-1} k_x^2 \exp(2\sigma_y^2/v_x \Omega_x) \left( \sum_{r=1}^{N_r} \alpha_{k,r}^2 \right)^{1/v_x} \Omega_x N_r^{(1-1/v_x)} F|\theta_x|^\alpha_x \right) \]

(formulated as follows [28])

\[ \mathcal{F}_{\gamma_k}(\gamma_a) = \left[ 1 - e^{-\frac{\gamma_a}{\gamma_k}} \sum_{r=1}^{N_r} \left( \frac{\gamma_a}{\gamma_k} \right)^{r-1} \frac{N_t}{(r-1)!} \right]^{N_i} \]

\[ = 1 - \sum_{i=1}^{N_t} \left( \frac{N_t}{i} \right) (-1)^{(i-1)} e^{-\gamma_0/\gamma_k} \]

\[ \times \sum_{r_1=1}^{N_r} \sum_{r_2=1}^{N_r} \sum_{r_3=1}^{N_r} \cdots \sum_{r_1=1}^{N_r} \frac{(\gamma_0/\gamma_k)^{r_{\text{sum}}}}{\prod_{p=1}^{K}(r_p - 1)!} \]

Where (10) follows from substituting (9) into (8), and \( \mathcal{J}_1 \) can be evaluated in terms of MGF as follows

\[ \mathcal{J}_1 = E_y \left[ e^{\frac{-i\gamma_x \gamma_0}{\gamma_k} \gamma_x k} \right] = \left( \frac{\gamma_k}{\gamma} \right)^{j} E_y \left[ e^{\frac{-i\gamma_x \gamma_0}{\gamma_k} \gamma_x k} \right] \]

By using the property of Laplace transform

\[ \mathcal{J}_1 = \left( \frac{\gamma_k}{\gamma} \right)^{j} (-1)^j \left[ \mathcal{M}_{\gamma_x \gamma_0 \gamma_x k} \right] \]

Defining \( P = \frac{i\gamma_x \gamma_0}{\gamma_k} \) and inserting it into (12) yields

\[ \mathcal{F}_{\gamma_k}(\gamma_0) = 1 - \sum_{i=1}^{N_t} \left( \frac{N_t}{i} \right) (-1)^{(i-1)} e^{-i\gamma_0/\gamma_k} \]

\[ \times \sum_{r_1=1}^{N_r} \sum_{r_2=1}^{N_r} \sum_{r_3=1}^{N_r} \cdots \sum_{r_1=1}^{N_r} \frac{(\gamma_0/\gamma_k)^{r_{\text{sum}}}}{\prod_{p=1}^{K}(r_p - 1)!} \]

\[ \times \sum_{j=0}^{r_{\text{sum}}} \left( \frac{\gamma_k}{\gamma} \right)^{j} (-1)^j \left[ \mathcal{M}_P(\gamma_0) \right] \]

Inserting (13) in (19) yields

\[ \mathcal{F}_{\gamma_k}(\gamma_0) = 1 - \sum_{k=1}^{K} \sum_{i=1}^{N_t} \left( \frac{N_t}{i} \right) (-1)^{(i-1)} e^{-i\gamma_0/\gamma_k} \]

\[ \times \sum_{r_1=1}^{N_r} \sum_{r_2=1}^{N_r} \sum_{r_3=1}^{N_r} \cdots \sum_{r_1=1}^{N_r} \frac{(\gamma_0/\gamma_k)^{r_{\text{sum}}}}{\prod_{p=1}^{K}(r_p - 1)!} \]

\[ \times \sum_{j=0}^{r_{\text{sum}}} \left( \frac{\gamma_k}{\gamma} \right)^{j} (-1)^j \left[ \mathcal{M}_P(\gamma_0) \right] \]

Inserting expression of (20) in (18) gives the upper bound outage probability

\[ \mathcal{F}_{\gamma_k}(\gamma_0) = 1 - \sum_{k=1}^{K} \sum_{i=1}^{N_t} \left( \frac{N_t}{i} \right) (-1)^{(i-1)} e^{-i\gamma_0/\gamma_k} \]

\[ \times \sum_{r_1=1}^{N_r} \sum_{r_2=1}^{N_r} \sum_{r_3=1}^{N_r} \cdots \sum_{r_1=1}^{N_r} \frac{(\gamma_0/\gamma_k)^{r_{\text{sum}}}}{\prod_{p=1}^{K}(r_p - 1)!} \]

\[ \times \sum_{j=0}^{r_{\text{sum}}} \left( \frac{\gamma_k}{\gamma} \right)^{j} (-1)^j \left[ \mathcal{M}_P(\gamma_0) \right] \]

C. Asymptotic Outage Probability

In order to understand the limiting factors affecting the multi-hop system with TAS/MRC experiencing interference.
model discussed earlier. We provide the asymptotic analysis of the multi-hop outage probability. This is evaluated using the Taylor series expansion of the end to end upper bound outage probability (20), where at high SNRs, \( F_{\text{ub}}(\gamma_0) = \sum_{k=1}^{K} F_{\gamma_k}(\gamma_0) \). This implies that the asymptotic behavior is dominated by the statistical properties of the weakest hop. We can write the asymptotic behavior as

\[
P_{\text{out}}^\infty \approx \sum_{k=1}^{K} \frac{\gamma_k^{N_t} \gamma_k}{(N_t!)^{N_r} \gamma_k} \left( \frac{\gamma_0}{\gamma_k} \right)^{\min\left( \frac{\gamma_0}{\gamma_k}, N_r \right)} \tag{21}
\]

From the above expression (21), we conclude that the diversity order is the minimum of number transmit antennas, number of receive antennas, and path loss exponent between the tagged nodes and interferers. It is shown that \( v \) is inversely proportional to \( \gamma_k \), which is a function of \( \lambda \). This provides an insight to the effect of interferers which implies that the more the number of interferers increase the worse the diversity gain is. It is clear from the asymptotic expression that the behavior depends on the path loss parameter \( v \) of the interferers. It can be noticed that the coding gain is affected by interfere powers.

IV. NUMERICAL RESULTS

In this section, we provide the numerical results for the upper bound, and exact analytic outage probability for \( K \) multi-hop system for flat Rayleigh channel between nodes. We consider balanced relaying scenarios, where average SNR \( \gamma_k \) is equal among all hops in presence of co-channel interference that varies between each receive antenna. We have provided the effect of various system parameters such as number of transmit antennas, number of receive antennas, and number of hops. We consider the same path loss exponent among all hops \( v = 4 \) and \( \gamma_0 = 0 \). The number of interferers are determined based on the value of \( \lambda \) which is assumed at the value of \( 10^{-3} \). Figures 2, 3, and 4 plot exact form, the upper bound, and asymptotic outage probability at \( \gamma_0 \). From these figure, we can show that the exact and Monte-Carlo simulations results as well as the upper bound are in exact agreements. In addition, the derived asymptotic results match the exact and simulation results at high SNR which reveal the accuracy of our derivation.

Fig. 2 plots the effect of changing the number of receive antennas. It can be noticed that when the number of receive antenna increases, the outage probability decreases.

Fig. 3 plots the effect of changing the number of transmit antenna. It can be noticed that when the number of transmit antenna increases, the outage probability decreases.

Fig. 4 plots the effect of changing the number of hops. It can be noticed that when the number of hops increases, the outage probability increases. Both exact and upper bound derived formulas accurately coincides the asymptotic one.
In this paper, we have investigated the impact of random number of interferes distributed in the 2D plane and additive white Gaussian noise on the performance a multi-hop MIMO system employing TAS/MRC per hop over Rayleigh fading channels. We have derived compact expressions for the exact and upper bound outage probability of the proposed system model. Furthermore, a simple expression for the asymptotic outage probability is derived based on which the diversity order and coding gain are obtained. We demonstrated that we have improvement in outage probability when increasing the number of transmit or receive antennas. We have demonstrated that at high SNR, the system depends on the path loss exponent value for the interferers.

**V. CONCLUSION**

The paper covers the investigation of random number of interferes distributed in the 2D plane and additive white Gaussian noise on the performance of a multi-hop MIMO system employing TAS/MRC per hop over Rayleigh fading channels. Compact expressions for the exact and upper bound outage probability of the proposed system model are derived, with a simple asymptotic expression for high SNR. The behavior of the system at high SNR is shown to depend on the path loss exponent value for the interferers.

**REFERENCES**


