ENVIRONMENTALLY ADAPTIVE SIGNAL PROCESSING IN SHALLOW WATER

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Abstract - Low-frequency signals propagating in shallow water are conveniently represented as a set of acoustic normal modes individually characterized by different dependencies of pressure on depth. These differences have been exploited to form spatial filters which selectively enhance single modes of propagation. The performance of these filters is degraded by errors in the knowledge of the calculated depth function. This paper discusses an alternative approach to filter construction, in which filter coefficients are adaptively extracted from measurements of naturally-occurring acoustic noises. When the acoustic array densely samples the water column, the modal eigenfunctions can be obtained by diagonalization of the cross-spectral density matrix formed by averaging over an ensemble in which the modal components add incoherently. In the experimentally more practical case, in which the receiving array does not span the water column, useful spatial filters can be constructed from noise cross-spectral density matrices.

I. INTRODUCTION

Acoustic propagation at low frequencies in shallow water is conveniently described by normal-mode theory [1]. The acoustic pressure $P(r, z)$ at range $r$ from the acoustic source and depth $z$ is given by the expression:

$$P(r, z) = \sum_{n=1}^{N} a_n(r) U_n(z) e^{ik_n z}.$$

where $N$ is the number of normal modes in the field, $a_n$ is the modal amplitude, $U_n$ is the normal-mode depth function, and $k_n$ is the horizontal component of the wavenumber associated with the $n$th normal mode field component. Range dependencies due to cylindrical spreading and attenuation are contained in the term $a_n(r)$. Differences in the depth dependence of different modes have been exploited by using vertical arrays as spatial filters for selectively enhancing or suppressing the contributions of individual modal terms [2-5]. If the array spans the water column and consists of uniformly-spaced sensors located at depths $z_i$, shading the array by weights $w_i(m) = f_m U_m(z_i)$ proportional to the $m$th normal mode depth function, i.e.,

$$\sum_{i=1}^{L} w_i(m) P(r, z_i) = f_m \sum_{n=1}^{N} a_n(r) e^{ik_n r} \sum_{i=1}^{L} U_n(z_i) U_n(z_i),$$

allows approximation of the orthogonality condition

$$\sum_{i=1}^{L} U_n(z_i) U_n(z_i) \simeq \int_{0}^{\infty} U_n(x) U_n(x) dx = \delta_{mn}$$

to selectively enhance a single mode:

$$\sum_{i=1}^{L} w_i(m) P(r, z_i) \simeq f_m \sum_{n=1}^{N} a_n(r) e^{ik_n r} \delta_{mn}$$

Experimental tests [2,3] of this filtering technique, employing 12-element vertical arrays nearly spanning the water column and filter weights derived from calculated normal-mode depth functions, achieved enhancements of the amplitude of the selected mode, relative to that of other mode orders, by as much as 20 decibels (dB).

If the array does not span the water column, the degree to which the normal-mode weights approximate the orthogonality condition is significantly degraded and the direct use of the normal-mode depth functions as filter weights becomes less effective. If we let the $L$-component vectors $V_n$ represent the $N$ normal-mode depth functions $U_n(z)$ sampled at the $L$ depths of the array's sensors, the $V_n$ associated with different mode orders $n$ will be linearly independent. From the $N$ vectors $V_n$ we form an $L$-by-$N$ column matrix $V$. If $P$ is an $L$-component vector of pressures sampled by the array and $a$ is an $N$-component vector of the phased modal amplitudes, we may write

$$P = Va.$$ 

Tindle, et al. [4] noted that this equation may be solved for $a$ by multiplying by a pseudo-inverse matrix $(V^TV)^{-1}V^T$:

$$a = (V^TV)^{-1}V^TP \equiv XP,$$
where the \( N \) rows of the \( N \) by \( L \) pseudo-inverse matrix \( X \) are the spatial weights for single-mode filters for the vertical array. Pseudo-inverse filters obtained from calculated normal mode amplitude vectors \( V_n \) were found to selectively enhance single modes of propagation by about 15 dB [4,5]. Performance of the pseudo-inverse filters was reported to be limited by accuracy of the knowledge of the environment used in the normal mode calculations, inaccuracies in knowledge of the sensor depths, and deviations of the array from a vertical line.

In this paper we consider alternative approaches for using vertical arrays for selective mode enhancement. The techniques considered do not require the use of computed normal mode functions, but instead use acoustic data from the vertical array to extract the needed information. By employing a technique that does not require knowledge of the waveform transmitted by the source, the technique can be used with the fields of uncontrolled or unknown noise sources, such as passing ships. Further, in situ autocalibration of the array reduces or removes errors arising from inaccurate knowledge of the hydrophone sensor positions.

We begin by considering the cross-spectral density matrix formed from \( PP^\dagger \) and its eigenvalue-eigenvector expansion. We then consider the mathematically simple case of an array which densely samples the entire water column, so that the orthogonality condition of (3) is approximated. Finally we consider the more complex case in which the array spans only part of the water column and (3) is not satisfied.

II. CROSS-SPECTRAL DENSITY MATRIX

The cross-spectral density matrix \( C \) of the narrowband signal of (5) is:

\[
C = PP^\dagger = Vaa^\dagger V^\dagger. \tag{7}
\]

Since the cross-spectral density matrix is self-adjoint, it may be diagonalized to yield eigenvalues \( \lambda_i \) and orthonormal eigenvectors \( e_i \). Thus, the cross-spectral density matrix may be expressed as

\[
C = \sum_{i=1}^{N} \lambda_i e_i e_i^\dagger. \tag{8}
\]

where \( N \), the rank of \( C \), is the number of nonzero eigenvalues. When the cross-spectral density matrix is formed from a single pressure sample or the average of a number of identical pressure samples, the rank of the matrix is unity. The eigenvector, assumed normalized to unity \((e_i^\dagger e_i = 1)\), is \( P/(PP^\dagger)^{1/2} \) and the only nonzero eigenvalue is \( PP^\dagger \). If we form the cross-spectral density matrix from different realizations of the pressure, the rank of the cross-spectral density matrix increases and consequently the number of terms in the eigenvalue-eigenvector expansion increases.

We consider the case in which a signal of small, but finite bandwidth is used. The bandwidth is assumed sufficiently small so that the normal mode vectors \( V \) and the magnitude of the normal mode amplitudes \( a \) are constant over the band. The phases of the components of \( a \), which are of the form \( \exp(ik_n r) \), are allowed to change over the frequency band. Denoting the ensemble average of \( C \) by \( \langle C \rangle \) and noting the constancy of \( V \) over the ensemble we have

\[
\langle C \rangle = V(aa^\dagger)V^\dagger. \tag{9}
\]

The diagonal elements of \( aa^\dagger \) are of the form \(|a_j|^2 \) and the off-diagonal elements are of the form \(|a_n||a_l| \exp[i(k_n - k_l)r] \). The off-diagonal elements are small if, over the ensemble in the frequency band \( \delta f \), the argument of the \( \exp(\cdot) \) ranges over \( 2\pi \). By expanding the wavenumbers in a Taylor’s series about the center frequency \( f_0 \) and noting that \( 2\pi \delta f/\delta f = v_n \), the group speed of the \( n \)th mode, the condition for the vanishing of the \( n \), \( l \)th off-diagonal element is:

\[
r(v_n^{-1} - v_l^{-1}) > (\delta f)^{-1}. \tag{10}
\]

The left side of (10) is the time difference of arrival of the \( l \)th and \( n \)th mode at the receiver and the right side is the duration of a gated sine wave pulse of bandwidth \( \delta f \). Thus the criterion for vanishing of an off-diagonal element of \( aa^\dagger \) is that the modal components of a pulse signal are temporally resolved by group velocity dispersion. Under the conditions in which the bandwidth and range are sufficient to cause all the off-diagonal elements to vanish, it follows that

\[
\langle C \rangle = \sum_{n=1}^{N} a_n^2 V_n V_n^\dagger = \sum_{n=1}^{N} \lambda_n e_n e_n^\dagger. \tag{11}
\]

III. ARRAY SPANNING WATER COLUMN

If the number of elements in the array (dimension of \( C \), \( V \), and \( e \)) is at least as large as the number of mode orders \( N \) and if the sampled modal depth functions are mutually orthogonal (i.e., \( V_n^\dagger V_m = \delta_{n,m} \), the Kronecker delta function), then the two expansions in (11) are equivalent. By diagonalizing the matrix \( (C) \) to obtain the mutually orthogonal set of eigenvectors \( e_n \), we obtain the (mutually orthogonal) modal depth functions. The condition that the sampled modal depth functions \( V_n \) are mutually orthogonal can be achieved, to a good degree of approximation, by densely sampling the entire water column, provided the normal modes arriving at the array do not significantly penetrate the ocean bottom.

The prediction that diagonalizing the cross-spectral density matrix would yield eigenvectors which were proportional to the modal eigenfunctions was experimentally tested [6]. A 25-element equi-spaced hydrophone array having total aperture 16.4 meters was deployed vertically.
from the Naval Ocean Systems Center Oceanographic Tower, which stood in water of depth 18 meters. An acoustic source was deployed from an anchored vessel at a range of 3.7 kilometers and driven by a 100 Hertz bandwidth pseudorandom sequence signal at center frequency 400 Hertz. The signal was periodically continued to provide a continuous transmission lasting about 30 minutes. The signals received, which had a nominal signal-to-noise ratio of 20 dB, were processed by performing 32-point discrete Fourier transforms (dft's), having processing bandwidth 80 Hertz per Fourier component, on the individual hydrophone signals. Throughput limitations of the computer used allowed a duty cycle of about 10% in performing the processing (300 points were skipped during processing of each 32 point sample). The data were calibrated by performing identical processing on a recorded random-noise signal of known power and accounting for the hydrophone sensitivities. Normal-mode group speed calculations verified that the 80-Hertz bandwidth signal processed met the criterion of (10). The complex dft component corresponding to center frequency 400 Hertz was selected and the dimension-25 cross-spectral density matrix was averaged over an ensemble of size 1000. Water temperature profiles were determined by using a calibrated thermistor array suspended from the tower. The temperature profile observed showed a negative gradient with surface temperature 21°C and bottom temperature 17°C.

The 25 eigenvalues of the cross-spectral density matrix, ordered by size from the largest to smallest and expressed in decibel measure, are shown in Fig. 1. We note that the first (largest) four eigenvalues depart from the trend of the remaining 21, which reflect ambient and system noise. In Fig. 2, the magnitudes of the eigenvectors corresponding to the largest four eigenvalues are compared with the magnitudes of the first four normal-mode depth functions calculated with a fluid normal-mode model using archival ocean bottom information. The shapes of the measured eigenvectors and the calculated normal-mode depth functions are in good agreement. It is worth noting that the energy in the fourth mode, whose shape is well-determined, is 19 dB below that of the first mode. The eigenvectors associated with the second, third, and fourth modes are found to have nearly 180 degree phase reversals (not shown here) at the amplitude minima. Magnitudes of eigenvectors associated with the fifth and higher-order eigenvalues do not have recognizable resemblance to normal-mode depth functions.

We note that, in carrying out the processing of the acoustic signal, no use (such as replica correlation) was made of knowledge of the waveform of the continuously transmitted signal, so similar processing could have been carried out on an uncontrolled waveform. The result shown above suggests that environmentally adaptive single-mode filters can be obtained by processing naturally-occurring noise sources, if the receiving array densely spans the water column.

IV. ARRAY PARTIALLY SPANNING WATER COLUMN

If the vertical receiving array only partially spans the water column, the one-to-one correspondence of normal-mode functions and eigenvectors of the cross-spectral density matrix cannot be made. The normal-mode depth functions will be merely linearly independent, not orthogonal, while the cross-spectral density matrix eigenvectors will still be mutually orthogonal. If we again consider the case in which the normal-mode components add incoherently and the array contains a number of elements larger than the number of propagating modes, then the rank of the cross-spectral density matrix will equal the number of modal components. If the signal-to-noise ratio is high, we expect to find a number of large eigenvalues. The rank of the cross-spectral density matrix is equal to the number of large eigenvalues. The eigenvectors associated with the large eigenvalues span the signal space of the propagating modes. The remaining eigenvectors, corresponding to the small eigenvalues are orthogonal to the eigenvectors spanning the signal space, and hence to the signals themselves.

This result can potentially be exploited to form environmentally adaptive filters which reject surface noise. Under downward-refracting sound speed conditions, at sufficiently high frequency, low-order normal modes will be refracted away from the surface and will not couple to near-surface sound sources. A deeper sound source will couple to both low- and high-order modes. We therefore anticipate that under downward-refracting conditions, the rank of the cross-spectral density matrix of a near-surface source will be smaller than that of a deeper source. As a result we expect that some of the eigenvectors associated with small eigenvalues of the near-surface-source cross-spectral density matrix will have significant projections on lower order modes. Consequently, these
The magnitude of the eigenvectors of the cross-spectral density matrix (C): (- -) The calculated normal-mode depth function, (---) the measured eigenvectors.

eigenvectors might be useable to form filters to enhance deep-source signal to surface-generated noise ratios.

This hypothesis is explored by using normal-mode model calculations. The cross-spectral density matrix of a near-surface source (5 meters deep) is calculated for the case of the downward-refracting profile and 20-element vertical receiving array. The water column is 100 meters deep and has sound speed $c(x) = 1500$ meters/second for depth $0 \leq x \leq 30$ meters, linearly decreases to 1480 meters/second at $x = 35$ meters, and is constant for $35 < x < 100$ meters. The vertical line array (VLA) has the top element at $x = 50$ meters and bottom element at $x = 90$ meters. Due to refractive effects, the lowest three mode orders did not significantly contribute to the cross-spectral density matrix, which was formed by adding the modal contributions incoherently and adding a small amount of uncorrelated noise.

It can be shown that the seventh eigenvector of the cross-spectral density matrix, which has an eigenvalue only slightly larger than that of the added noise power, has a significant projection on the first mode and is effective as a filter for enhancing the signals of deeper sources relative to those of shallower sources. The degree of enhancement achieved is illustrated in Fig. 3: compare the curve marked with a $\triangle$ and the curve marked with a $\bigcirc$.

to produce this figure, the signal fields of a source at a number of individual different source depths are calculated. The response of the array, which is weighted by the seventh eigenvector of the near-surface cross-spectral density matrix, to each of these fields is determined and the result was plotted as a function of source depth. Similar calculations were performed using a constant weighting function and a weight proportional to the first order mode (which was incompletely sampled in this case due to the shortness of the array aperture). While all three arrays show a higher response to the deeper source than to shallower sources (suggestive of the degree to which they would provide relative rejection of surface-generated noise), the eigenvector-based array's performance is some 10 to 15 dB better, apparently due to performance degradation accompanying sidelobe leakage associated with the other array weights.

To test the robustness of the eigenvector filter against perturbations of the environment, we consider the effect of changes in the sound velocity profile (SVP) and ocean depth upon the filter response. It is observed that small changes in depth, on the order of 5 meters, have negligible effect. In contrast, the filter response curve is more strongly dependent upon sound speed variations. For example, we consider two SVP's which differ only in the value of $c(x)$ for $x < 30$ meters. For the first environment, denoted $E_1$, $c_1(x) = 1500$ meters/second while the second environment $E_2$ has $c_2(x) = 1495$ meters/second for $x < 30$ meters. The sound speed for the lower layer and transition region are the same as previously defined.

The environment $E_1$ is more strongly downward refracting than $E_2$. A shallow source at $z = 5$ meters and a distance 2 kilometers from the VLA is used to calculate the eigenvectors of (C) in $E_2$, the less downward refracting environment. The performance of the filter response in $E_1$ and $E_2$ using the seventh eigenvector $e_7$ and the constant filter are shown in Fig. 3. First, we note that $e_7$ performs better than the constant filter function. Second, the constant filter does not show much dependence upon the SVP chosen; a 2-3 dB gap is seen for the shallower
source depths (i.e., $z_s < 20$ meters). Finally, it appears that the filter $e_7$ obtained in $E_2$ actually performs better in $E_1$ than in $E_2$. Consequently, it appears possible that no significant loss in performance occurs if the filter is used in an environment which is more downward refracting than the one used to obtain it. In contrast, if a filter is used in an environment which is less strongly downward refracting than the one in which it was obtained, the performance has been observed, but is not shown here, to decrease.

In the results shown in Fig. 3, the use of a single eigenvector provided performance as good as that obtained from various linear combinations of eigenvectors of small eigenvalues of the cross-spectral density matrix of a near-surface sound source. In some of the cases we investigated, filter performance could be improved by using constrained linear combinations of eigenvectors of small eigenvalues. Fig. 4 illustrates one example.

The filter response in all three curves of Fig. 4 is calculated in environment $E_1$, whereas the cross-spectral density matrix is calculated in three different ways. We use a shallow source depth of $z_s = 5$ meters in environment $E_2$ to calculate the first cross-spectral density matrix and the same source depth in environment $E_1$ to calculate the second cross-spectral density matrix. The third cross-spectral density matrix arises from using a source depth of $z_s = 30$ meters in environment $E_1$. As source depth increases, the portion of the signal energy propagated in the first mode increases. We observe in simulations that this energy distributes itself among several eigenvectors. Thus, as anticipated, we find for the third cross-spectral density matrix that a judiciously chosen linear combination of eigenvectors will provide a better filter response than any single eigenvector.

We construct this constrained linear combination of eigenvectors in two stages. First, we take the spatial Fourier transform of each eigenvector. Using this information, we select an index set $I$ of eigenvectors based upon their low spatial frequency content. For the example in Fig. 4 (with $z_s = 30$ meters), it is found that $I = \{3, 4, \ldots, 7\}$. Second, the linear combination of eigenvectors

$$F(z_l) = \sum_{n \in I} c_n e_n(z_l) \quad (12)$$

is chosen to maximize the bias objective function $B$ defined as

$$B(F) = \frac{1}{J} \left| \sum_{l=1}^{J} F(z_l) \right| \quad (13)$$

subject to the constraint

$$\sum_{n \in I} (c_n)^2 = 1. \quad (14)$$

The bias objective function provides one measure of the suitability of the filter. A filter which has a large bias

![Image](https://via.placeholder.com/150)

**Fig. 4.** Filter response in environment $E_1$ as a function of source depth for (△) $e_7$ of (C) formed in environment $E_2$ with source depth $z_s = 5$ m; (○) $e_7$ of (C) formed in environment $E_1$ with source depth $z_s = 5$ m; (▲) the filter calculated from $e_3$-$e_7$ of (C) formed in environment $E_1$ with a source depth of $z_s = 30$ m.

will have a strong low spatial frequency component and thus be nearly orthogonal to the middle and high order modes. For brevity, we only discuss the bias objective function here, though we have developed and tested other objective functions.

We now present a preliminary investigation of the effect of mode-coupling upon the performance of the filtering method. A basic assumption of this method is that a shallow source will primarily excite high-order modes in a downward refracting environment and subsequently propagate energy to a VLA with minimal coupling to the lower-order modes. To evaluate the impact of violating this premise, we consider the following simple simulation. We evaluate the correlation, as measured by

$$20 \log \left| \frac{(F, U_n)}{F} \right|$$

between the normal modes $U_n$ and the filter $F$, where the norms $\| \cdot \|$ are taken over the water column. We consider the following two filters: $F_1$ is eigenvector 7 of the cross-spectral density matrix due to a source at 5 meters in environment $E_1$ and $F_2$ is the filter which minimizes the bias of a linear combination of eigenvectors 3 through 7 of the cross-spectral density matrix due to a source at $z = 30$ m. As source depth increases, more of the lower-order modes are excited, thus giving an indication of what may happen when mode-coupling results in energy leaking into the low-order modes. As seen in Fig. 5, the filter $F_1$ rejects the mid-range modes (modes 5–12) better than $F_2$, although $F_2$ still reduces the contribution from the higher-order modes.

Thus, although mode-coupling appears to reduce the signal-to-noise ratio, this filtering method still presents significant advantages.
V. CONCLUSIONS

Broadband signals received on vertical arrays can be processed adaptively by using eigenvalue-eigenvector decomposition of the cross-spectral density matrix to yield mode-selective filters. Although the waveform transmitted need not be controlled, or even known, to use the techniques presented here, the signal bandwidth and range to the source must be sufficiently large that the normal-mode components of the cross-spectral density matrix add incoherently. If the vertical array densely samples the entire water column, the filter coefficients obtained are the modal depth functions, which can be used as single-mode filters. In the case of shorter arrays, application of the eigenvectors is less straightforward. Simulations indicate that, under negative gradient sound speed profile conditions, refraction of low-order modes away from the ocean surface can be exploited via adaptive processing to strongly reject surface-generated noise.

REFERENCES