REAL TIME PREDICTION OF SHIP RESPONSE TO OCEAN WAVES USING TIME SERIES ANALYSIS

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ABSTRACT

This paper describes the results of a feasibility study of using time series analysis methods to predict the real time motion of ships up to 10 seconds in advance. Potential application of this work are aircraft landing on ships, and motion compensation devices for handling loads at sea.

It uses a statistical method that finds a time domain model which best fits an input wave sensor time history to the ship response time history. It is called a leading indicator method using an Auto-Regressive Moving Average (ARMA) model. It determines the relationship in situ, and can be redetermined as ship heading and sea conditions change.

Tests of the algorithm on recorded ocean test data show good predictions of phase and amplitude for 2 to 4 seconds in advance and phase for 8 to 10 seconds in 8 second waves.

SUMMARY

This is a paper on the applications and feasibility of real time ship motion prediction. The applications section describes potential applications of motion prediction. The background section describes some of the basic concepts and gives the history of the work in this area. The methods section describes the equations used and the relationship of a least squares analysis to Kalman Filtering. The ocean testing section describes the experiments used to verify the methodology. The results section contains examples of predictions from computer analysis of the ocean test data. The final section presents conclusions about the feasibility of using ARMA for motion prediction.

BACKGROUND

This section provides a summary of the mathematical concepts used followed by a historical summary of previous investigations.

Imagine an input time series, x(t), which can be wave measurements and an output time series, y(t), such as ship heave. x(t) and y(t) are related by a convolution or response operator b(t). y(t) is contaminated by the noise e(t). The diagram below shows b(t) as a black box.

In discrete form, and assuming equal time increments At.

\[ y(t) = \sum_{j=0}^{\infty} b(j)x(t-jAt) + e(t) \]

\[(1)\]

where for typographical reasons, write

\[ \sum_{j=0}^{\infty} = \sum \]

and * is a convolution operator. If the system has damping, the impulse response b(t) goes to zero as t goes to infinity. So the series can be truncated. For example, a truncated series of length p, and assuming t=1, is

\[ y(t) = b(0)x(t) + b(1)x(t-1) + \ldots + b(p)x(t-p) + e(t) \]

\[(2)\]

This is called the Moving Average (MA) model of order p.
Alternatively, the input and output can be modeled by an Auto-Regressive (AR) model which uses previous values of the the output, \( y(t) \), and a single value of the input, \( x(t) \). The AR model of order \( p \) is

\[
y(t) + a(1)y(t-1) + \ldots + a(p)y(t-p) = b(0)x(t) + e(t)
\]

(3)

If the input is unknown, the \( x(t) \) term is dropped, and the relationship for a single time series AR model is obtained.

The AR coefficients converge to zero much more quickly than the MA coefficients.

An Auto-Regressive Moving Average (ARMA) model is a mixture of AR and MA models. For order \( p \), it is

\[
y(t) + a(1)y(t-1) + \ldots + a(p)y(t-p) = b(0)x(t) + b(1)x(t-1) + \ldots + b(p)x(t-p)
\]

(4)

The coefficients of the MA, AR and ARMA models can be estimated by analyzing time histories in situ (in place) or by precalculation of the impulse response function by theoretical models or wave tank measurements. It is simple to transform MA coefficients to ARMA or vice versa.

There are several methods of prediction. A self predictor would use the past history of ship motion to predict itself. It would use the AR model. A wave predictor would use past history of the wave and ship motion to predict the ship motion with the use of the ARMA model.

Historically, use of time series analysis for real time ship motion prediction is not new. Enochson, in 1963 (Ref. 2), described the analytic methodology for estimating ARMA models from data. He felt the main problem of using time series analysis was a requirement for inverting a large matrix, which could only be done on the large computers of his time. He proposed predicting ship roll as part of an anti-roll system. He did not use ocean data.

Dalzell, in 1965 (Ref. 3), proposed prediction of carrier deck motions by installing a wave sensor at the bow and using MA coefficients that were determined by wave tank experiments. Kaplan, in 1968 (Ref. 4 & 5), attempted to verify this method aboard a carrier. The bow mounted radar wave sensor was damaged in high seas. After it was repaired, the ensuing wave conditions were very small. The results were not conclusive.

Sidar and Doolin, in 1975 (Ref. 6), used a Kalman Filter (see Method section on Kalman Filtering) to predict heave and pitch. They were able to show good prediction of up to 15 seconds for the phasing. Their work was based upon perfectly uni-directional seaways that were mathematically generated. They did not use ocean testing.

The author, in 1978 (Ref. 7), experimented with a six foot model in wind generated seas and was able to get good predictions up to 1 1/2 seconds in one second small seas. He was able to demonstrate that new techniques made time series analysis more practical. He used an iterative method of solving for the coefficients that was much quicker than a matrix inversion. He used a statistical criterion for finding the best order \( p \).

Current work is being sponsored by the Navy Vertical Takeoff and Landing Capability Development (NAVTOLAND) program, a landing guidance system for helicopters and STOL aircraft on ships. Bathys of DTNSRD is working with the AR method of self-prediction. The author is studying the application of the ARMA method.

METHOD

The method of prediction is called the Leading Indicator method from its use in economics. This method is described in Ref. 8. Basically, this method analyzes the time histories of wave and ship motion to find the best statistical fit using time domain coefficients. It then uses these coefficients to predict the response from

![Figure 1. Wave and Heave Motion](image-url)
the most recent measurements of the wave and ship motion. The advantage of this method is that the relationship is determined in situ, and it is empirically derived. When conditions change (i.e., different ship heading or sea conditions), the new relationship can be easily recalculated. The procedure is as follows:

1. Take 10 to 20 minutes of ship motion and wave height data.
2. Calculate auto-covariances and cross-covariances of the ship and wave data.
3. Iteratively calculate the time domain coefficients that provide the best fit between the wave input and ship motion output data.
4. Iteratively find the time domain model for the wave input.
5. Using the most recent wave data, predict the wave.
6. Using the most recent ship motion data and predicted wave, predict the ship motion.

The input time history, \(x(t)\), and ship motion history, \(y(t)\), are recorded at time intervals \(\Delta t\). Figure 1 is an example of the wave and heave motion time histories of the Semi-Submerged (SSP) KAIMALINO. The wave height was obtained by measuring water pressure and subtracting ship heave. The ship heave displacement was obtained by double integrating a heave accelerometer. Figure 2 is a sketch of the sensor locations on the SSP. Notice that the ship heave is predominately a eight to nine second motion. The wave record has higher frequencies. The ship tends to filter out the higher frequencies.

The procedure is based upon the following equations which are derived:

\[
\begin{align*}
\text{ACOUSTIC} & \\
\text{WAVE HEIGHT SENSOR} & \\
\text{SHIP ACCELEROMETER} & \\
\text{WAVE RIDER BUOY} & \\
\text{PRESSURE SENSOR} & \\
\end{align*}
\]

Figure 2. SSP KAIMALINO Transducer Locations

The covariance function is a measure of the correlation of neighboring values of a time series as a function of the amount of separation between values defined as lag, \(\tau\). The covariance between neighboring values of a single time series is defined as an auto-covariance function, \(r_{xx}\). The covariance between neighboring values of two time series is defined as cross-covariance function, \(r_{xy}\). Any mean value must be removed from the time series.

The auto and cross-covariances are estimated by:

\[
\begin{align*}
\hat{r}_{xx}(\tau) &= \frac{1}{N-\tau} \sum_{t=0}^{N-\tau} [x(t)-\bar{x}][x(t-\tau)-\bar{x}] \\
\hat{r}_{yy}(\tau) &= \frac{1}{N-\tau} \sum_{t=0}^{N-\tau} [y(t)-\bar{y}][y(t-\tau)-\bar{y}] \\
\hat{r}_{xy}(\tau) &= \frac{1}{N-\tau} \sum_{t=0}^{N-\tau} [x(t)-\bar{x}][y(t-\tau)-\bar{y}]
\end{align*}
\]

where: \(N\) = number of points

\(r_{xx}(\tau)\) = input auto-covariance

\(r_{yy}(\tau)\) = output auto-covariance

\(r_{xy}(\tau)\) = input-output cross-covariance

\(\bar{x}\) = average \(x\)

\(\bar{y}\) = average \(y\)

The ARMA coefficients are found by fitting the input time series into the output time series (regression analysis). The criterion of fit is a minimization of the sum of the squares of the errors (least square error).
Using the definition of an ARMA model (equation 4) and rewriting it as:

\[ P \sum_{j=0}^{p} a(j)y(t-j) = \sum_{j=0}^{p} b(j)x(t-j) + c(0)e(t) \]

where: \( c(0)e(t) \) = error or noise

Assuming \( a(0) = 1 \) and \( b(0) = 0 \), because a physically real impulse = 0 when \( t = 0 \), solving for \( c(0)e(t) \):

\[ P \sum_{j=0}^{p} a(j)y(t-j) = \sum_{j=0}^{p} b(j)x(t-j) + c(0)e(t) \]

The summation of errors squared over \( t = N \) points is

\[ E = \sum_{t=0}^{N} [c(0)e(t)]^2 = \sum_{t=0}^{N} [y(t)+a(1)y(t-1)+b(1)x(t-1)\]

where \( c(0)e(t) \) = error or error

Assume \( b(0) = 0 \), and solve for the error \( c(0)e(t) \).

\[ c(0)e(t) = y(t) + a(1)y(t-1) - b(1)x(t-1) \]

The summation of errors squared over \( t = N \) points is

\[ E = \sum_{t=0}^{N} [c(0)e(t)]^2 = \sum_{t=0}^{N} [y(t)+a(1)y(t-1)\]

Taking partials with respect to the coefficients and setting to 0 for a minimum.

\[ \frac{\partial E}{\partial a(1)} = 0 \Rightarrow \sum_{t=0}^{N} [2y(t-1)y(t)+a(1)y(t-1)\]

or

\[ \frac{\partial E}{\partial b(1)} = 0 \Rightarrow \sum_{t=0}^{N} [2y(t-1)x(t-1)] \]

Although the limits of summation are slightly smaller, each of the components is approximated by the covariances

\[ r_{yy}(-1) = \sum [y(t)y(t-1)] \]
\[ a(1)r_{yy}(-1)r_{yy}(0) = \sum [a(1)y(t-1)y(t-1)] \]
\[ b(1)r_{xy}(-1)r_{xy}(0) = \sum [b(1)x(t-1)y(t-1)] \]

to give

\[ r_{yy}(-1) = a(1)r_{yy}(0)-b(1)r_{xy}(0) \]

Likewise, the partial of \( E \) with respect to \( b(1) \) is

\[ \frac{\partial E}{\partial b(1)} = 0 \Rightarrow -r_{xy}(0) - a(1)r_{xy}b(1)r_{xx}(0) \]

use the symmetry relationships for covariances.

\[ r_{yy}(-1) = r_{yy}(1) \]
\[ r_{xy}(1) = r_{yx}(-1) \]

and rewrite in matrix form

\[ \begin{bmatrix} r_{yy}(1) \\ r_{xy}(1) \end{bmatrix} = \begin{bmatrix} C(0) \\ -r_{xy}(0) \end{bmatrix} \begin{bmatrix} b(1) \\ a(1) \end{bmatrix} \]

In a more general form, it can be shown that for any \( p \):

\[ \begin{bmatrix} r_{yy}(1) \\ r_{xy}(p) \end{bmatrix} = \begin{bmatrix} C(0) \\ -r_{xy}(p) \end{bmatrix} \begin{bmatrix} b(1) \\ a(p) \end{bmatrix} \]

where \( C(k) = \begin{bmatrix} r_{xx}(k) \\ -r_{xy}(k) \end{bmatrix} \begin{bmatrix} r_{yy}(k) \\ -r_{xy}(k) \end{bmatrix} \)

is a 2x2 sub-matrix using indicator \( k \).

Likewise, the matrix equation for an AR model is

\[ \begin{bmatrix} r_{yy}(1) \\ r_{xy}(p) \end{bmatrix} = \begin{bmatrix} r_{xy}(0) \\ -r_{xy}(p) \end{bmatrix} \begin{bmatrix} a(1) \\ a(p) \end{bmatrix} \]

The AR coefficients can be found by the Yule-Walker iterative algorithm. An iterative algorithm uses \( p \) coefficients and \( p+1 \) covariances to find the \( p+1 \) coefficients. It is much faster than inverting the matrix. A computer program and description of the method is found in Ref. 9. Akaike expanded the Yule-Walker for ARMA coefficients (see Ref. 10).

A big problem of this type of analysis is knowing how large a \( p \) to use. As the order \( p \) is increased, the fit between the input and output is better, but increasing order means increasing the statistical uncertainty.

Fortunately, several criteria have been developed for selection of the best order. One of the easiest to use is the Final Prediction Error (FPE) which was developed by Akaike in 1968. It is described in Ref. 9. It is the product of the residue, \( S_e/N \) (the amount of error of fit) multiplied by a measure of the
statistical uncertainty. For an ARMA model, the residue is:

\[
P_{s/n} = \sum_{k=0}^{P} b(k) r_{xy}(k-l) - \sum_{k=0}^{P} a(k) r_{yy}(k-l)
\]

The FPE is:

\[
FPE(p,n) = \frac{P_{s/n}}{N+p+1}
\]

The minimum value of FPE give the best order.

An iterative determination of coefficients is ideal for using this criterion. The computation continues for higher orders until a minimum FPE is found.

Prediction involves using a chain process on the model. It uses the most recent time histories of the input and output to predict the output.

To illustrate the application to the ARMA model, rearrange equation (4) into

\[
y(t) = a(1)y(t-1) + ... + a(p)y(t-p) + b(1)x(t-1) + ... + b(p)x(t-p)
\]

Increment t by one time step to form a prediction of \(y(t+1)\), where \(^\prime\) means a predicted value.

\[
y(t+1)' = a(1)y(t) + ... + a(p)y(t-p+1) + b(1)x(t) + ... + b(p)x(t-p+1)
\]

Similarly, \(y(t+2)\)' is generated by adding another time step and using the prediction \(y(t+1)\)' and \(x(t+1)\)'.

Obviously, this can be continued indefinitely. Predicted values of the input are found by a similar process using the AR model. The prediction equation for \(x(t+1)\) is

\[
x(t+1)' = a_x(1)x(t) + ... + a_x(p_x)x(t-p_x+1)
\]

where \(a_x(\cdot)\) are AR coefficients for input

\(p_x\) = best order of AR input model

This least squares regression analysis can be shown to be a subset of Kalman Filtering. Kalman Filtering is mainly used for navigation, but can be used for prediction. A Kalman Filter requires a State vector, which are equations relating the variables such as the equations of motion, and a relationship for the noise. If the State vector is unknown and the noise relationship is unknown, the Kalman Filter reduces to the normal equations that are used in this analysis (see Ref. 11)

OCEAN TESTING

The main testing was conducted with the Semi-Submerged Platform (SSP) KAIMALINO. The SSP has its main buoyancy in two pontoons below the surface. It has a small waterplane area and thereby is nonresponsive to short period waves. It has the motion characteristics of a very large ship, although it is small (length=87.8 ft (26.8m), displacement=210 tons).

Testing was conducted off the island of Kauai in the Hawaiian Islands in Jan. 1979. A Humphrey's Vertical Stabilized Accelerometer was used to measure ship motions. Wave motion was measured by three different sensors. Water pressure sensors in both hulls at a depth of 13 feet (4m) were combined with ship heave displacement signal to give wave height. An acoustic range sensor measured the vertical distance from the bow to the wave surface. A Waverider buoy measured the waves directly. The seastate varied from 4 to 5 with a maximum significant wave height of 10.2 ft (3.1m). Test speeds were from dead in the water to 15.5 knots. The relative wave directions were head, beam, quartering and following seas.

RESULTS

Figures 3 and 4 are examples of the ability of the algorithm to predict vertical displacements of the SSP with recorded data. The author purposely chose examples which are difficult to predict. It is easy to predict time series which have constant amplitude and period. Sudden changes in amplitude are more difficult.

Figure 3 is a comparison of the predictions between the AR method (no wave data) to the ARMA method (wave input). Both are of the same time period and start the prediction at the same instant. The confidence band is based upon the probability of the prediction. The 50% band would encompass the actual points 50% of the time. The confidence bands are proportional to the standard deviation of the prediction. The standard deviations are calculated from the residue and coefficients (see Ref. 8). The table contains the standard deviation of the predictions for increasing one second time steps into the future. Up to 5 seconds, the ARMA method’s standard deviations are smaller than the AR method, which means that there is higher confidence in the ARMA method up to a five second prediction.

Figure 4 illustrates the performance of the algorithm on SSP dead in water data. The example is a stretch of data that has a sudden quiescent period during large motions. The prediction is presented in a format that allows a better understanding of the performance of the predictor. The predictor is a running constant prediction interval. For example, the top plot uses all the data up to a point: predicts 1 second in advance; then uses all the data up to the next point and predicts 1 second again. Similarly, plots are made for constant two, four, and eight second intervals. A root mean square (RMS) error ratio is used as a measure of the accuracy.
Figure 3. Comparison of Predictions Given by AR and ARMA Methods

<table>
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<th>STEP</th>
<th>PREDICTION STANDARD DEVIATION</th>
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</thead>
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<tr>
<td></td>
<td>AR</td>
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<tr>
<td>1</td>
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<tr>
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</tr>
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<td>.883</td>
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<td>7</td>
<td>.905</td>
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</table>
Figure 4. Comparison Between Predicted and Actual Heave Motion

**SSP - DEAD IN WATER PRESSURE INPUT**

--- ACTUAL
--- PREDICTED

PREDICTION = 1.00 sec  
RMS ERROR = 7.16%

PREDICTION = 2.00  
RMS ERROR = 16.29%

PREDICTION = 4.00 sec  
RMS ERROR = 24.52%

PREDICTION = 8.00 sec  
RMS ERROR = 33.56%
of the predictor. It is the ratio of the square root of the sum of the error between the predicted and actual values squared to the square root of the actual values squared.

\[
\text{RMS Error} = \sqrt{\frac{\sum_{t=0}^{N} [y(t) - \hat{y}(t)]^2}{\sum_{t=0}^{N} [y(t) - \bar{y}]^2}}
\]

A 10% ratio means an average one ft. error in an RMS ten ft. motion. The plots show good predictions of amplitude and phase up to 4 seconds in advance and good predictions of phase up to 8 seconds in advance.

The predictions are a half of a cycle for amplitude and one cycle for phase and are not as good as the predictions of Sidar (Ref. 6) (two cycles for phase) and Yumori (Ref. 7) (1 1/2 cycles for amplitude). There are several reasons for this. The testing used seas that were not perfectly uni-directional, as Sidar used. The seas had more of a spread of frequencies than the Yumori studies. The wave sensor was not as far into the wave as the Yumori studies (1/10 of a wavelength vs. 1/2 of a wavelength).

Use of the pressure sensor was compared to the acoustic sensor for a wave input. The results were very similar. An attempt was made to position the SSP at a constant distance from the Waverider to give a wave signal 1/2 of a wavelength ahead. The prediction was not good because a constant interval could not be maintained.

CONCLUSIONS

1. This feasibility study has shown that it is possible to predict the motion of a ship in ocean waves up to 2 to 4 seconds in advance. The most likely application of predictions of this range will be for motion compensation equipment and automatic control landing of aircraft on ships.
2. The leading indicator method (ARMA) is better that the self-predictor (AR) for up to 5 seconds.
3. There was no difference in the quality of the prediction between pressure and acoustic inputs. An acoustic or radar remote sensor may give better predictions if it could sense waves at a distance from the ship.

REFERENCES