Numerical Modeling of Solitary and Cnoidal Waves Propagating over a Submerged Bridge Deck

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Abstract—An initial component in the study of tsunami and storm surge wave forces on a submerged bridge deck is the development of a two-dimensional numerical model of solitary and cnoidal waves propagating over a submerged plate based on the Navier-Stokes (N-S) equations. Results are compared with numerical models based on a non-linear dispersive shallow water wave theory known as the Green-Naghdi (G-N) theory of water waves. Solitary and cnoidal waves are modeled using the open source computational fluid dynamics library OpenFOAM which solves the N-S equations by use of the finite volume method. Solitary waves are initiated by implementing a user defined velocity boundary condition based on the G-N theory and cnoidal waves are initiated using Waves2Foam, a wave generation toolbox developed for OpenFOAM.

Results for the surface elevation on and around the bridge deck obtained by solving the N-S equations are compared with the numerical results obtained through the G-N theory. Even though these are two different solvers, solving two different sets of equations, preliminary analysis shows generally good agreement between the N-S and G-N solvers, with the propagation speed, wave amplitude and soliton fission being nearly identical. By solving these problems by two different methods, we are able to increase our confidence in the accuracy of our predictions of wave loads on coastal bridges.

I. INTRODUCTION

Coastal communities such as those located in the Hawaiian Islands are particularly vulnerable to both storm surge and tsunami events. Storm surge is the temporary rise in sea level during a major storm or hurricane. This increases the likelihood of a larger number of waves and waves with greater amplitude impinging on coastal structures which are normally located above the wave crest. During a tsunami event, large volumes of water may propagate onshore at high speeds. Coastal bridges, typically not designed for such severe loads, may partially or completely collapse as witnessed during Hurricanes Ivan, (2004) and Katrina, (2005), and the Indian Ocean, (2004) and Tohoku, (2011) tsunamis.

Modeling solitary and cnoidal waves propagating over a submerged plate provides insight into the interaction tsunami and storm surge waves may have with a submerged bridge deck during such an event. These two cases are solved using two solvers, one based on the N-S equations and one based on the G-N equations and the results for surface elevation are compared. Both the N-S and G-N equations have been successful in modeling non-linear, dispersive effects, wave-wave interaction and shallow water effects (Goring (1978) [4], Ertekin and Wehausen (1986) [3], and Seabra-Santos et al. (1987) [6]).

II. NUMERICAL MODELING

A. Numerical Solvers

Solitary and cnoidal waves are modeled by use of the open source computational fluid dynamics library OpenFOAM which solves the Navier-Stokes equations by use of the finite volume method. The solver used, interFoam, solves for two incompressible, isothermal immiscible fluids by use of a volume of fluid phase-fraction based interface capturing approach, making it ideal for modeling solitary and cnoidal waves. The N-S equations

\[ \frac{\partial (\rho U)}{\partial t} + \nabla \cdot (\rho U U) = F_b - \nabla P + \Delta (\mu U) \]  

are coupled with the continuity equation for incompressible flow

\[ \nabla \cdot U = 0 \]

where \( \nabla \) is the gradient vector, \( U = (u,v,w) \) is the velocity field, \( \rho \) is the density, \( P \) is pressure, \( \mu \) is dynamic viscosity and \( F_b \) represents the body forces. The N-S equations are discretized by use of the Euler method for time discretization and central difference method for convection and diffusion, providing a second-order numerical approximation. In this work, only laminar flow is considered, and turbulence is left for future studies.

The two dimensional G-N equations for conservation of mass and momentum are given by

\[ \eta_t + (h + \eta - \alpha) u_x = \alpha_t, \]  
\[ \frac{u + gn_x + \frac{\hat{p}_x}{\rho}}{\eta} = -\frac{1}{6} [2\eta + \alpha] \alpha_x^2 + (4\eta - \alpha) \eta_x + (h + \eta - \alpha) (\delta + 2\eta)_x, \]

where \( \eta(x,t) \) is the free surface measured from the still-water level \( h \), \( \hat{p}(x,t) \) is free surface pressure, \( u \) and \( w \) are horizontal and vertical velocity components, \( \alpha(x,t) \) is the vertical location of the bottom of the fluid sheet, \( \rho \) is the density of the fluid and the single and double superposed dots represent the first and second material time derivative, respectively [5]. These equations are valid for incompressible and inviscid fluid. The three dimensional version of the equations can be found...
in Ertekin (1984) [1] and Ertekin et al. (1986) [3]. The G-N equations are solved numerically by use of the finite difference method and the modified Euler method for time discretization, providing a second-order numerical approximation.

B. Wavemaker

The solitary wave is initiated by implementing a user defined velocity boundary condition based on the following solution to equations (3) and (4) for a solitary wave:

\[
\eta(x') = A_s\text{sech}^2 \left( \frac{3A_s}{4h^2(h + A_s)} x' \right),
\]

and

\[
u = C \frac{\eta}{h + \eta},
\]

where \(A_s = \frac{c^2}{2} - h\), \(x' = x - Ct\), and \(C\) is wave celerity where \(C > \sqrt{gh}\). The derivation for these equations can be found in Ertekin (1984) [1].

Cnoidal waves are generated by the wave generation toolbox Waves2Foam developed for OpenFOAM. Waves2Foam includes a library of both linear and non-linear wave generation models coupled with a relaxation zone in the domain to reduce reflected waves from the outlet and wavemaker boundaries. The cnoidal wave theory used in Waves2Foam is based on the first approximation of the Korteweg-deVries (KdV) equations [8]. For the G-N equations, the solution for cnoidal waves can be obtained analytically (see, Sun (1991) [7]) and is very similar to the solution obtained by Keulegan and Patterson.

C. Model

In order to assess the accuracy of the generated wave, OpenFOAM calculations for a solitary wave propagating over a flat bottom are compared with analytical results obtained through the G-N theory. The waves are then allowed to propagate over a submerged plate, or bridge deck. Results for surface elevation and forces on the bridge deck are then compared with numerical results obtained by the G-N theory.

The numerical wave tank is 300m long and 4.5m high and filled with 3m of sea water. A plate of 15m length is located 150m to the right of the left-hand tank wall and is submerged 1.2m below the water surface.

III. RESULTS

The solitary wave produced by the wavemaker in OpenFOAM is compared with analytical results obtained by the G-N equations (5) and (6). Figure 2 shows excellent agreement between the OpenFOAM numerical results and the G-N analytical results, with OpenFOAM producing a slightly smaller wave. This is likely due to viscous terms included in the N-S equations and not present in the G-N equations.

Allowing the solitary wave to propagate over a submerged plate, similar to an abrupt change in water depth, causes some of the waves to be reflected, and the remaining wave deforms and splits into several waves traveling at different velocities in Fig. 3. This is known as soliton fission see e.g. Ertekin and Wehausen. (1986) [3]. Figure 3 compares numerical results for this case obtained by OpenFOAM and the G-N theory at three locations: two plate lengths prior to the plate leading edge, the plate center, and two plate lengths after the plate trailing edge. Generally good agreement is observed between the two solvers, with propagation speed, wave amplitude and soliton fission being nearly identical. Differences can be seen in the reflected waves at wave gauge I and a larger wave amplitude produced by OpenFOAM at wave gauge II. This again may be the result of viscous effects taken into account by solving the N-S equations.

The cnoidal wave produced by OpenFOAM by solving the KdV equations is compared with the cnoidal wave produced by solving the G-N equations, seen in Fig. 4. Again, there is excellent agreement between the two solvers with wave amplitude and wave steepness nearly identical. Figure 5 shows the results produced by OpenFOAM for a cnoidal wave propagating over a submerged plate, or bridge deck. Effects due to reflection can be seen prior to the leading edge of the plate in the top figure. The second figure shows the wave amplitude at the center of the plate, where effects of dispersion are indicated. Lastly, the waves become highly irregular downstream from the trailing edge of the plate.
IV. CONCLUSIONS

A two dimensional numerical model of solitary and cnoidal waves propagating over a submerged plate was developed by OpenFOAM. The solutions of the Navier-Stokes equations are compared with the inviscid solutions of the Green-Naghdi equations. Results for surface elevation are compared with generally good agreement between the two solvers, with wave amplitude, reflection and soliton fission being nearly identical in the solitary wave case. Preliminary results for cnoidal waves propagating over a submerged plate indicates reflection prior to the leading edge of the plate, deformation of the wave over the plate and highly irregular waves at the trailing edge of the plate.

Further study is required to determine the forces on the plate under solitary and cnoidal waves, and results will again be compared with those obtained by the G-N theory. In addition, turbulence will be added to the model as well as variations in submergence depth and the addition of girders to the plate. Finally the results will be compared with experimental data when available.

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